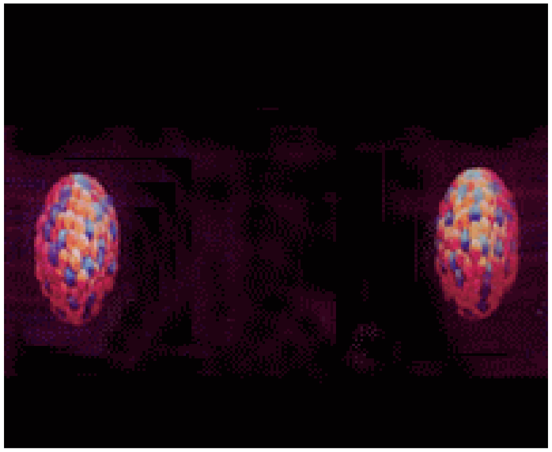
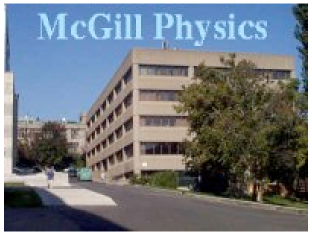
International Conference on Topics in Heavy Ion Collisions

Montreal, 25-28 June 2003

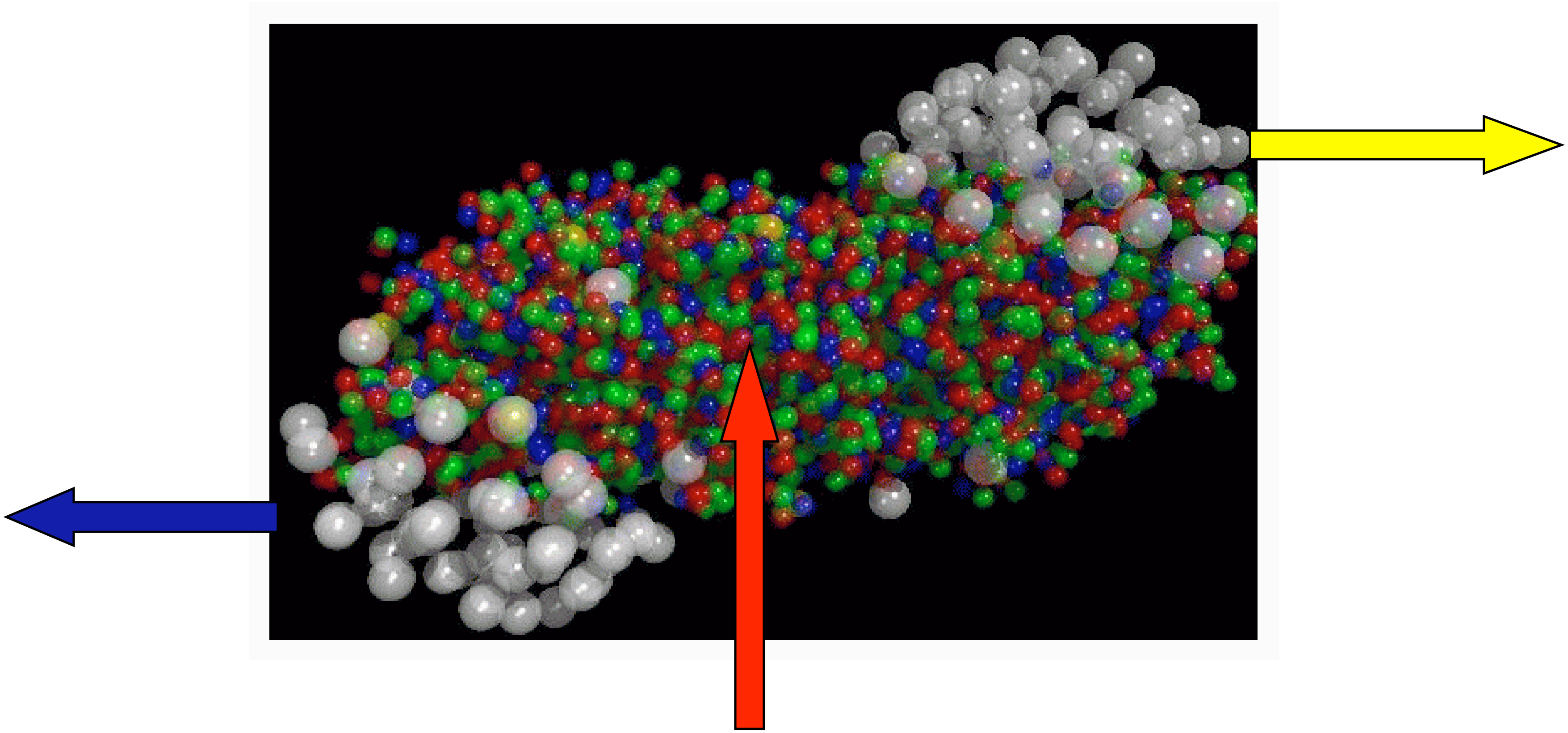
Chiral Condensate Dynamics

Jørgen Randrup, LBNL, Berkeley





[UrQMD: 60 GeV/N Au + 60 GeV/N Au]



Central rapidity region:

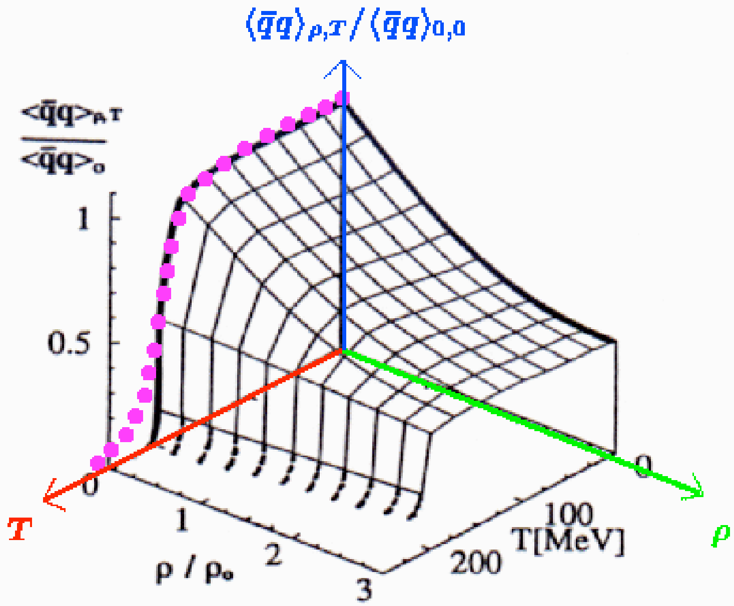
* Highly ***excited*** strongly interacting matter
* Rapid ***expansion*** (primarily longitudinally)

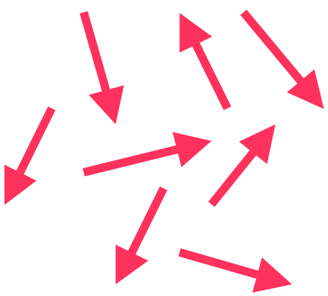
QCD ⇒ Chiral Condensate

$$q = \begin{pmatrix} u \\ d \end{pmatrix} : \quad \mathcal{L}_{QCD} = \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

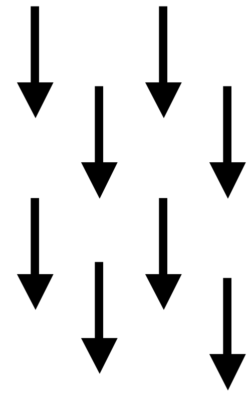
The *QCD* vacuum is a condensate of quark-antiquark pairs:

$$\langle \bar{q}q \rangle \equiv \langle u^\dagger \gamma^0 u + d^\dagger \gamma^0 d \rangle \neq 0$$





Hot



Cold

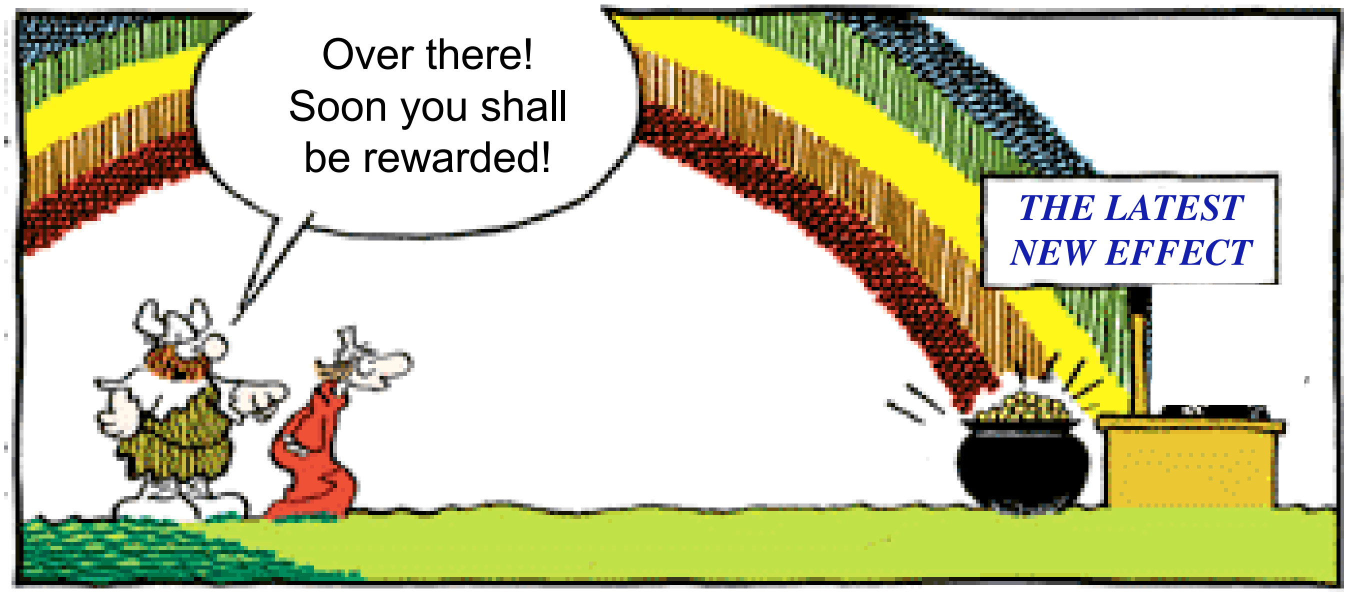
[T. Waas, R. Brockmann and W. Weise: Phys. Lett. B405 (1997) 215]

Chiral Condensate Dynamics

- Supercritical fields? $\mu_\pi^2 \ < \ 0$

- Disoriented chiral condensates? $\langle \bar{q}\tau\gamma_5 q \rangle \ \neq \ 0$

- Chiral antenna? $\mu_\pi(t)$


Over there! Soon you shall be rewarded!
THE LATEST NEW EFFECT

``Emergence of coherent long-wavelength oscillations after a quench''

[K. Rajagopal & F. Wilczek: Nucl. Phys. B404 (1993) 577]

$$\phi(\mathbf{r},t) = (\sigma, \vec{\pi}): \qquad L = \int d^3\mathbf{r}\,dt \left[\tfrac{1}{2}\partial^i \phi^\alpha \partial_i \phi_\alpha - \tfrac{1}{4}\lambda \left(\phi^\alpha \phi_\alpha - v^2\right)^2 + H\sigma \right]$$

Eq of motion for pion field

$$\frac{d^2}{dt^2}\pi_k(t) = -\omega_k^2(t)\,\pi_k(t) = -\left[k^2 + \mu_\pi^2(t)\right]\pi_k(t)$$

Effective pion mass

$$\mu_\pi^2(t) = \lambda\left[\langle\phi^2\rangle(t) - v^2\right]$$

When $\langle\phi^2\rangle < v^2$ then $\mu_\pi^2 < 0$ so $\pi_k \sim e^{\pm\int_0^t dt'|\omega_k(t')|}$

Spontaneous pion-pair production

Quench => ***Super-critical*** => ***Explosive growth!***

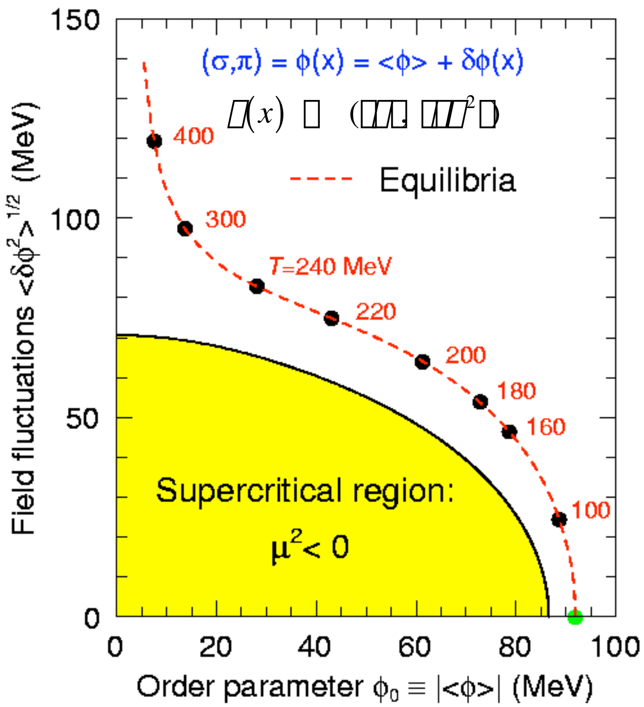
How to analyze a dynamical chiral field?

EoM: $\left[\partial_t^2 - \nabla^2 + \lambda(\phi\circ\phi - v^2)\right]\phi_\alpha = He_\alpha$ => $\phi(\mathbf{r},t) = \left(\sigma(\mathbf{r},t), \vec{\pi}(\mathbf{r},t)\right)$

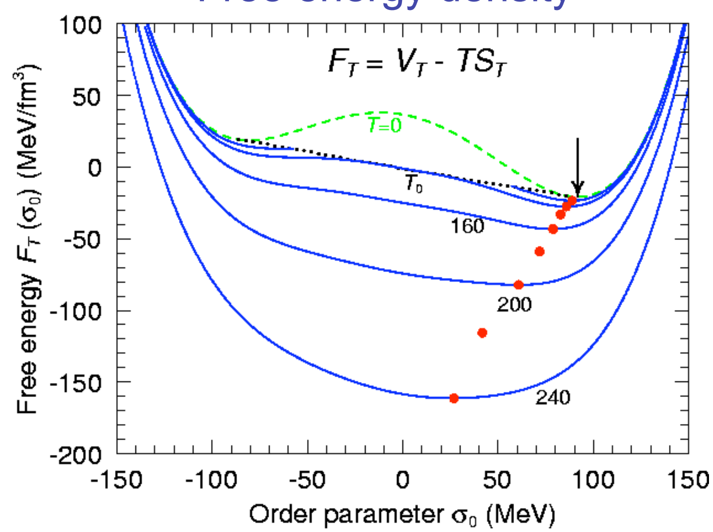
Project the dynamical state onto a chiral phase diagram

$$\begin{cases} \langle\phi_\alpha\rangle = \Omega^{-1}\int d^3\mathbf{r}\,\phi_\alpha(\mathbf{r},t) \\ \langle\delta\phi_\alpha^2\rangle = \Omega^{-1}\int d^3\mathbf{r}\left(\phi_\alpha(\mathbf{r},t) - \langle\phi_\alpha\rangle\right)^2 \end{cases}$$

Chiral Road Map



Free energy density



$$\mu_\alpha^2 = \lambda\left[\langle\phi\rangle\circ\langle\phi\rangle + \langle\delta\phi\rangle\circ\langle\delta\phi\rangle + 2\langle\delta\phi_\alpha^2\rangle - v^2\right]$$

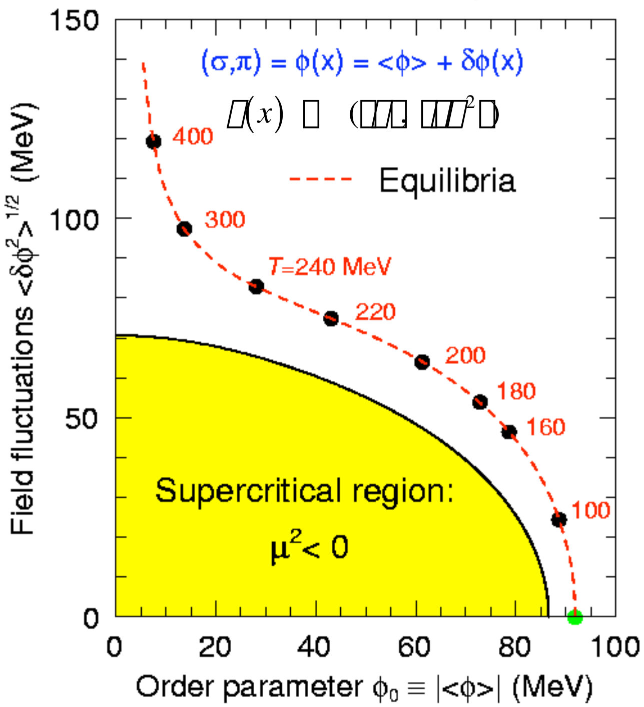
Effective mass μ

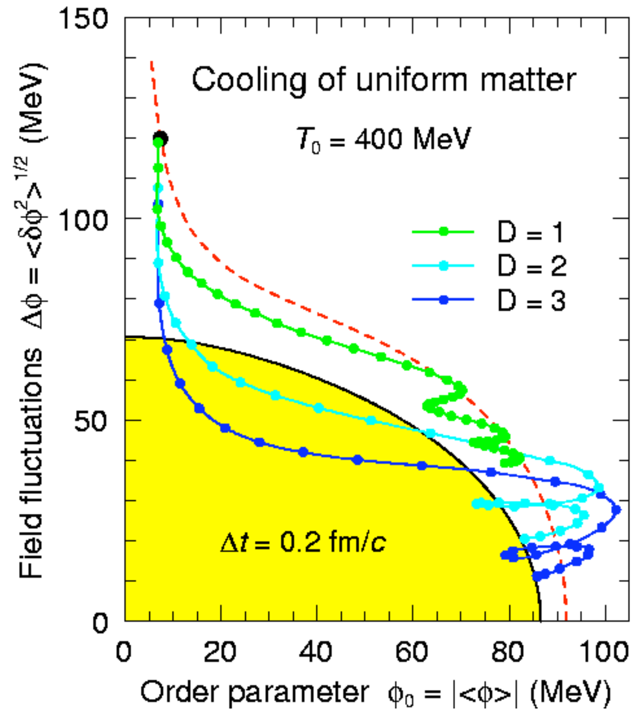
Will a quench occur?

Emulate a D-dimensional Bjorken scaling expansion of a hot uniform system

$$\left[\partial_t^2 - \nabla^2 + \lambda\left(\phi\circ\phi - v^2\right)\right]\phi_a - He_a = -\frac{D}{t}\partial_t\phi_a$$

Chiral Road Map





Supercriticality is NOT likely to occur!

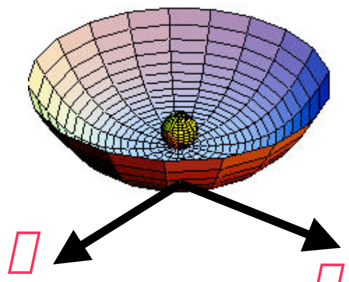
Chiral Condensate Dynamics

Supercritical fields?

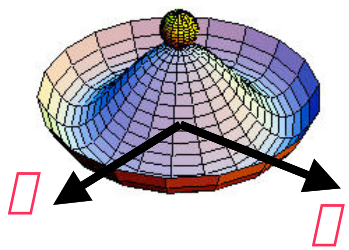
The early expansion is predominantly longitudinal => $\mu_\pi > 0$

Disoriented chiral condensates?

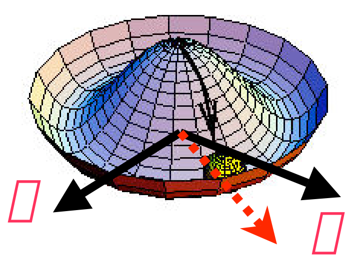
Disoriented Chiral Condensates?



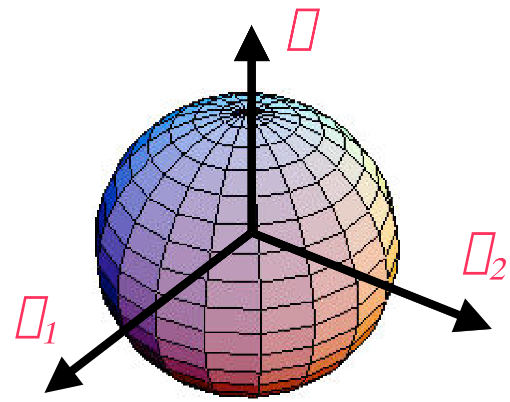
The environment is hot:
O(4) symmetry is favored

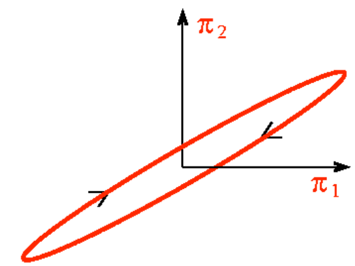


The environment cools:
O(4) symmetry disfavored



The order parameter grows
in a disoriented direction



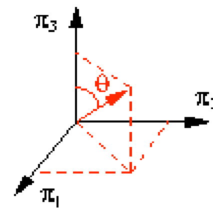


Isospin-directed
field oscillations

Neutral pion fraction: very idealized

Pion field: $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$

$n_0 \sim |\pi_3|^2 = |\vec{\pi}|^2 \cos^2\theta$



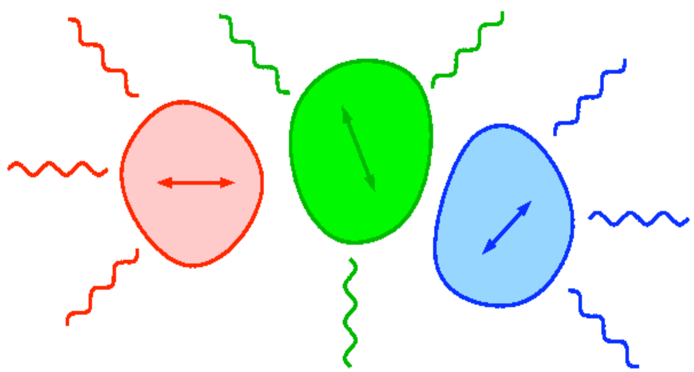
$\pi_\pm \sim (\pi_1 \pm i\pi_2)$

$\pi_0 = \pi_3$

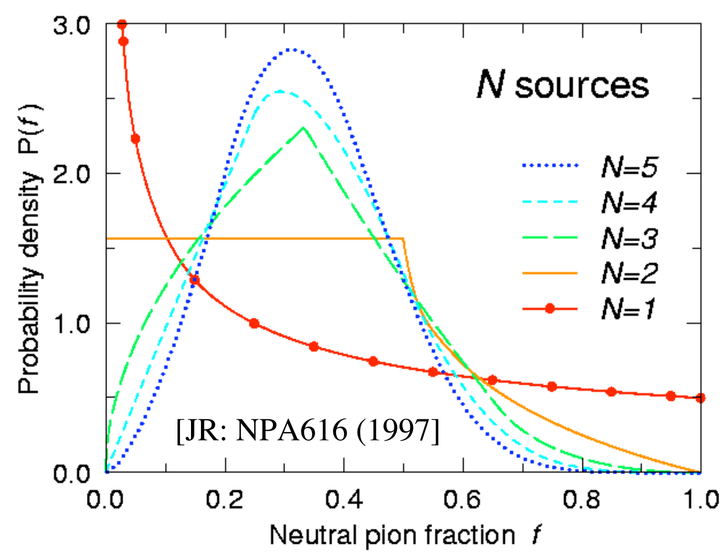
$$\text{Neutral pion fraction : } f = \frac{n_0}{n_- + n_0 + n_+} = \cos^2\theta$$

[AA Amsden & MG Ryskin: Phys Lett B266 (1991) 482]

$$P(f)df = d\cos\theta \quad \Rightarrow \quad P(f) = \frac{d\cos\theta}{df} = \left(\frac{df}{d\cos\theta}\right)^{-1} = (2\cos\theta)^{-1} = \frac{1}{2\sqrt{f}}$$

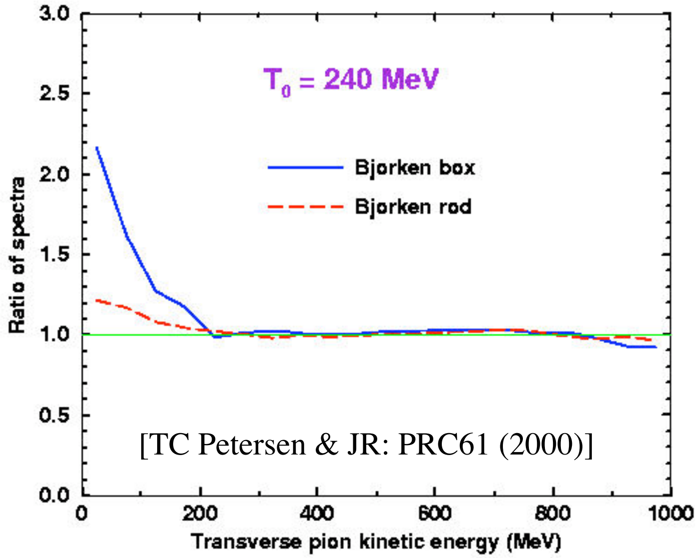


Domains of Disoriented Chiral Condensates

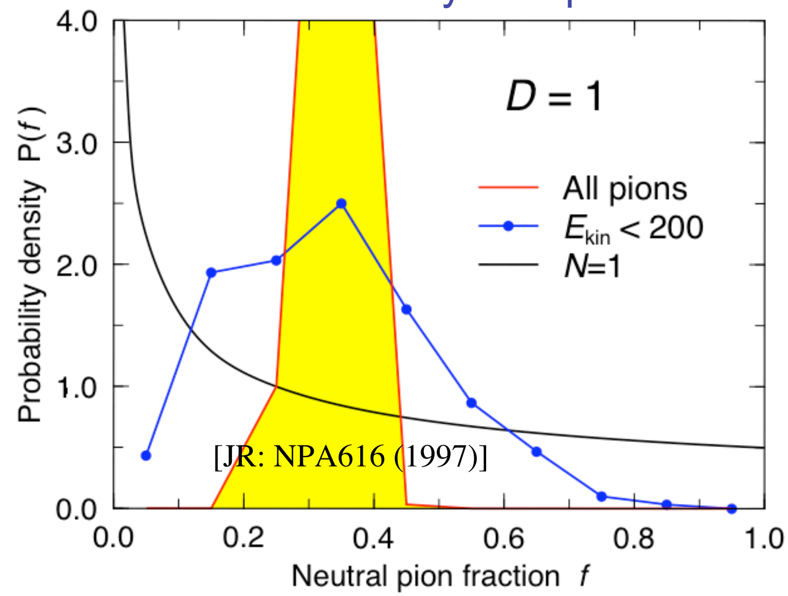


Neutral pion fraction: less idealized

Transverse pion spectrum:



From **all** to only **soft** pions:



=> The chiral dynamics affects only the soft pion modes!

$\pi^0 \to 2\gamma$ cannot be cut experimentally!

Experimental searches have found no anomaly in $P(f_\pi)$:

- ☹ MiniMax Collababoration: PRD55 (1997)
- ☹ WA98 Collaboration: PLB420 (1998)

Chiral Condensate Dynamics

Supercritical fields?

The early expansion is predominantly longitudinal => $\mu_\pi > 0$

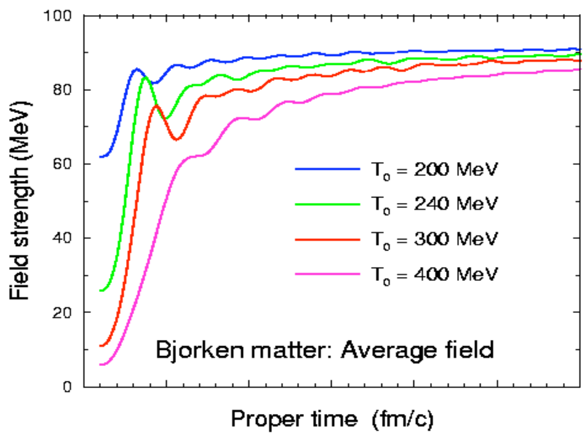
Disoriented chiral condensates?

Measuring the *soft* neutral pion fraction is impractical: $\pi^0 \rightarrow 2\gamma$

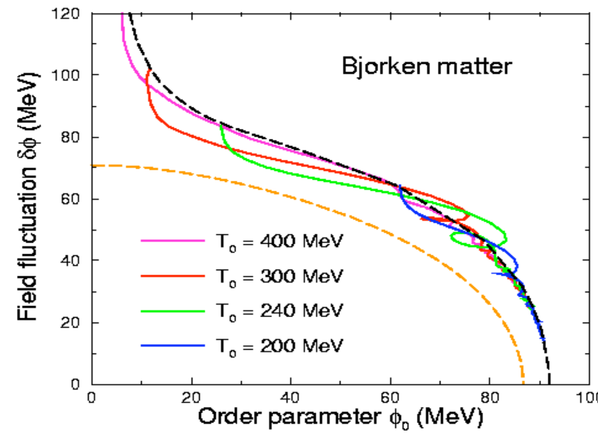
Chiral antenna?

Time-dependent chiral environment

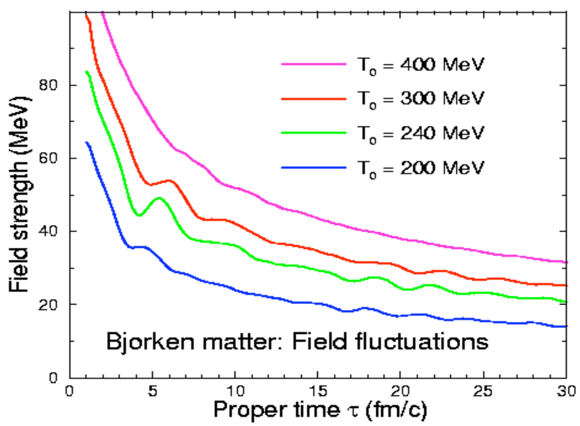
Order parameter <□>



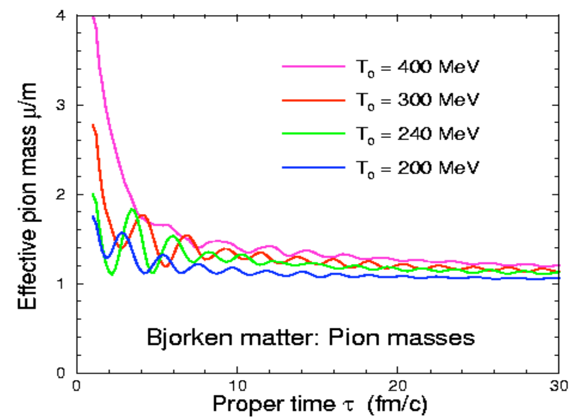
Phase evolution (□,□□)



Fluctuation □□



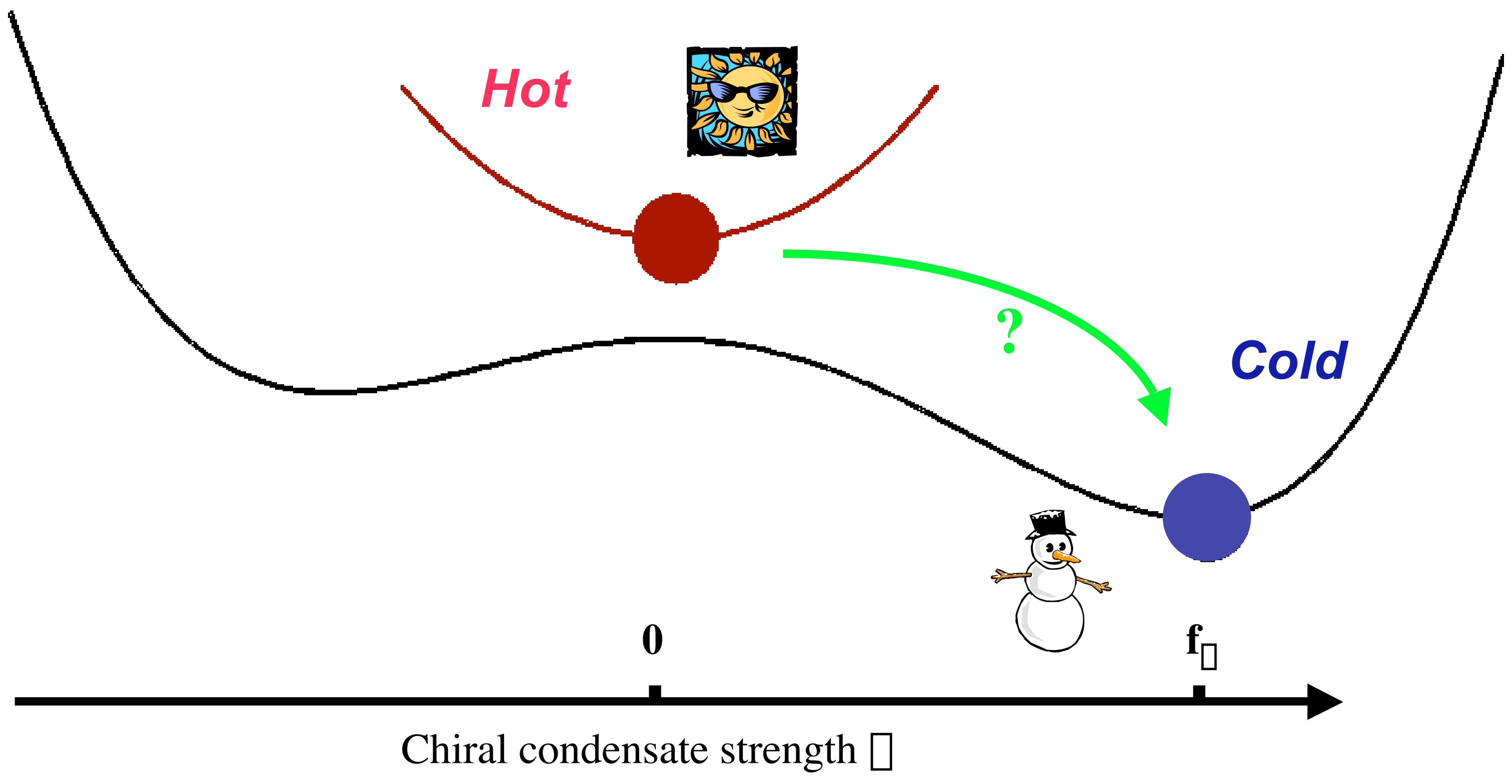
Effective mass □(*t*)

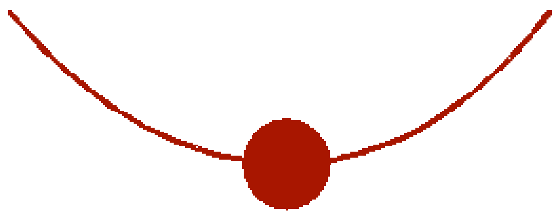


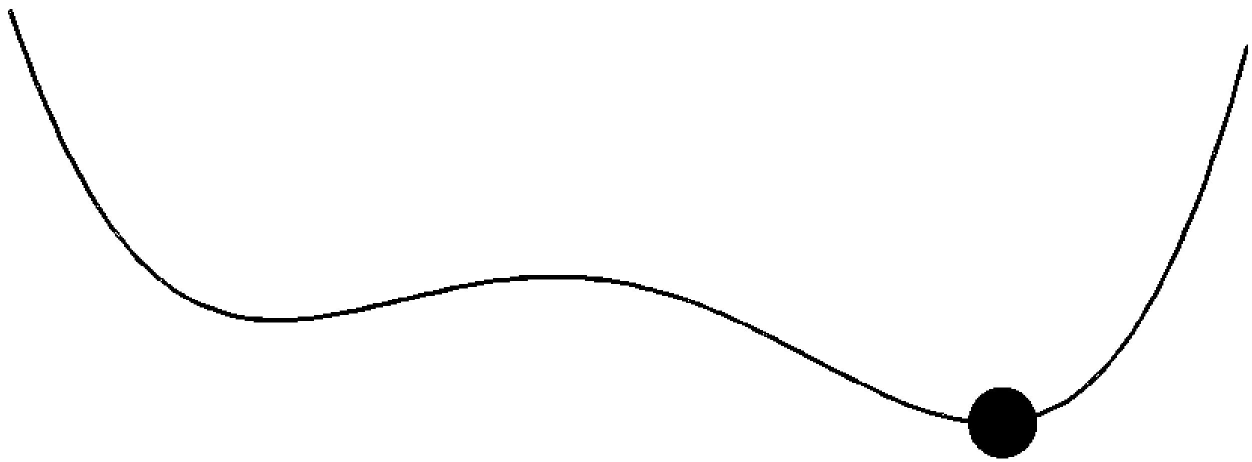
$$\square_i^2(t) = \square\left[\square_0^2 + \square\square\square^2\square + 2\square\square\square_i^2\square\square v^2\right]$$

Chiral condensate dynamics: Non-equilibrium relaxation

The effective potential evolves from hot to cold form,
bringing the condensate out of equilibrium:

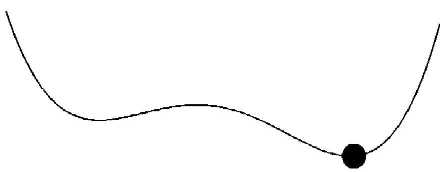




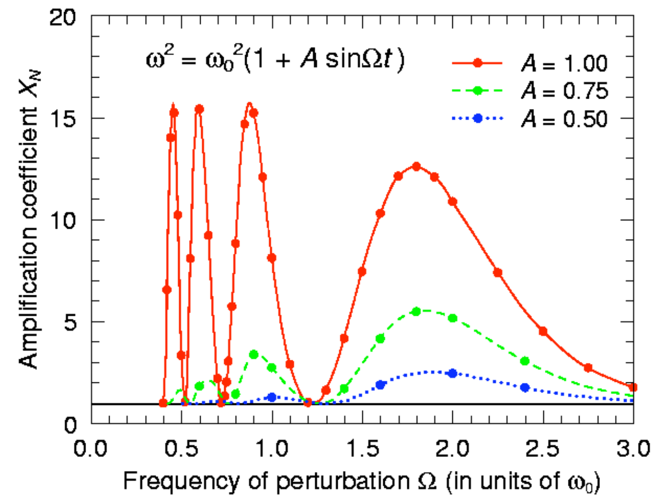


Frequency modulation => pair creation

$$m^2(t) \propto \lambda\left[\phi(t)^2 - v^2\right] : \quad \omega^2(t) = k^2 + m^2(t)$$



$$\left[\partial_t^2 + \omega^2(t)\right]\phi(t) = 0$$



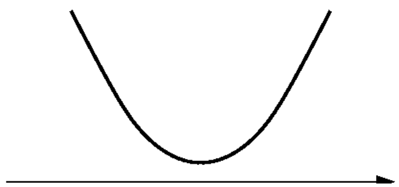
Mean particle number:
vacuum fluctuations help

$$\langle n\rangle \equiv \langle a^{+}a\rangle : \quad \langle n(t)\rangle = X(t)\left[n(0)+\tfrac{1}{2}\right] - \tfrac{1}{2}$$

Number fluctuations
are *enhanced*

$$\sigma_n^2 \equiv \langle n^2\rangle - \langle n\rangle^2 : \begin{cases} n(0)=0: \ \sigma_n^2(t) = 2\langle n(t)\rangle\left(\langle \bar{n}(t)\rangle + 1\right) \\ n(0) << 1: \ \sigma_n^2(t) \approx X(t)^2\left[\sigma_n^2(0)+\tfrac{1}{2}\right] - \tfrac{1}{2} \end{cases}$$

The number of quanta changes by ***two*** at a time!



$$H(t) = \tfrac{1}{2}p^2 + \tfrac{1}{2}\omega^2(t)q^2$$

$$\begin{cases} q = \frac{1}{\sqrt{2\omega_0}}\left[a + a^+\right] \\ p = -i\sqrt{\frac{\omega_0}{2}}\left[a - a^+\right] \end{cases}$$

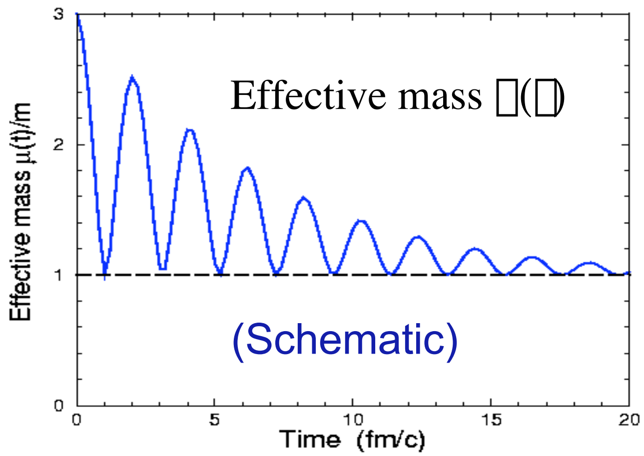
$$H(t) = \tfrac{1}{2}\Omega_+(t)\left[aa^+ + a^+a\right] + \tfrac{1}{2}\Omega_-(t)\left[a^2 + \left(a^+\right)^2\right] \qquad \Omega_\pm(t) = \left[\omega^2(t) \pm \omega_0^2\right]/2\omega_0^2$$

OBS: The particles are created pairwise

In a wave mechanical picture, this is a direct consequence of the reflection symmetry, which conserves the wave function's parity:

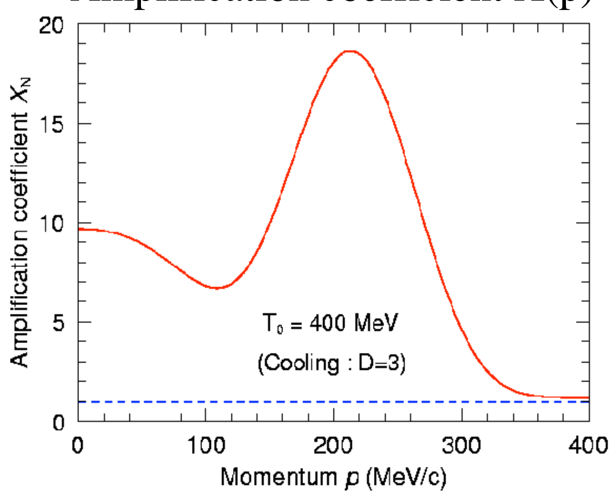
An even wave function will remain even and an odd wave function will remain odd

General character of the mass evolution

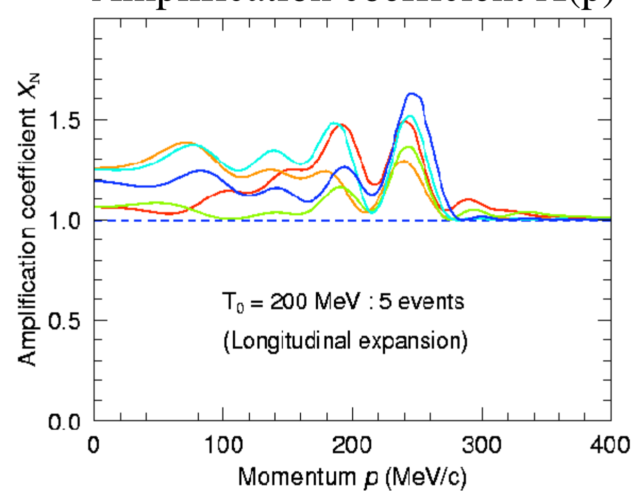


- Expansion: overall *decrease* of □
- Non-equilibrium: *oscillations* of □

Amplification coefficient X(p)



Amplification coefficient X(p)



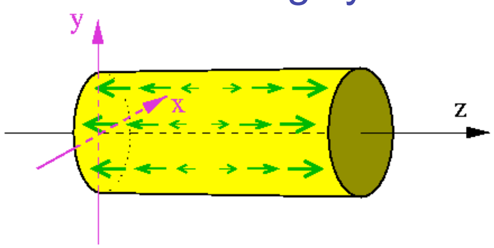
Fluctuations in the number of soft pions

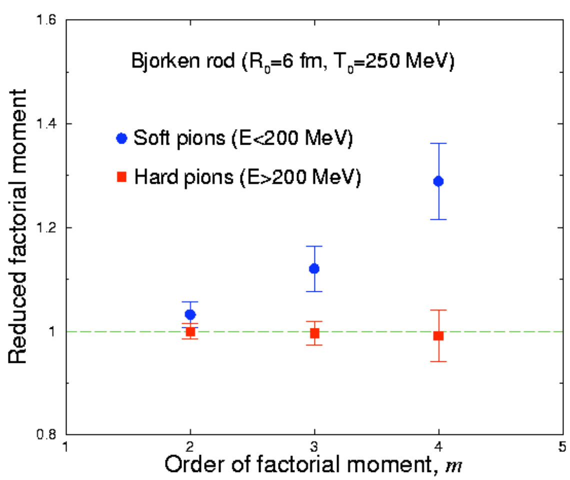
Factorial moments of the multiplicity distribution:

$$M_m \equiv \langle N(N-1)\cdots(N-m+1)\rangle$$

-- are enhanced by the time modulation

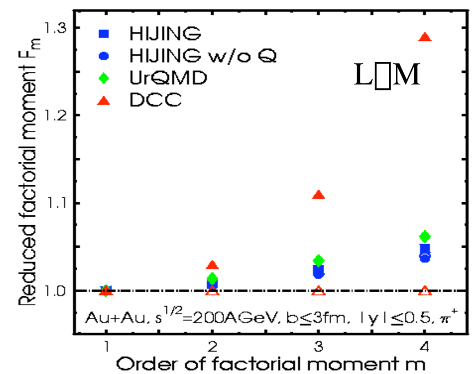
Linear σ model simulations for a stretching cylinder:

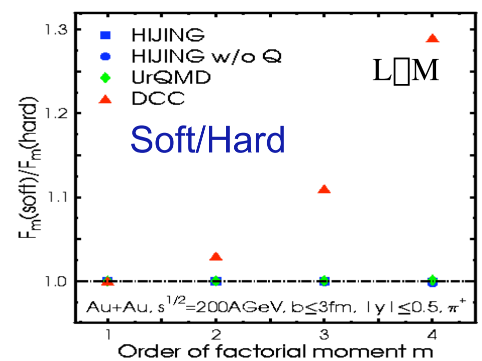




[TC Petersen & JR: PRC61 (2000)]

Comparison with HIJING & UrQMD:

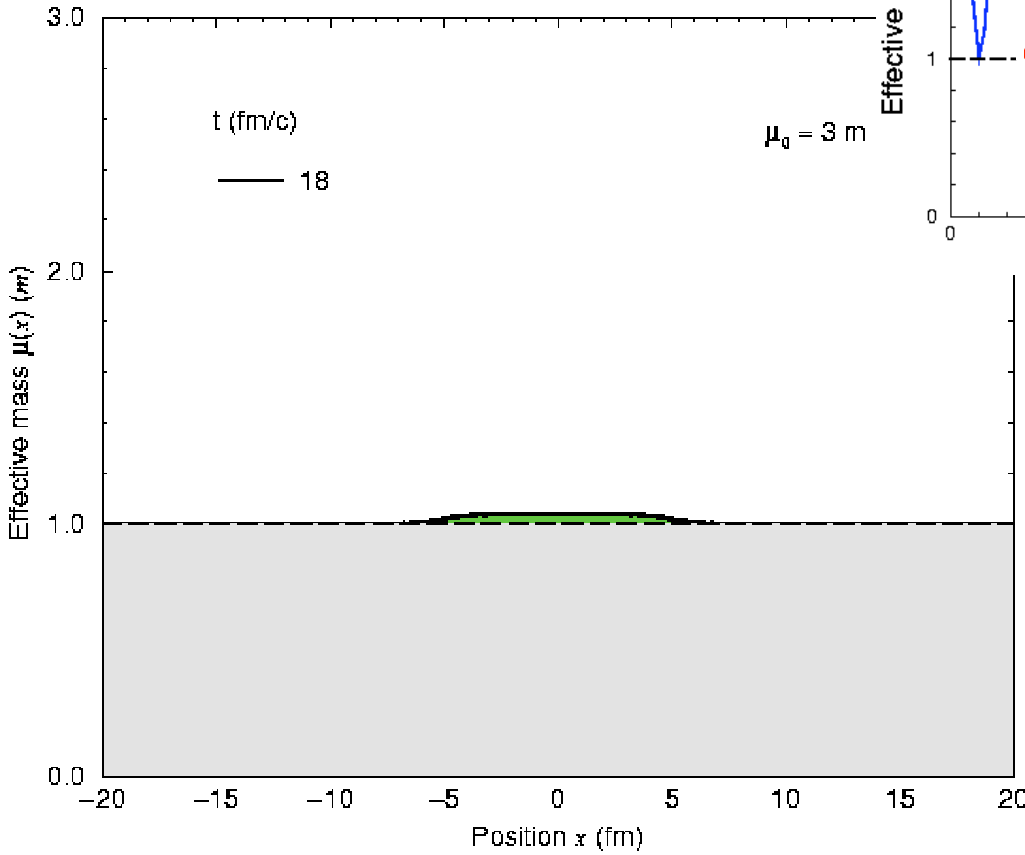


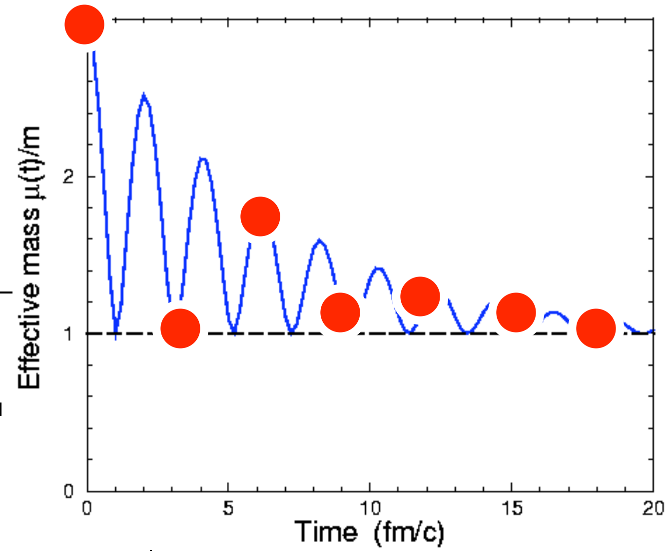


<-- Soft = Hard

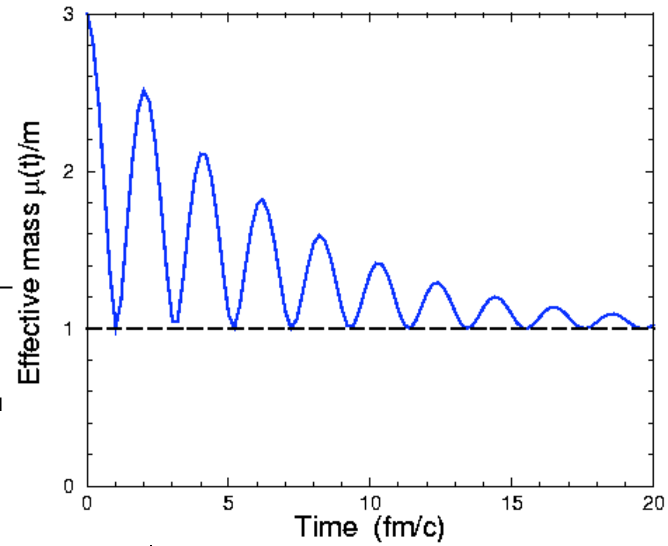
[M Bleicher et al: PRC62 (2000)]

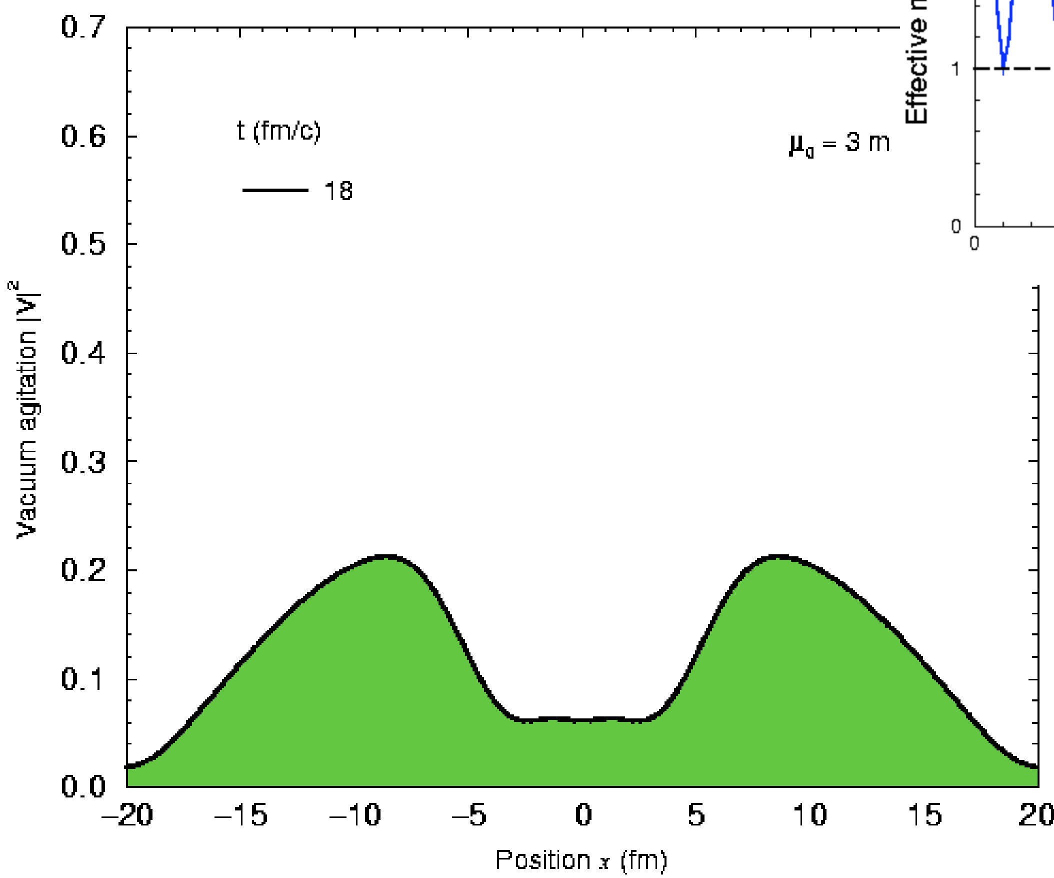
Effective mass function □(x,t):



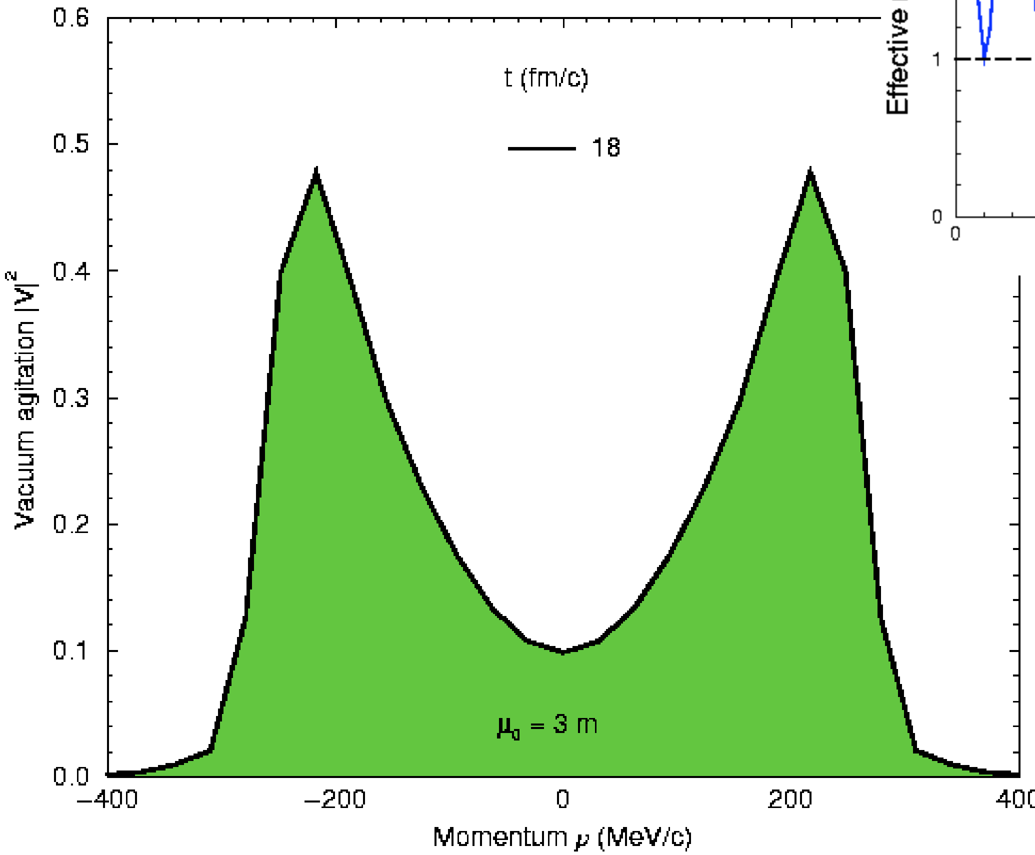


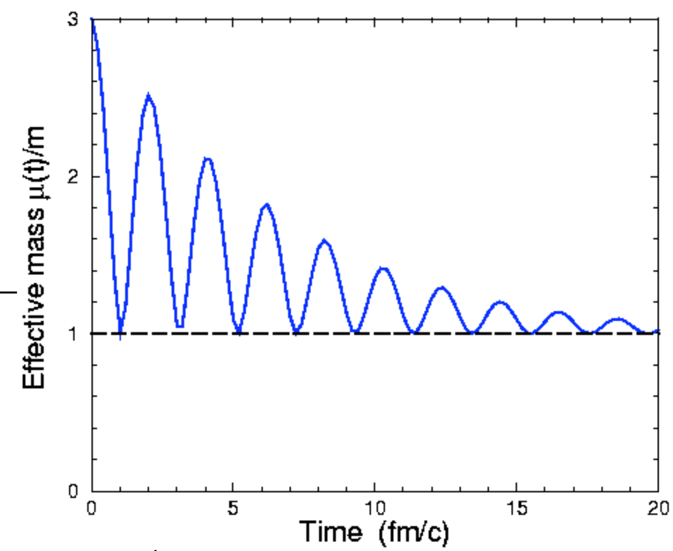
Spatial density of quasiparticles:





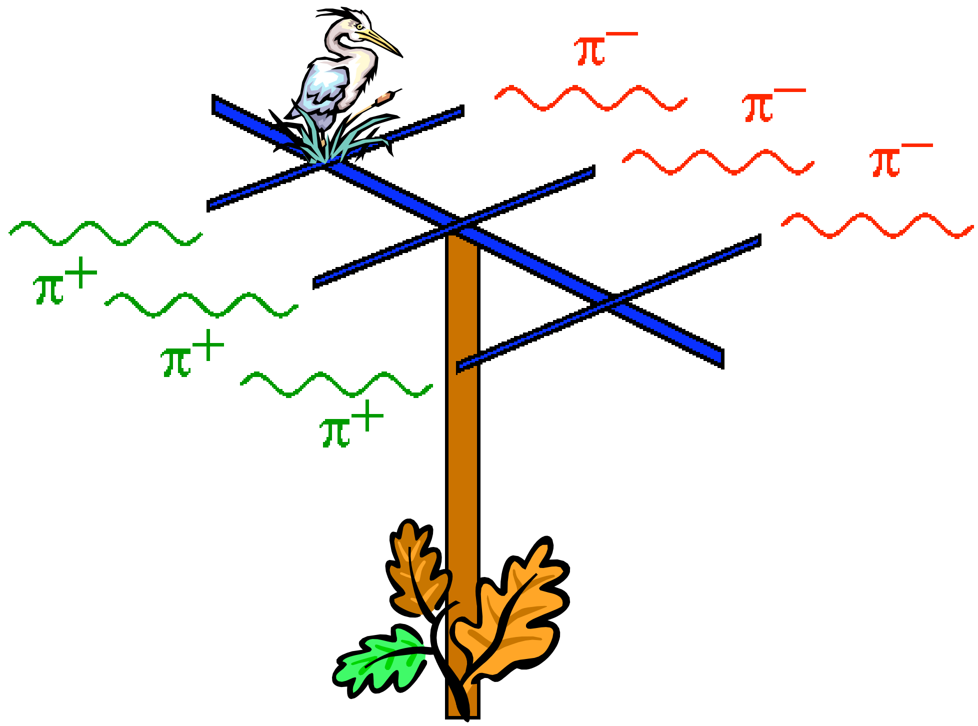
Momentum spectrum of quasiparticles:





Chiral antenna

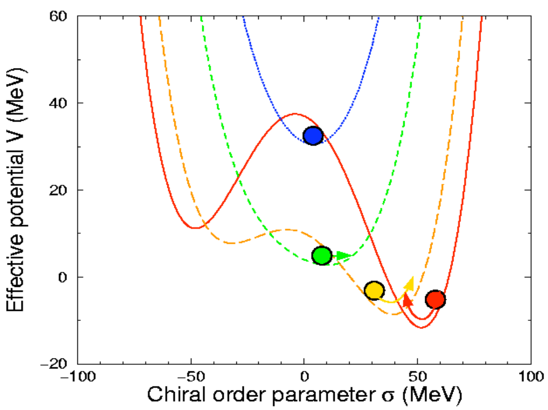
Soft charge-conjugate back-to-back pion pairs

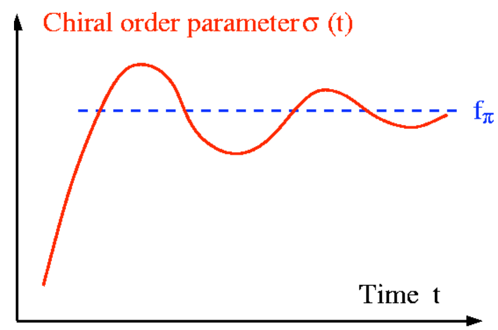


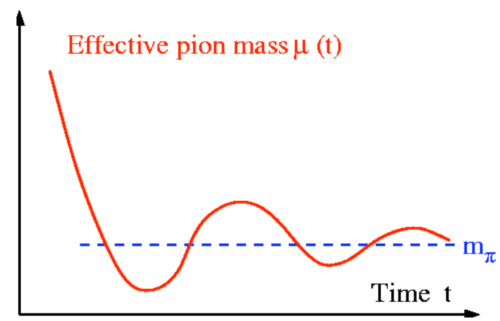
Relaxation dynamics of the chiral order parameter

[J. Randrup, Physical Review C63 (2001) 061901(R); C65 (2002) 054906]

The potential changes fast from hot to cold => The order parameter shows non-equilibrium relaxation => The pion mass exhibits an oscilatory evolution





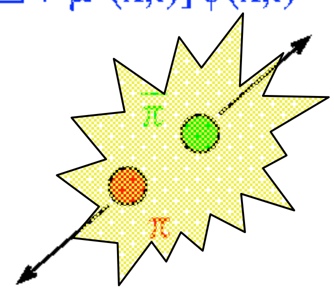


The rapid change of V leads to non-equilibrium relaxation of □

The pionic degrees of freedom see an undulating environment

Emission of soft back-to-back charge-conjugate pion pairs

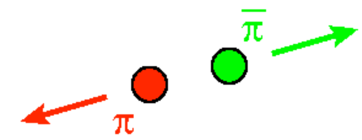
$$[\Box + \mu^2(x,t)]\,\phi(x,t) = 0$$

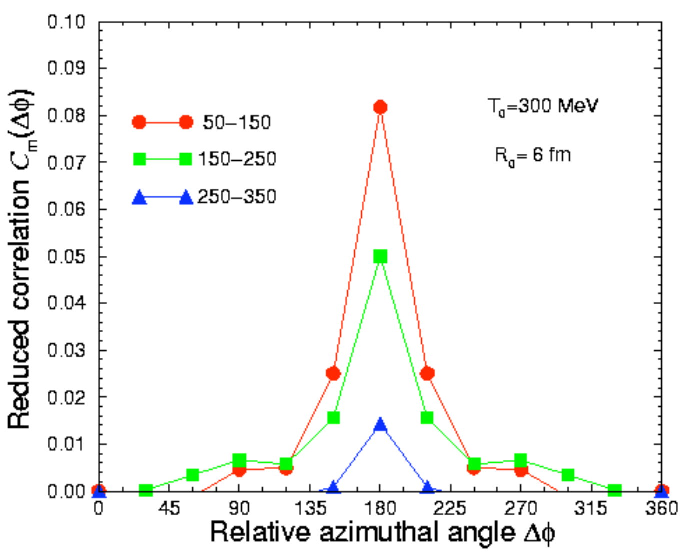


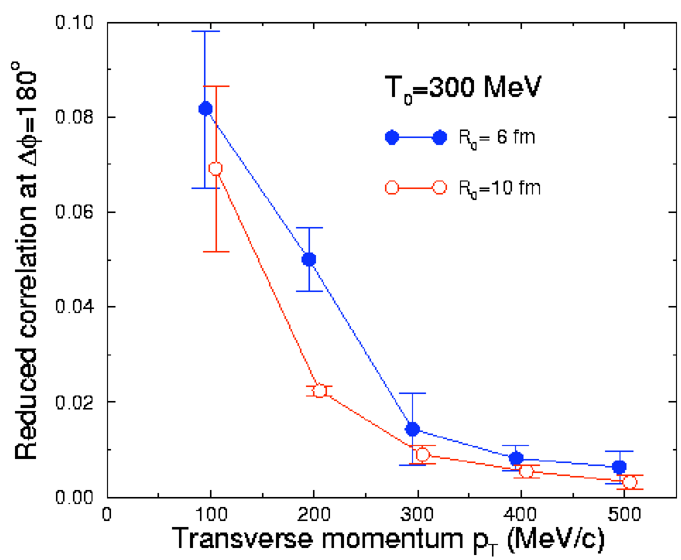
Correlations among the soft pions

The dynamical change of the effetive mass creates back-to-back pairs of soft charge-conjugate pions:

$$[\Box + \mu^2(x,t)]\,\phi(x,t) = 0$$







Chiral Antenna

The antenna signal is sensitive to the properties of interest, such as the initial degree of chiral symmetry restoration and the subsequent rate of expansion/cooling.
So, if identified, it may be a useful probe of these features.

But the calculations should not be taken at face value,
as they are subject to a number of <u>caveats</u>:

Signal strength: additional background?

L□M underpredicts the pion multiplicity => Correlation signal is reduced

Competing mechanisms: Meson decays?

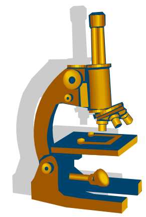
$K_s^0(500) \to 2\pi$	$\rho(770) \to 2\pi$	$\omega(780) \to 3\pi$
just above threshold	well above threshold	not back-to-back

Signal degradation: collisions?

Baked Alaska (Bjorken): Signal pions are late & slow, so don't collide
[Input from transport models needed!]

Linear □ model is inadequate?

Chiral condensate dynamics

		
Novel phenomena:	*Observational consequences:*	Status:
Supercritical strong field?	*Spontaneous pion pair creation*	
Isospin-directed chiral condensates?	*Anomalous distributions of neutral π (and K) fractions*	
Parametric amplification?	*Enhanced fluctuations and correlations*