Summary of EH chapter 3

- can introduce complex shifted momenta $\hat{p}_i = p_i + z r_i$ s.t. momentum conservation holds and all are on shell $\hat{p}_i^2 = 0$.

- consider $\hat{A}_n(z)$
  - holomorphic in z
  - no branch cuts
  - only has simple poles

  } Tree level

- the real amplitude is $A_n = \hat{A}_n(0) = -\sum_{\text{poles}} \text{Res}\left[\hat{A}_n(z)/z\right] + B_n$

- the boundary term is the $O(z^0)$ term in the $z \to \infty$ lim of A_n - $B_n = 0$ is a requirement for many recursion relations.

- at the poles in z the amplitude factorizes and if $B_n = 0$, the amplitude is determined by its poles:

$$A_n = \sum_{\text{diagrams } I} \hat{A}_L(z_I) \frac{1}{P_I^2} \hat{A}_R(z_I)$$

this is the general recursion formula.

- BCFW: choose r_i and r_j s.t. $|\hat{i}] = |i] + z|j]$, $|\hat{j}\rangle = |j\rangle + z|i\rangle$ and no other spinors shifted. and use the above recursion formula

- gluon amplitudes and B_n:

$[i,j\rangle$-shift	$[-,-\rangle$	$[-,+\rangle$	$[+,+\rangle$	$[+,-\rangle$
$\hat{A}_n(z) \sim$	z^{-1}	z^{-1}	z^{-1}	z^{3}

here i & j are adjacent. the amp is suppressed by an add power of z^{-1} if i & j are non-adjacent.

Problems: 3.1, 3.2, 3.5, 3.6 + Prove Parke-Taylor using non-adj shift

3.1 $\quad -\not{p} = |p\rangle[p| + |p]\langle p|$

$$\hat{i}] = i] + z\, j] \qquad \hat{j}] = j] \qquad \hat{i}\rangle = i\rangle \qquad |\hat{j}\rangle = j\rangle - z|i\rangle$$

$$-\hat{\not{p}} = -\not{p} + z\not{r}$$
$$= \underbrace{|p\rangle[p| + |p]\langle p|}_{-\not{p}} + z\left(|r\rangle[r| + |r]\langle r|\right)$$

$$-\hat{\not{i}} = \hat{i}]\langle\hat{i} + \hat{i}\rangle[\hat{i}$$
$$= i]\langle i + z\, j]\langle i + i\rangle[i + z\, i\rangle[j$$
$$= -\not{i} + z(\, j]\langle i + i\rangle[j\,)$$

$$-\hat{\not{j}} = -\not{j} - z(\, i\rangle[j + j]\langle i\,)$$

Choose r_i s.t. $\quad \not{r}_i = j]\langle i + i\rangle[j$

$\quad\quad\;\; r_j$ s.t. $\quad \not{r}_j = -\, i\rangle\langle j + j]\langle i$

(recall that the λ & $\tilde{\lambda}$ of $\not{p}$, $\not{p}$ are ind for complex momenta)

now, we want to extract the μ components from $\not{r}_{i,j}$

$$r_{a\dot{a}} = r_\mu \sigma^\mu_{a\dot{a}} \qquad \text{and} \qquad r_\mu = -\tfrac{1}{2}\, r_{a\dot{a}}\, \bar{\sigma}_\mu^{\dot{a}a}$$

3.2 $\quad A_n[1^- 2^- 3^+ \cdots n^+] = \dfrac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$

$$|\hat{1}] = |1] + z\,|2] \qquad |\hat{1}\rangle = |1\rangle$$
$$|\hat{2}] = |2] \qquad |\hat{2}\rangle = |2\rangle - z\,|1\rangle$$

$$\langle 12\rangle \to \langle\hat{1}\hat{2}\rangle = \langle 12\rangle$$
$$\langle 1k\rangle \to \langle\hat{1}k\rangle = \langle 1k\rangle \qquad k = 3, \cdots n$$
$$\langle 2k\rangle \to \langle\hat{2}k\rangle = \langle 2k\rangle - z\langle 1k\rangle \qquad k = 3, \cdots n$$

$$\hat{A}_n = \frac{\langle 12\rangle^4}{\langle 12\rangle(\langle 23\rangle - z\langle 13\rangle)\langle 34\rangle\cdots\langle n1\rangle} \sim \frac{1}{z} \quad \text{for large } z$$

Other shifts:

1) $[1, k\rangle$ for $k = 3, \cdots, n-1$

$$|\hat{1}] = |1] + z\,|k] \qquad |\hat{1}\rangle = |1\rangle$$
$$|\hat{k}] = |k] \qquad |\hat{k}\rangle = |k\rangle - z\,|1\rangle$$

$$\langle\hat{k}j\rangle = \langle kj\rangle - z\langle 1j\rangle \quad \text{for any } j$$

appears 2x in denom $\Rightarrow$ $\hat{A}_n \sim 1/z^2$ at large z

2) $[k, 1\rangle$ for $k = 3, \dots, n-1$

$|\hat{k}] = |k] + z|1]$ $\qquad$ $|\hat{k}\rangle = |k\rangle$

$|\hat{1}] = |1]$ $\qquad$ $|\hat{1}\rangle = |1\rangle - z|k\rangle$

$\langle \hat{k}\hat{j}\rangle = \langle kj\rangle$

$\langle \hat{1}\hat{j}\rangle = \langle 1j\rangle - z\langle kj\rangle$

$$\hat{A}_n \to \frac{\langle \hat{1}\hat{2}\rangle^4}{\langle \hat{1}\hat{2}\rangle\langle \hat{2}\hat{3}\rangle\cdots\langle \widehat{k-1},\hat{k}\rangle\langle \hat{k},\widehat{k+1}\rangle\cdots\langle \hat{n}\hat{1}\rangle}$$

$$= \frac{(\langle 12\rangle - z\langle k2\rangle)^3}{\langle 23\rangle \cdots\cdots (\langle n1\rangle - z\langle nk\rangle)} \sim z^2 \text{ at large } z.$$

3). $[k, j\rangle$ adj & non-adj where $k, j \neq 1, 2$

$|\hat{k}] = |k] + z|j]$ $\qquad$ $|\hat{j}] = |j]$

$|\hat{k}\rangle = |k\rangle$ $\qquad$ $|\hat{j}\rangle = |j\rangle - z|k\rangle$

$\langle \hat{k}\hat{j}\rangle = \langle kj\rangle$

$\langle \hat{k} i\rangle = \langle ki\rangle$ for $i \neq j, 1, 2$

$\langle \hat{j} i\rangle = \langle ji\rangle - z\langle ki\rangle$ for $i \neq k, j, 1, 2$

$$\hat{A}_n \to \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle k-1, \hat{k}\rangle\langle \hat{k}, k+1\rangle\cdots\langle j-1, \hat{j}\rangle\langle \hat{j}\, j+1\rangle\cdots\langle n1\rangle}$$

$$= \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle k-1\, k\rangle\langle k, k+1\rangle\cdots(\langle j-1\, j\rangle - z\langle j-1\, k\rangle)(\langle j\, j+1\rangle - z\langle k\, j+1\rangle)\cdots\langle n1\rangle}$$

$\sim \frac{1}{z^2}$ for large z

if k & j were adj the $\langle kj\rangle$ bracket would remain unshifted so $A_n \sim 1/z$ for large z.

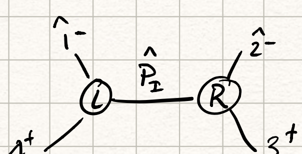
3.5 $\quad M_4(1^- 2^- 3^+ 4^+) = ?$ $\qquad$ $[1, 2\rangle$ - shift.

$M_3(1^- 2^- 3^+) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$ $\qquad$ $\hat{1}] = 1] + z\, 2]$

$\qquad$ $\hat{2}] = 2]$

$M_3[1^+ 2^+ 3^-] = \frac{[12]^6}{[13]^2[23]^2}$ $\qquad$ $\hat{1}\rangle = 1\rangle$

$\qquad$ $\hat{2}\rangle = 2\rangle - z\, 1\rangle$

$M_4[1^- 2^- 3^+ 4^+) =$  $+ \; (3 \leftrightarrow 4)$

$$= \hat{M}_3[\hat{1}^- \; \hat{P}_I^{h_I} \; 4^+] \; \frac{1}{P_I^2} \; \hat{M}_3[-\hat{P}_I^{-h_I} \; \hat{2}^- \; 3^+] \; + (3 \leftrightarrow 4)$$

Sum over h_I

$\hat P_I = -\hat P_{14}$ $\to$ on shell cond. $\hat P_{14}^2 = 2\hat p_1\cdot p_4 = \langle \hat 1 4\rangle[\hat 1 4] = \langle 14\rangle[\hat 1 4] = 0$

$\Rightarrow$ $[\hat 1 4]=0$ for generic momenta

$|\hat P_{14}\rangle[\hat P_{14} 4] = -\hat P_{14}|4] = -(\hat p_1+p_4)|4] = -\hat p_1|4] = |\hat 1\rangle[\hat 1 4] = 0$

$\Rightarrow$ $[\hat P_{14} 4]=0$ since $|\hat P_{14}\rangle \neq 0$

$\hat P_I = \hat P_{23}$ $\to$ $\hat P_{23}^2 = 2\hat p_2\cdot p_3 = \langle\hat 2 3\rangle[\hat 2 3] = \langle \hat 2 3\rangle[23]=0$

$\Rightarrow$ $\langle\hat 2 3\rangle = 0$ for general momenta

$\Rightarrow$ $\langle \hat P_{23} 2\rangle = 0$ by a similar argument to that above.

when $h_I=+$ then $\hat M_3[\hat 1^- -\hat P_{14}^+ 4^+] = \left(\frac{[P_{14}4]^3}{[P_{14}\hat 1][4\hat 1]}\right)^2 \to 0$

$$\hat M_3[-\hat P_{23}^- \hat 2^- 3^+] = \left(\frac{\langle \hat P_{23}\hat 2\rangle^3}{\langle \hat P_{23} 3\rangle\langle \hat 2 3\rangle}\right)^2\to 0$$

$$M_4[1^-2^-3^+4^+] = \hat M_3[\hat 1^- -\hat P_{14}^- 4^+]\frac{1}{P_I^2}\hat M_3[\hat P_{14}^+ \hat 2^- 3^+] + (3\leftrightarrow 4)$$

$$= \left(\frac{\langle \hat 1 \hat P_{14}\rangle^3}{\langle\hat 1 4\rangle\langle\hat P_{14}4\rangle}\right)^2 \frac{1}{\langle 14\rangle[14]}\left(\frac{[\hat P_{14}3]^3}{[\hat P_{14}\hat 2][3\hat 2]}\right)^2 + (3\leftrightarrow4)$$

$$\begin{aligned}\langle\hat 1\hat P_{14}\rangle[\hat P_{14}3] &= -\langle\hat 1|\hat P_{14}|3] \\ &= -\langle \hat 1|\hat 1+4|3] \\ &= -\langle\hat 1|4|3] \\ &= -\langle 1|4|3] \\ &= -\langle 14\rangle[43]\end{aligned}$$

$$\begin{aligned}\langle\hat P_{14}4\rangle[\hat P_{14}\hat 2] &= -\langle 4|\hat P_{14}|\hat 2] \\ &= -\langle 4|\hat 1+4|\hat 2] \\ &= -\langle 4|\hat 1|\hat 2] \\ &= +\langle 4\hat 1\rangle[\hat 1\hat 2] \\ &= \langle 41\rangle[12] \\ &= -\langle 14\rangle[12]\end{aligned}$$

$$M_4 = \frac{(-\langle 14\rangle[43])^6}{\langle\hat 1 4\rangle^2(-\langle 14\rangle[12])^2[3\hat 2]^2\langle 14\rangle[14]} + (3\leftrightarrow4)$$

$$= \frac{\langle 14\rangle^6[13]^6}{\langle 14\rangle^5[12]^2[32]^2[14]} + (3\leftrightarrow 4)$$

$$= \frac{[43]^6}{[12]^2[32]^2}\frac{\langle 14\rangle}{[14]} + (3\leftrightarrow4)$$

$$= \frac{[43]^6}{[12]^2[23]^2}\frac{\langle14\rangle}{[14]} + \frac{[34]^6}{[12]^2[24]^2}\frac{\langle 13\rangle}{[13]} \qquad -S_{12} = [12]\langle 12\rangle$$

$$= \frac{[34]^6}{[12]^2}\left(\frac{\langle14\rangle}{[14][23]^2} + \frac{\langle13\rangle}{[24]^2[13]}\right)$$

$$= \underbrace{\frac{[34]^6}{[12]}\frac{\langle12\rangle}{[14][23][13][24]}}\left(\underbrace{\frac{\langle14\rangle[24][13]}{\langle12\rangle[23][12]} + \frac{\langle13\rangle[14][23]}{\langle12\rangle[24][12]}}\right)$$

$-S_{12}\, A_4[1^-2^-3^+4^+]\, A_4[1^-2^-4^+3^+]$ X

Parke-Taylor

$$M_4[1^- 2^- 3^+ 4^+] = -S_{12}\ A_4[1^- 2^- 3^+ 4^+]\ A_4[1^- 2^- 4^+ 3^+]$$

$$= -\langle 12\rangle[12]\ \frac{[34]^4}{[34][41][12][23]}\ \frac{[34]^4}{[43][31][12][24]}$$

$$= +\langle 12\rangle[12]\ \frac{[34]^8}{[34]^2[12]^2[41][23][31][24]}$$

$$= \frac{\langle 12\rangle[34]^6}{[12]\quad [14][23][13][24]}$$

$$= \frac{[34]^6}{[12]}\left(\frac{\langle 12\rangle}{[14][23][13][24]}\right)$$

We want to show that $\circledast = 1$

$$\circledast = \left(\frac{\langle 14\rangle[24][13]}{\langle 12\rangle[23][12]} + \frac{\langle 13\rangle[14][23]}{\langle 12\rangle[24][12]}\right)$$

$$= \frac{\langle 14\rangle[24][13]}{\langle 14\rangle[34][12]} - \frac{\langle 13\rangle[14][23]}{\langle 13\rangle[34][12]}$$

$$= \frac{[24][13] - [14][23]}{[34][12]}$$

$$= \frac{[12][34]}{[34][12]}$$

$$= 1$$

1) $|1\rangle[1| + |2\rangle[2| + |3\rangle[3| + |4\rangle[4| = 0$

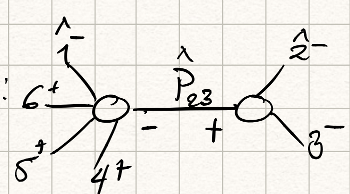
$\langle 12\rangle[23] = -\langle 14\rangle[43]$

$\langle 12\rangle[24] = -\langle 13\rangle[34]$

$[12][34] + [13][42] + [14][23] = 0$

$\Rightarrow -[13][24] + [14][23] = -[12][34]$

$\Rightarrow [13][24] - [14][23] = [12][34]$

3.6 we are to show that the diagram: 

$= \circledast$

does not contribute to $A_6[1^- 2^- 3^- 4^+ 5^+ 6^+]$ with a $[1,2\rangle$ shift

$$\circledast \propto A_3[\hat{P}_{23}^+, \hat{2}^-, 3^-] \qquad \langle \hat{2}\hat{P}_{23}\rangle\langle 3\hat{P}_{23}\rangle$$

$$= \frac{\langle \hat{2}3\rangle^3}{\langle \hat{2}\hat{P}_{23}\rangle\langle 3\hat{P}_{23}\rangle}$$

on-shell cond: $\hat{P}_{23}^2 = 0 = \langle \hat{2}3\rangle[\hat{2}3] = \langle \hat{2}3\rangle[23] = 0 \Rightarrow \langle \hat{2}3\rangle = 0$ since for general momenta $[23] \neq 0$

$$|\hat{P}_{23}]\langle \hat{P}_{23}|\hat{2}\rangle = -\hat{P}_{23}|\hat{2}\rangle = -(\hat{p}_2 + p_3)|\hat{2}\rangle = -p_3|\hat{2}\rangle = |3]\langle 3\hat{2}\rangle = 0$$

$\Rightarrow \langle \hat{P}_{23}\hat{2}\rangle = 0$ as $|\hat{P}_{23}] \neq 0$ for general momenta

$$|\hat{P}_{23}]\langle \hat{P}_{23}|3\rangle = -\hat{P}_{23}|3\rangle = -\hat{p}_2|3\rangle = -|\hat{2}]\langle \hat{2}3\rangle = 0$$

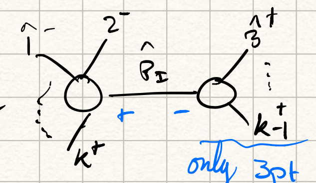
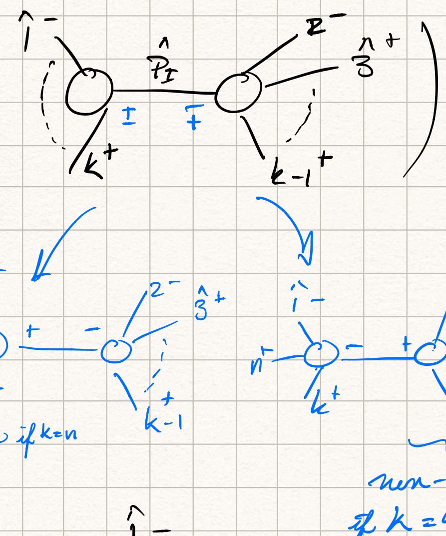
$\Rightarrow \langle \hat{P}_{23}|3\rangle = 0$ as $|\hat{2}] \neq 0$ in general

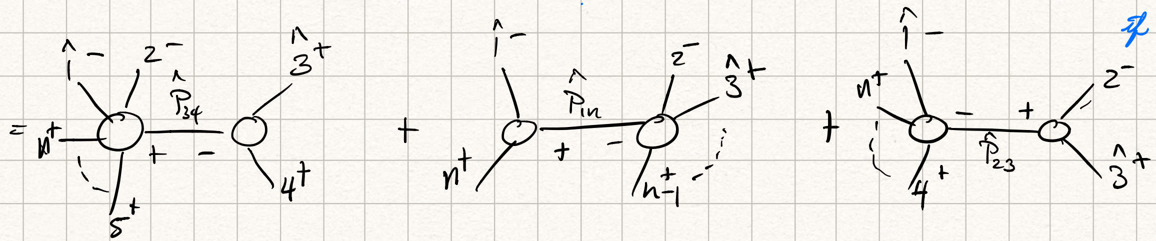
So $A_3 \sim \frac{0^3}{0^2} \sim 0$ and the P_{23} channel does not contribute.

Parke-Taylor for non-adj shifts

$[1, 3\rangle$ shift: $\quad \hat{1}] = 1] + z\,3] \qquad |\hat{1}\rangle = |1\rangle$

$\qquad\qquad\qquad\;\; \hat{3}] = 3] \qquad\qquad\quad |\hat{3}\rangle = |3\rangle - z\,|1\rangle$

$A_n[1^- 2^- 3^+ \dots n^+] = \sum_{k=5}^{n}$  $+ \sum_{k=4}^{n}$ 

$=$ 

$$= \hat{A}_{n-2}[\hat{1}^- 2^- \hat{P}_{34}^+ 5^+ \dots n^+] \frac{1}{P_{34}^2} \hat{A}_3[-\hat{P}_{34}^-, \hat{3}^+ 4^+]$$
$$+ \hat{A}_3[\hat{1}^-, -\hat{P}_{1n}^+, n^+] \frac{1}{P_{1n}^2} \hat{A}_{n-2}[\hat{P}_{1n}^-, 2^-, \hat{3}^+, \dots, n^+-1]$$
$$+ \hat{A}_{n-2}[\hat{1}^- \hat{P}_{23}^- 4^+ \dots n^+] \frac{1}{P_{23}^2} A_3[-\hat{P}_{23}^+, 2^-, \hat{3}^+]$$

diagram 2: the only diagram when $\hat{P}_I^2$ gives a [ ] constraint which is what we need to see if one of the anti MHV A_3's are zero.

$$\hat{P}_{1n}^2 = 0 = \langle \hat{1} n\rangle[\hat{1} n] = \langle 1 n\rangle[\hat{1} n] \Rightarrow [\hat{1} n] = 0$$

$$\hat{A}_3[\hat{1}^-, -\hat{P}_{1n}^+, n^+] = \frac{[\hat{P}_{1n}\, n]^4}{[\hat{P}_{1n}\, \hat{1}\,][n\,\hat{1}]}$$

$$|\hat{P}_{1n}\rangle[\hat{P}_{1n}\, n] = -\hat{P}_{1n}|n] = -\hat{p}_1|n] = |\hat{1}\rangle[\hat{1} n] = 0$$
$$\Rightarrow [\hat{P}_{1n}\, n] = 0$$
$$\Rightarrow \hat{A}_3[\hat{1}^-, -\hat{P}_{1n}^+, n^+] \to 0.$$

$$A_n[1^- 2^- 3^+ \dots n^+] = \hat{A}_{n-2}[\hat{1}^- 2^- \hat{P}_{34}^+ 5^+ \dots n^+] \frac{1}{P_{34}^2} \hat{A}_3[-\hat{P}_{34}^-, \hat{3}^+ 4^+] \leftarrow ①$$
$$+ \hat{A}_{n-2}[\hat{1}^- \hat{P}_{23}^- 4^+ \dots n^+] \frac{1}{P_{23}^2} A_3[-\hat{P}_{23}^+, 2^-, \hat{3}^+] \leftarrow ②$$

$$
① = \frac{\langle \hat{1} 2\rangle^4}{\langle \hat{1} 2\rangle \langle 2 \hat{P}_{34}\rangle \langle \hat{P}_{34} 5\rangle \langle 56\rangle \cdots \langle n \hat{1}\rangle} \; \frac{1}{\langle 34\rangle [34]} \; \frac{[\hat{3} 4]^3}{[\hat{3} \hat{P}_{34}][4 \hat{P}_{34}]}
$$

$$
= \frac{-\langle 12\rangle^4 \cancel{[34]^3}}{\langle 12\rangle \langle 2\hat{3}\rangle \cancel{[43]} \langle 54\rangle \cancel{[43]} \langle 56\rangle \cdots \langle n\hat{1}\rangle} \; \frac{1}{\langle 34\rangle \cancel{[34]}}
$$

$$
= \frac{+\langle 12\rangle^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \cdots \langle n1\rangle} \left(\frac{\langle 23\rangle}{\langle 2\hat{3}\rangle} \right)
$$

$$
\langle 2 \hat{P}_{34}\rangle [4 \hat{P}_{34}] = \langle 2|\hat{P}_{34}|4] = -\langle 2\hat{3}\rangle [34]
$$

$$
\langle \hat{P}_{34} 5\rangle [\hat{3} \hat{P}_{34}\rangle = -\langle 5|\hat{P}_{34}|\hat{3}] = \langle 54\rangle [43]
$$

$$
② = \frac{\langle \hat{1} \hat{P}_{23}\rangle^{\cancel{4}3}}{\langle \cancel{\hat{1} \hat{P}_{23}}\rangle \langle \hat{P}_{23} 4\rangle \langle 45\rangle \cdots \langle n\hat{1}\rangle} \; \frac{1}{\langle 23\rangle [23]} \; \frac{[\hat{P}_{23} \hat{3}]^3}{[\hat{P}_{23} 2][32]}
$$

$$
= \frac{\cancel{-} \langle 12\rangle^3 \cancel{[23]^3}}{\langle \hat{3} 4\rangle \cancel{[23]} \langle 45\rangle \cdots \langle n1\rangle \langle 23\rangle \cancel{[23]} \cancel{[32]}}
$$

$$
= \frac{\langle 12\rangle^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \cdots \langle n1\rangle} \left(\frac{\langle 34\rangle}{\langle \hat{3} 4\rangle} \right)
$$

$$
\langle \hat{1} \hat{P}_{23}\rangle [\hat{P}_{23} \hat{3}] = -\langle \hat{1}|\hat{P}_{23}|\hat{3}] = -\langle \hat{1}|2|\hat{3}] = +\langle \hat{1} 2\rangle [2\hat{3}] = \langle 12\rangle [23]
$$

$$
\langle \hat{P}_{23} 4\rangle [\hat{P}_{23} 2] = +\langle 4|\hat{P}_{23}|2] = -\langle 4\hat{3}\rangle [\hat{3} 2] = -\langle \hat{3} 4\rangle [23]
$$

$$
A_n = ① + ② = \frac{\langle 12\rangle^4}{\langle 12\rangle \langle 23\rangle \cdots \langle n1\rangle} \Big(\underbrace{\frac{\langle 34\rangle}{\langle \hat{3} 4\rangle} + \frac{\langle 23\rangle}{\langle 2\hat{3}\rangle}}_{\circledast \text{ must be } 1} \Big)
$$

$$
\left.\begin{aligned}
&\hat{P}_{23}^2 = 0 = \underbrace{\langle 2\hat{3}\rangle}_{=0} [23] \\
&\langle 2\hat{3}\rangle = \langle 23\rangle - z\langle 21\rangle = 0 \Rightarrow z_{23} = \langle 23\rangle / \langle 21\rangle \\
&\langle \hat{3} 4\rangle = \langle 34\rangle - z_{23}\langle 14\rangle = \langle 34\rangle - \frac{\langle 23\rangle}{\langle 21\rangle} \langle 14\rangle
\end{aligned}\right\} \text{for } ②
$$

$$
\left.\begin{aligned}
&\hat{P}_{34}^2 = 0 = \langle \hat{3} 4\rangle [\hat{3} 4] = \underbrace{\langle \hat{3} 4\rangle}_{=0} [34] \\
&\langle \hat{3} 4\rangle = \langle 34\rangle - z_{34}\langle 14\rangle = 0 \Rightarrow z_{34} = \langle 34\rangle / \langle 14\rangle \\
&\langle 2\hat{3}\rangle = \langle 23\rangle - z_{34}\langle 21\rangle = \langle 23\rangle - \frac{\langle 34\rangle}{\langle 14\rangle} \langle 21\rangle
\end{aligned}\right\} \text{for } ①
$$

$$
\circledast = \frac{\langle 34\rangle}{\langle \hat{3} 4\rangle} + \frac{\langle 23\rangle}{\langle 2\hat{3}\rangle}
$$

$$
= \frac{\langle 34\rangle}{\langle 34\rangle - \frac{\langle 23\rangle}{\langle 21\rangle}\langle 14\rangle} + \frac{\langle 23\rangle}{\langle 23\rangle - \frac{\langle 34\rangle}{\langle 14\rangle}\langle 21\rangle}
$$

$$
= \frac{\langle 34\rangle \langle 21\rangle}{\langle 21\rangle \langle 34\rangle - \langle 23\rangle \langle 14\rangle} + \frac{\langle 23\rangle \langle 14\rangle}{\langle 23\rangle \langle 14\rangle - \langle 34\rangle \langle 21\rangle}
$$

$$
= 1
$$