Neutrino Superfluids

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Subbal Festschrift!
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Superfluids/Superconductors

- Conventional superconductivity of elements
- Conventional superconductivity of compounds
- Heavy fermion superconductivity
- High temperature superconductivity
- Superconductivity in double-walled carbon nanotubes
- Superfluid He-3
- Dilute neutron matter $^1S_0$ superfluidity
- Dense neutron matter $^3P_2$-$^3F_2$ superfluidity
- Color superconductivity in quark matter

Neutrino Superfluids?

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$\nu_L - \nu_R$ pairing might occur due to due to attractive Higgs exchange; right-handed neutrinos, if they exist, do not couple to anything else in the standard model. This assumes that neutrinos are Dirac particles and that they obtain their mass via the usual Higgs mechanism.

Higgs field

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_0 + \sigma \end{pmatrix}$$

$$v_0 = 1\Big/\sqrt{\sqrt{2}G_F} = 246\,\mathrm{GeV}$$

$$L_{Yukawa} = h_\nu \bar{l}_L \Phi_c \nu_R + h.c. = \left(m_\nu + \frac{h_\nu}{\sqrt{2}}\sigma \right)\bar{\nu}\nu$$

$$m_\nu = h_\nu v_0 \Big/ \sqrt{2}$$

→

$$H_I = -\frac{h_\nu^2}{4m_\sigma^2}\left(\bar{\nu}\nu\right)\left(\bar{\nu}\nu\right)$$

Low energy contact interaction is attractive!

Express the neutrino field in Dirac representation as

$$\nu_L = \frac{1}{2}(1-\gamma_5)\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_L \\ -\psi_L \end{pmatrix}$$

$$\nu_R = \frac{1}{2}(1+\gamma_5)\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_R \\ \psi_R \end{pmatrix}$$

where ψ_L and ψ_R are two-component spinors.

$$\longrightarrow \quad H_I = -\frac{h_\nu^2}{4m_\sigma^2}\left[2\psi_{La}^{+}\psi_{Rb}^{+}\psi_{Lb}\psi_{Ra} + \psi_{La}^{+}\psi_{Lb}^{+}\psi_{Rb}\psi_{Ra} + \psi_{Ra}^{+}\psi_{Rb}^{+}\psi_{Lb}\psi_{La}\right]$$

Allow for condensation of spin-0 Cooper pairs of the form

$$\boxed{\left\langle \psi_L^a \psi_R^b \right\rangle = \varepsilon^{ab} D}$$

Using $\left\langle \psi_L^a \psi_R^b \right\rangle = \varepsilon^{ab} D$ and making the mean-field approximation

$$H_I^{MF} = \frac{h_\nu^2}{2m_\sigma^2}\left[D\psi_{La}^{+}\psi_{Rb}^{+} + D^{*}\psi_{Lb}\psi_{Ra}\right]\varepsilon^{ab}$$

In terms of the usual creation and annihilation operators

$$H_I^{MF} = -\frac{h_\nu^2}{4m_\sigma^2}\sum_{\mathbf{p}}\frac{m_\nu}{\varepsilon}\left\{\begin{array}{l} D\,\mathrm{e}^{2i\varepsilon t}\left[b^{+}(p,+)b^{+}(-p,-) - b^{+}(p,-)b^{+}(-p,+)\right] \\ + D^{*}\,\mathrm{e}^{-2i\varepsilon t}\left[b(-p,-)b(p,+) - b(-p,+)b(p,-)\right] \end{array}\right\}$$

where $\varepsilon = \sqrt{p^2 + m_\nu^2}$. The full Hamiltonian is $H = H_{free} + H_I^{MF}$.

$$H_{free} = \sum_{\mathbf{p}} \varepsilon\left[b^{+}(p,+)b(p,+) + b^{+}(p,-)b(p,-)\right]$$

The Hamiltonian can be diagonalized to the form

$$H = \sum_{\mathbf{p}} E\left[c^+(p,+)c(p,+) + c^+(p,-)c(p,-)\right]$$

$$E = \sqrt{(\varepsilon - \mu)^2 + (Km_\nu / \varepsilon)^2}$$

by making the canonical transformation

$$c(p,+) = \cos\theta\, \mathrm{e}^{-i(\alpha+\varepsilon t)}\, b(p,+) - \sin\theta\, \mathrm{e}^{i(\alpha+\varepsilon t)}\, b^+(-p,-)$$

$$c(p,-) = \cos\theta\, \mathrm{e}^{-i(\alpha+\varepsilon t)}\, b(p,-) + \sin\theta\, \mathrm{e}^{i(\alpha+\varepsilon t)}\, b^+(-p,+)$$

$$\tan\theta = \frac{Km_\nu}{\varepsilon(\varepsilon - \mu)}$$

$$D = |D|\, \mathrm{e}^{2i\alpha}$$

The gap equation is derived by demanding self-consistency between the assumed value of the condensate and the value obtained via the canonical transformation of creation and annihilation operators. One finds that either D=0 or α=π/2 with the magnitude of the gap determined by

$$\frac{h_\nu^2}{8m_\sigma^2}\int\frac{d^3p}{(2\pi)^3}\frac{m_\nu^2}{\varepsilon^2}\frac{1}{\sqrt{(\varepsilon-\mu)^2+(Km_\nu/\varepsilon)^2}}=1$$

In the limit of weak coupling the gap is $\Delta = Km_\nu/\mu$

The integral is divergent; it could be cut-off with an upper limit Λ of order m_σ or with the form-factor $m_\sigma^2\Big/\left(m_\sigma^2+4\vec{p}^{\,2}\right)$.

Put the gap equation in a form similar to that of conventional condensed matter superconductivity:

Gap equation

$$\frac{1}{2} g N(0) \int_{\xi_{\min}}^{\xi_{\max}} \frac{d\xi}{\sqrt{\xi^2 + \Delta^2}} = 1$$

(Relativistic) density of states

$$N(0) = \frac{p^2}{2\pi^2} \left(\frac{dp}{d\varepsilon} \right) \frac{m_\nu^2}{\varepsilon^2} \Bigg|_{\varepsilon=\mu} = \frac{m_\nu^2 v_F}{2\pi^2}$$

$$\xi_{\min} = -(\mu - m_\nu), \quad \xi_{\max} = \Lambda = \sqrt{m_\sigma^2 + m_\nu^2} - \mu$$

Solution

$$\boxed{\Delta = 2\sqrt{|\xi_{\min}|\,\xi_{\max}}\; \mathrm{e}^{-1/gN(0)}}$$

In terms of neutrino parameters
the solution to the gap equations is

$$\Delta = 2\sqrt{(\mu - m_\nu)\Lambda}\,\exp\left[-8\pi^2 m_\sigma^2 / h_\nu^2 m_\nu^2 v_F\right]$$

Since this is a typical BCS-type theory the critical temperature is

$$T_c = \frac{e^\gamma}{\pi}\Delta \approx 0.57\Delta$$

These formulae do not assume any connection between
the neutrino mass and the neutrino-Higgs coupling.

Light Neutrinos

For $m_\nu = 1\,\text{eV}$ and $m_\sigma = 110\,\text{GeV}$

$$\Delta = 2\sqrt{(\mu - m_\nu)\Lambda}\,\exp\left(-10^{46}/\nu_F\right) \quad !$$

Heavy Nonrelativistic Neutrinos

Gap
$$\Delta = \sqrt{\frac{2\Lambda}{m_\nu}}\left(\frac{3\pi^2\rho_\nu}{m_\nu}\right)^{1/3} \mathrm{e}^{-x}$$

London coherence length
$$\xi_{London} = \frac{v_F}{\pi\Delta} = \frac{1}{\pi\sqrt{2\Lambda m_\nu}} \mathrm{e}^{x}$$

$$x = \frac{(2\pi m_\sigma v_0)^2}{m_\nu^2\left(3\pi^2\rho_\nu m_\nu^2\right)^{1/3}}$$

Mass density
$$\rho_\nu = m_\nu n_\nu = m_\nu \frac{p_F^3}{3\pi^2}$$

Hubble Ultra Deep Field
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Cosmology

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Hubble Ultra Deep Field
Hubble Space Telescope • Advanced Camera for Surveys

Cosmology

$$\rho_\nu < 5 \text{ keV/cm}^3$$

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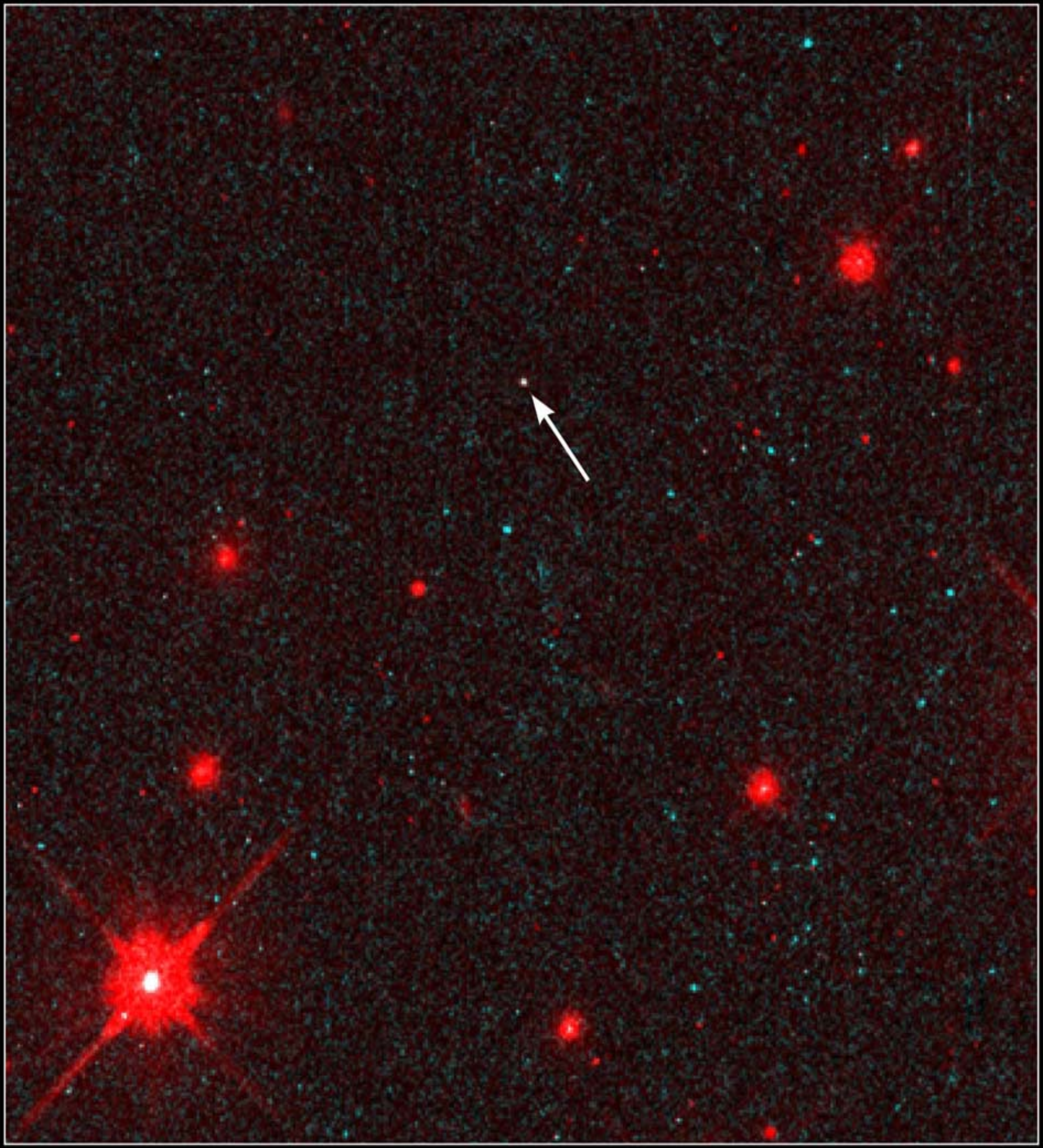
$$T_c \ll 2.7 \text{ K}$$

Neutron Stars



Isolated Neutron Star RX J185635-3754
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Neutron Star

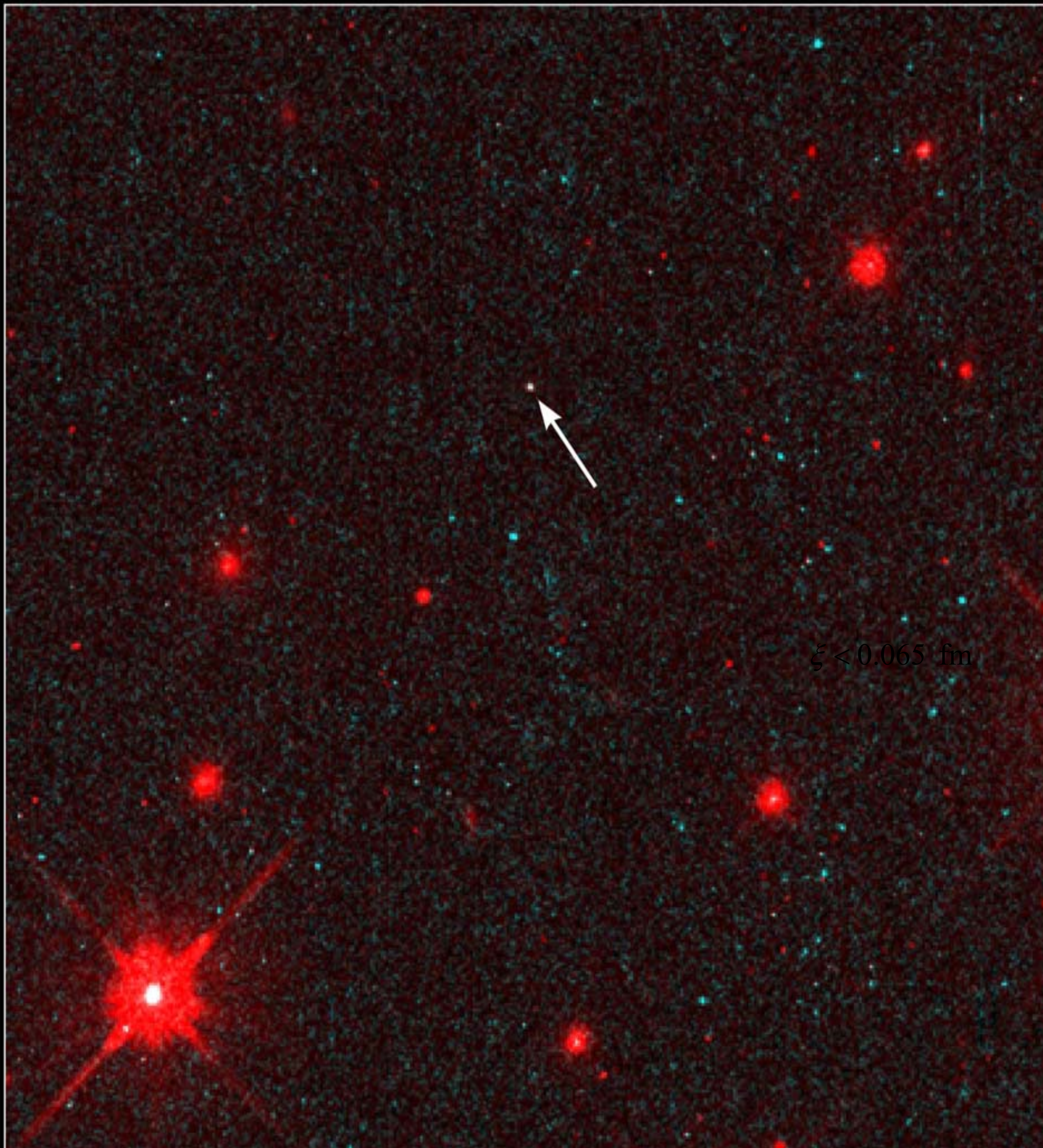
Choose a reference mass of 10 TeV and a reference energy density of 10 MeV/fm^3.

$$\Delta = 67.2\left(\frac{\rho_\nu}{10\ \mathrm{MeV/fm^3}}\right)^{1/3}\left(\frac{10\ \mathrm{TeV}}{m_\nu}\right)^{4/3} \mathrm{e}^{-x}\ \ \mathrm{keV}$$

$$\xi_{\mathrm{London}} = 5.71\times 10^{-4}\ \mathrm{e}^{x}\ \ \mathrm{fm}$$

$$x = 4.73\left(\frac{10\ \mathrm{MeV/fm^3}}{\rho_\nu}\right)^{1/3}\left(\frac{10\ \mathrm{TeV}}{m_\nu}\right)^{8/3}$$

Neutron Stars



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$$m_\nu > 10 \text{ TeV}$$

$$\downarrow$$

$$\xi > 0.065 \text{ fm}$$

$$T_c > 3.9 \times 10^6 \text{ K}$$

Supernovae

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Conclusion

- Neutrino superfluidity is a possibility if Dirac neutrinos exist with nonzero mass.
- If the neutrino coupled to a much lighter scalar boson than the Higgs, or if superheavy neutrinos exist, then neutrino superfluidity could conceivably be realized in nature.