Critical Behavior of Non Order Parameter Fields & Confinement

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A Simple Question

Is it possible to extract relevant information about, or even identify the onset of a phase transition using non order parameter fields?

Answer and Outcome

We predict a universal behavior of a non order parameter field induced by the order parameter field near and at the phase transition.

& determine relevant features of the deconfining phase transition by monitoring the critical properties of n.o.p. physical excitations.

Content

- The General Theory
- Time-Independent Order Parameter Field
- Comparison with Lattice
- Time-Dependent Order Parameter Field
- Induced Universal Properties and Deconfinement

A. M., F. Sannino and K. Tuominen, hep-ph/0301229
A. M., F. Sannino and K. Tuominen, hep-ph/0306069

General theory

⋆ Temperature driven phase transition & work at T_c

⋆ Renormalizable potential:

$$V(h,\chi) = \frac{m_h^2}{2}h^2 + \frac{m_\chi^2}{2}\chi^2 + \frac{\lambda}{4!}\chi^4 + \frac{g_1}{2}h\chi^2 + \frac{g_2}{4}h^2\chi^2 + \frac{g_3}{3!}h^3 + \frac{g_4}{4!}h^4$$

$h(\vec{x},t)$ non order parameter scalar singlet field (glueball H)

$\chi(\vec{x})$ (Polyakov loop l) or $\chi(\vec{x},t)$ (chiral condensate, Higgs)
real order parameter field respecting Z_2

⋆ Assume: $m_\chi \ll T \ll m_h$ $\qquad$ $m_\chi(T_c) = 0$

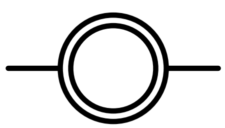
⋆ $H \longleftrightarrow h$ and $\ell \longleftrightarrow \chi \Longrightarrow$ couplings in $\mathcal{L}(H,\ell) \longleftrightarrow \mathcal{L}(h,\chi)$

Time-Independent Order Parameter Field: Probing Static Properties

For $\chi \equiv \chi(\vec{x})$ and $h \equiv h(\vec{x}, t)$ at finite T the h zero mode is relevant.

$$
\begin{aligned}
-\mathcal{L}_{3D} &= \frac{1}{2}\nabla h \nabla h + \frac{1}{2}\nabla\chi\nabla\chi + \frac{1}{2}m_h^2 h^2 + \frac{1}{2}m_\chi^2\chi^2 \\
&+ T\frac{\lambda}{4!}(\chi^2)^2 + \sqrt{T}\frac{g_1}{2}\,h\chi^2 + T\frac{g_2}{4}\,h^2\chi^2 + \sqrt{T}\frac{g_3}{3!}h^3 + T\frac{g_4}{4!}h^4
\end{aligned}
$$

IR Dominated Spatial Correlators

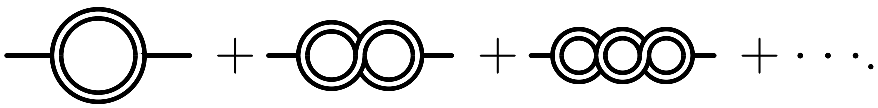


$$
= \quad T\,(\frac{g_1}{2})^2 \int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+m_\chi^2)^2} = T\,\frac{g_1^2}{32\pi m_\chi}
$$

$$
m_h^2(T) \quad = \quad m_h^2 - T\,\frac{g_1^2}{16\pi m_\chi} \quad \text{with} \quad m_\chi \propto |T_c - T|^\nu
$$

Healing the IR behavior

$T < T_c$: The Unbroken Phase

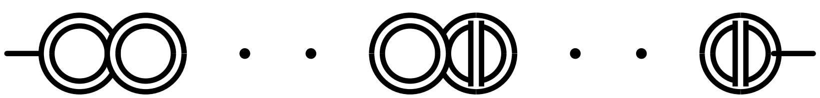


Exact in $O(N)$ for large N

S.R.Coleman, R.Jackiw and H.D.Politzer, PRD 10, 2491 (1974)

$$m_h^2(T) = m_h^2 - T\,\frac{g_1^2}{16\pi\, m_\chi + \lambda\, T} \quad \Longrightarrow \quad m_h^2(T_c) = m_h^2 - \frac{g_1^2}{\lambda}$$

$T > T_c$: The Broken Phase



Exactly Computable!

A.M., F.Sannino and K.Tuominen, hep-ph/0306069

$$m_h^2(T) = m_h^2 - \frac{g_1^2 \mathcal{I}}{2}\,\frac{1 + \frac{\lambda}{3}\mathcal{I}}{1 + \frac{\lambda}{2}\mathcal{I} + \frac{\lambda^2}{6}\mathcal{I}^2} \quad \text{with} \quad \mathcal{I} = \frac{T}{8\pi\sqrt{2}|m_\chi|}$$

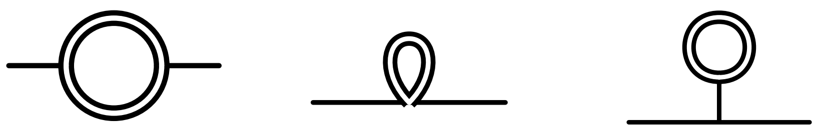
At $T = T_c$ the two sums agree.

Time Dependent Order Parameter Field

For $\chi = \chi(\mathbf{x}, t)$ and $h = h(\mathbf{x}, t)$ all modes are relevant.

$$\mathcal{L}_{4D} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_h^2}{2} h^2 - \frac{m_\chi^2}{2}\chi^2$$
$$- \frac{\lambda}{4!}\chi^4 - \frac{\hat{g}_1 m_h}{2} h\chi^2 - \frac{\hat{g}_2}{4} h^2\chi^2 - \frac{\hat{g}_3 m_h}{3!} h^3 - \frac{g_4}{4!} h^4$$

Relevant Diagrams :



- ◇ Thermal fluctuations for $\chi \rightarrow$ tendency to restore symmetry at high T
- ◇ Tadpoles are real $\propto T^2$
- ◇ Eye has Re and Im part

pole mass M $\equiv$ pole of the full two-point function

$$M^2 \simeq m_h^2 \left[1 + \left(\hat{g}_2 - \hat{g}_1 \hat{g}_3 - 2\hat{g}_1^2\right) \frac{T^2}{24\, m_h^2}\right]$$

not IR dominated

screening mass m_s $\equiv$ pole in the static propagator

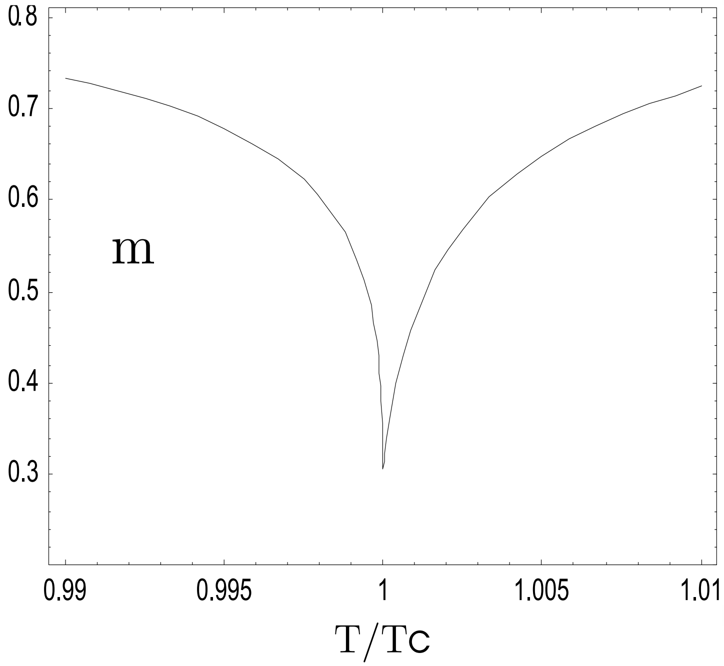
$$m_s^2 \simeq m_h^2 \left[1 - \frac{\hat{g}_1^2}{32\pi} \frac{T}{m_\chi} - (\hat{g}_1 \hat{g}_3 - \hat{g}_2) \frac{T^2}{24\, m_h^2}\right]$$

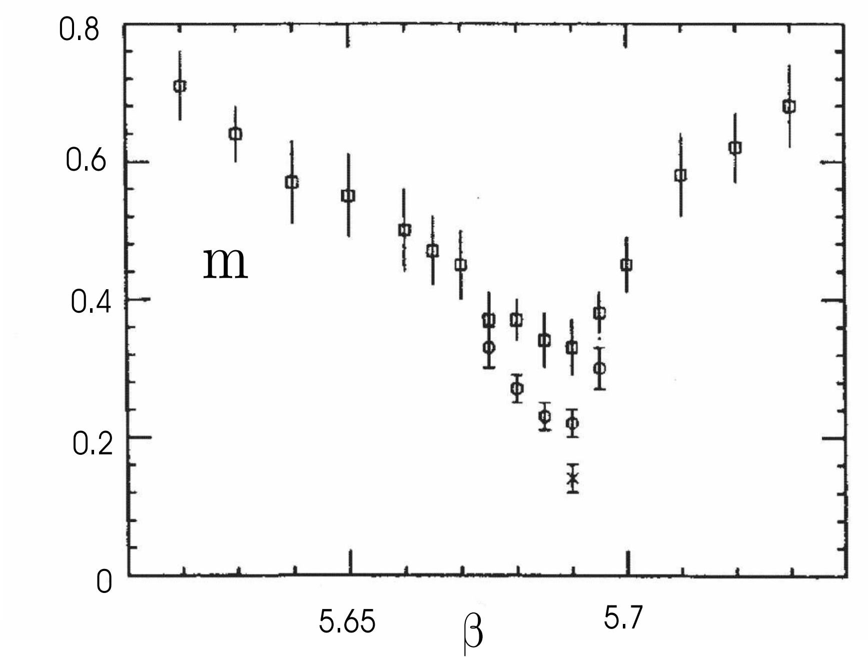
IR dominated

Static limit of h two-point function in 4D theory (time dependent o.p. field) displays same features as in 3D theory (time independent o.p. field).

Info about the Y-M phase transition via singlet field:

glueball screening mass behavior close to the phase transition





left panel: mass of singlet field vs. temperature near the phase transition

A. M. , F. Sannino and K. Tuominen, hep-ph/0301229

right panel: lattice data

P. Bacilieri *et al.* [Ape Collaboration], Phys. Lett. B **220**, 607 (1989)

THE PHYSICS OF THE SLOPE

We Define

$$\mathcal{D}^{\pm} \equiv \lim_{T \to T_c^{\pm}} \frac{1}{\Delta m_h^2} \frac{d\, m_h^2(T)}{dT} \qquad \text{with} \qquad \Delta m_h^2 = m_h^2(T_c) - m_h^2 = g_1^2/\lambda$$

From the Unbroken Side $\quad \mathcal{D}^{-} \propto \frac{d\, m_\chi}{dT}$

From the Broken Side $\quad \mathcal{D}^{+} \propto \frac{d|m_\chi|^2}{dT}$

$$\mathcal{D}^{+} = -6\, \frac{16\pi}{\lambda} \frac{|m_\chi|}{T_c} \mathcal{D}^{-}$$

$m_\chi \propto |T_c - T|^\nu \Longrightarrow$ $\mathcal{D}^{-}$ scales with the exponent $(\nu - 1)$
$\mathcal{D}^{+}$ scales with $(2\nu - 1)$

Critical behavior and Confinement

- ⋆ $SU(N)$ Yang-Mills th: global Z_N symmetry $\longrightarrow$ Polyakov loop $\ell(\vec{x})$ o.p.
  & hadronic states - trace anomaly $\longrightarrow$ H glueballs n.o.p

F. Sannino, PRD 66, 034013

- ⋆ Our renormalizable th. is a truncated version of the full glueball th.

$$H = \langle H \rangle \left(1 + \frac{h}{\sqrt{c}\langle H \rangle^{1/4}} \right) \quad \text{with} \quad \langle H \rangle = \frac{\Lambda^4}{e} \qquad \text{and} \quad \chi = \sqrt{\kappa}\ell$$

- ⋆ Info about the Y-M phase transition via singlet field:
  glueball screening mass behavior close to the phase transition

- ⋆ Present results: higher loop corrections to Sannino's glueball model.

Conclusions

Spatial correlators - screening mass, not pole mass - of n.o.p. fields are IR dominated. Divergence healed via resummation.

Universal results :

finite drop in the screening mass of any scalar singlet field (for time independent & time dependent o.p. field) *at the phase transition*

the drop itself is controlled by the ratio of the square of the relevant coupling of the singlet field to the o.p. and the coupling governing the self interactions of the o.p.

$\Rightarrow$ Info about the phase transition encoded in the behavior of the o.p. field transferred to, and obtainable from singlet field(s).

PREDICTIONS

Only one n.o.p. field, but many are expected to display a similar behavior.

?RHIC Monitoring the spatial correlators of heavy hadrons provides an efficient and sufficient experimental way to uncover the existence and features of the chiral/deconfining phase transition.

While the induced critical behavior is universal the quantitative details depend on the strength of the couplings between the fields.

LATTICE For the Y-M deconfining phase transition lattice simulations are able to determine the coupling strength of any glueball state to the Polyakov loop by following the temperature dependence of screening masses.