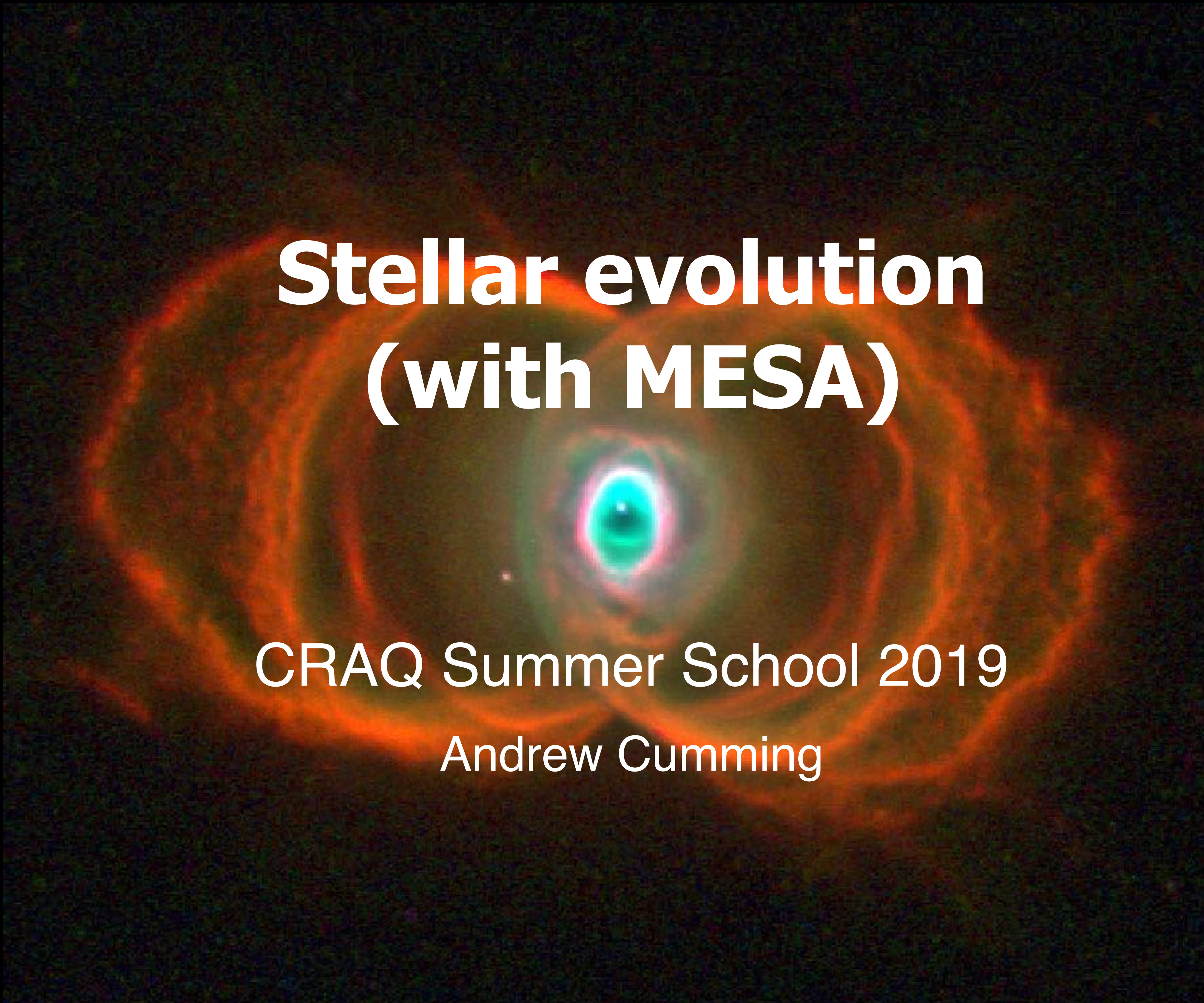


Stellar evolution (with MESA)

CRAQ Summer School 2019

Andrew Cumming

Goal for this session

- cover some of the basic ideas in stellar structure and evolution

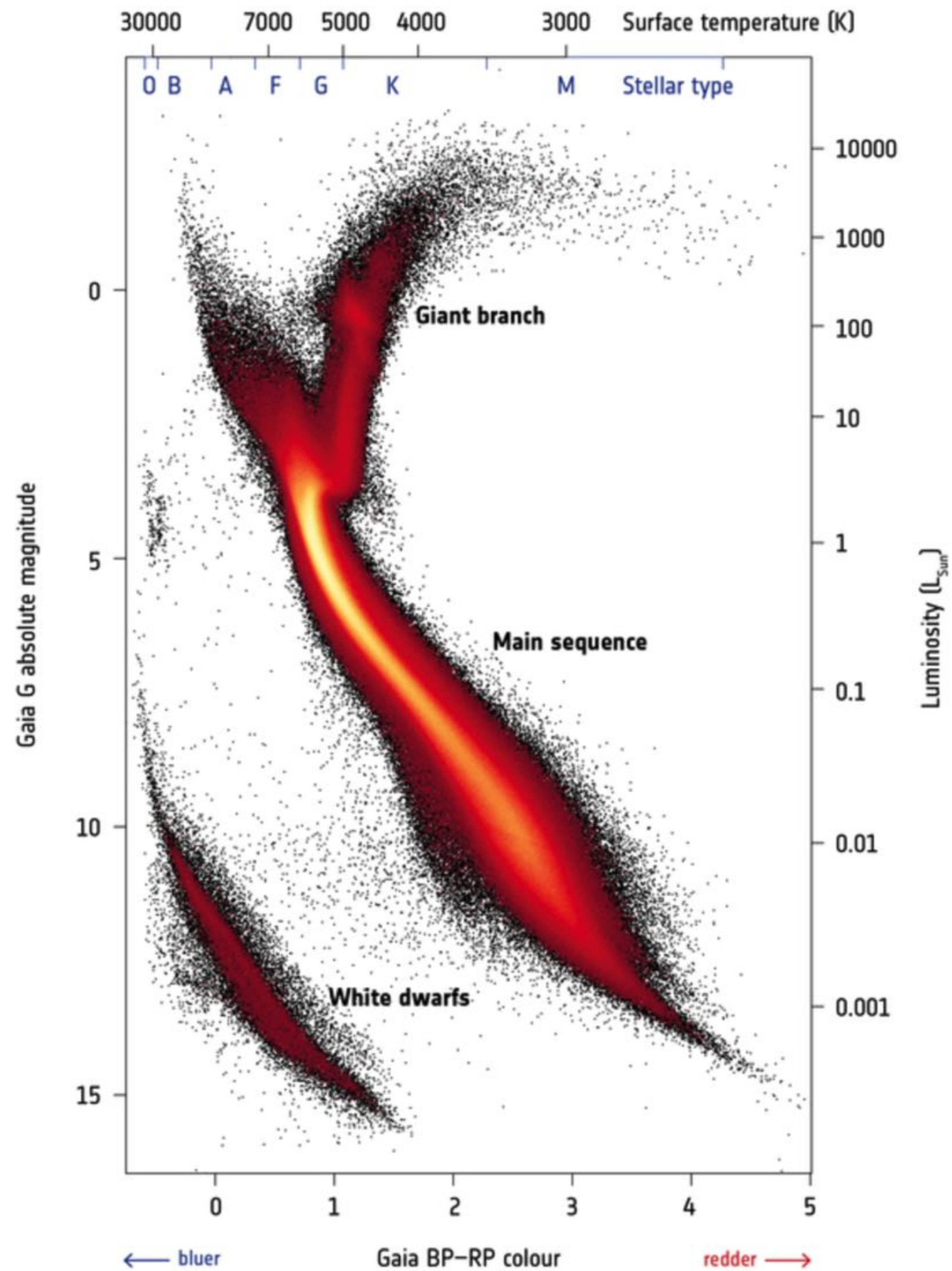
- how to make your own stellar models with the MESA stellar evolution code

Thursday June 13
9-10.20 am
10.40am-noon

HR diagram from Gaia

Stellar properties depend on a few parameters:
mass, age, composition

Understanding stellar structure and evolution tells us why stars look the way they do, and how they evolve through the HR diagram as they age



http://sci.esa.int/gaia/60198-gaia-hertzsprung-russell-diagram/

What is a star?

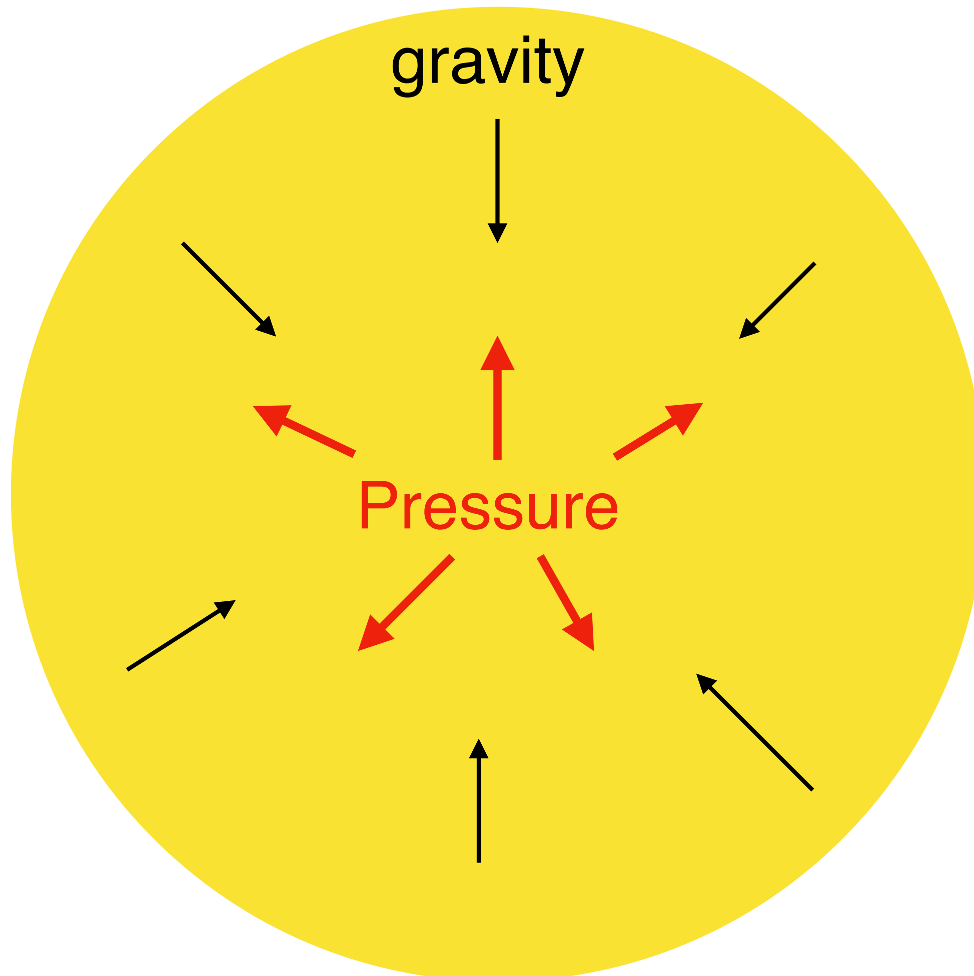
ball of gas (plasma)

internal pressure balances against the weight of the gas

if you know the mass and radius, then you can work out what the central pressure must be

hydrostatic balance

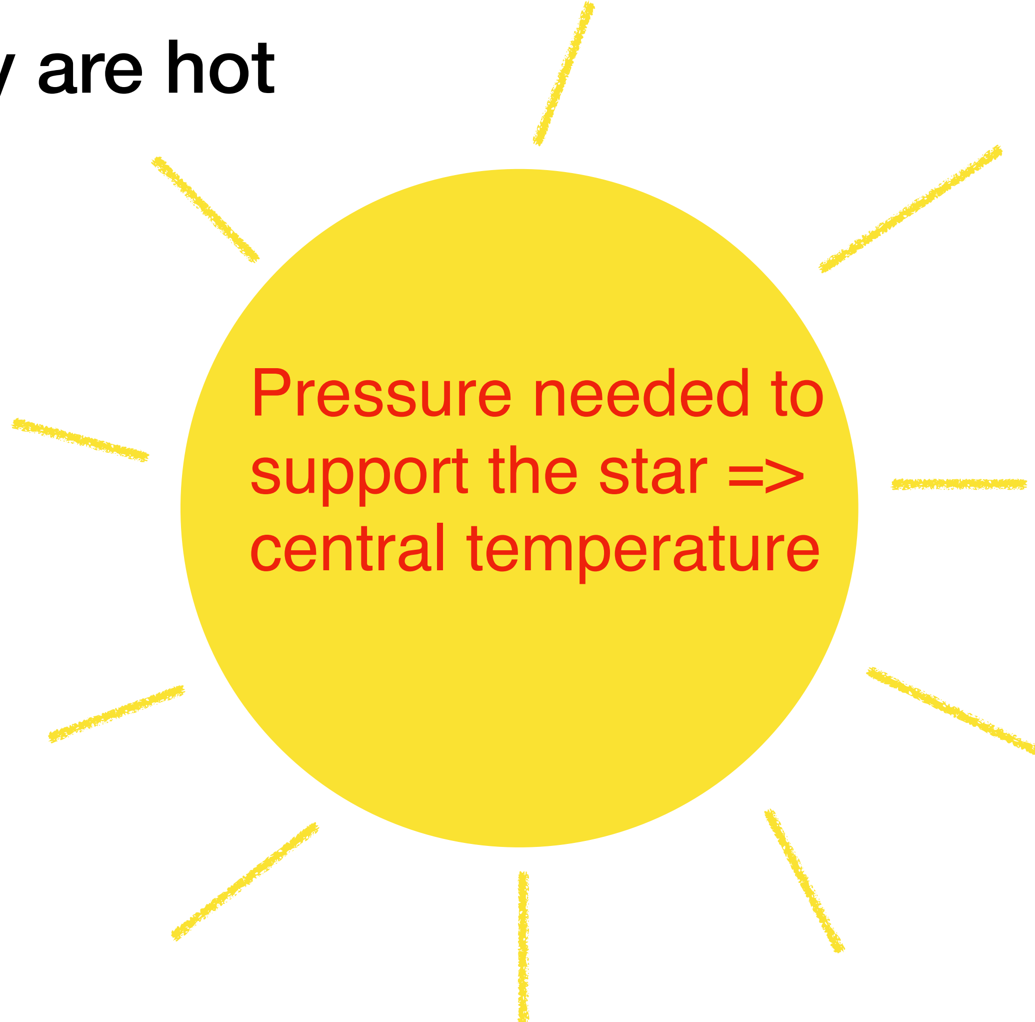
$$\frac{\partial P}{\partial r} = -\rho \frac{Gm}{r^2}$$



Stars shine because they are hot

A hot object will cool by emitting thermal radiation

In many cases, heat is transported internally by diffusion of photons



heat transport equation

$$L = -4\pi r^2 \frac{4acT^3}{3\kappa\rho}\frac{\partial T}{\partial r}$$

Luminosity at radial coordinate r

Thermal conductivity

Temperature gradient

κ = opacity (proportional to the cross-section for scattering or absorption of photons)

The life of a star is a continual battle against gravity

As energy is radiated away, the star will contract, unless there is an energy source inside the star

* **Pre-main sequence:** the star is too cold for nuclear burning; it contracts as it radiates away energy
* **Main sequence:** the central temperature is large enough that the energy released from hydrogen burning balances the luminosity: contraction halts
* **White dwarf:** once the star becomes very dense, degenerate electrons provide the pressure. Degeneracy pressure is independent of temperature => contraction halts once again and the white dwarf cools at fixed radius

energy equation

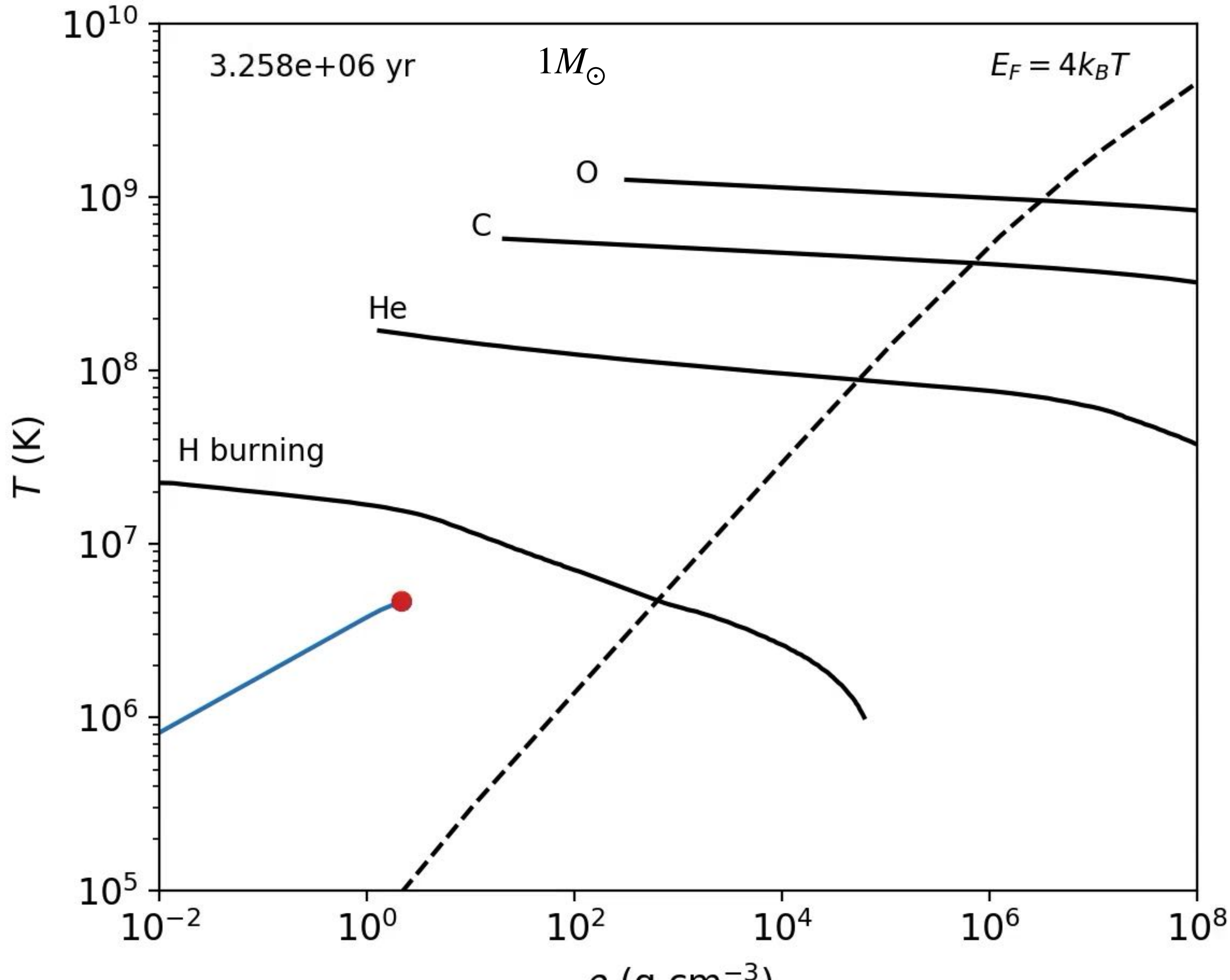
$$\frac{\partial L}{\partial m} = \epsilon_{\rm nuc} - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t}$$

Energy associated with time-changing temperature or pressure

How luminosity changes with mass coordinate

Energy per gram per second from nuclear burning

Tracks in central density and temperature



The equations of stellar structure

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \varrho} \,,$$

mass continuity

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \,,$$

hydrostatic balance

$$\frac{\partial l}{\partial m} = \varepsilon_{\mathrm{n}} - \varepsilon_{\nu} - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\varrho} \frac{\partial P}{\partial t} \,,$$

energy equation

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla \,,$$

heat transport

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\varrho} \left(\sum_j r_{ji} - \sum_k r_{ik} \right) \,, \quad i = 1, \ldots, I \,.$$

composition changes

Timescales in stellar evolution

Following the evolution of a star is a challenging calculation because of the large range of timescales involved

nuclear burning timescale $$t_{\rm nuc} \sim \frac{0.007 X M_\odot c^2}{L_\odot} \sim 70 \text{ Gyr}$$

can be much shorter: e.g. hours at the end of the life of a massive star, or during a thin shell flash

thermal time $$t_{\rm KH} \sim \frac{G M_\odot^2}{R_\odot L_\odot} \sim 30 \text{ Myr}$$

sets the evolution time when you run out of fuel

dynamical time $$t_{\rm dyn} \sim \left(\frac{R_\odot^3}{G M_\odot}\right)^{1/2} \sim 20 \text{ mins}$$

e.g. core collapse

Part 1 : Make a model of the Sun with MESA

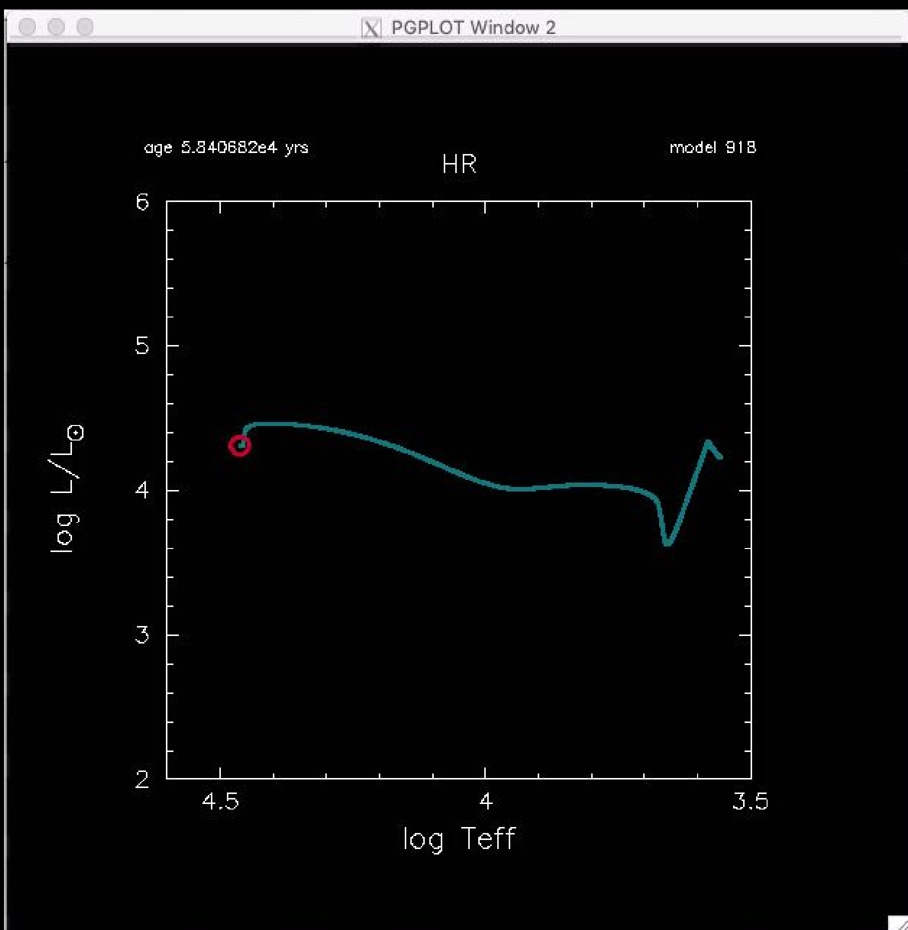
Make a new work directory:

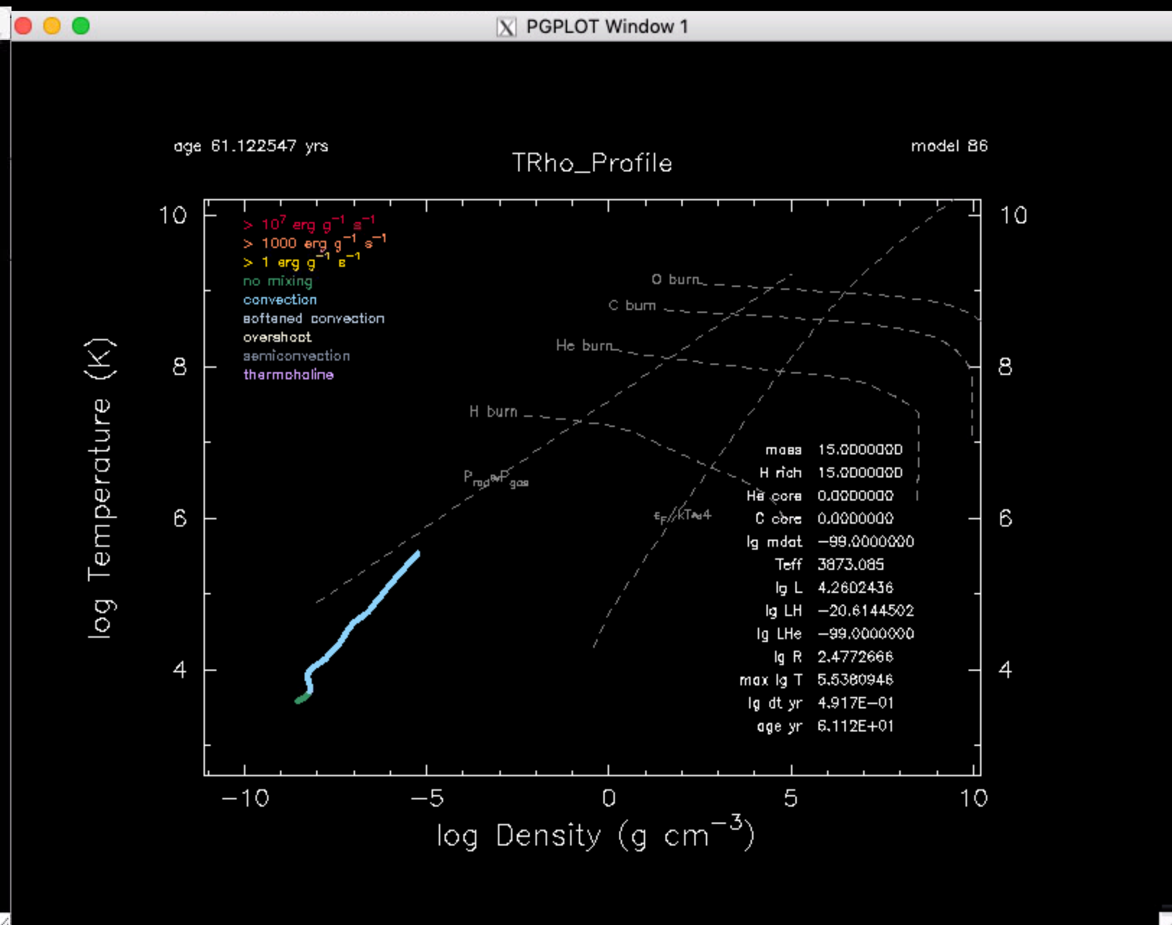
```
cp -r $MESA_DIR/star/work evolve
```

If you do

```
cd evolve
./mk
./rn
```

MESA should run and pop up two windows





Anatomy of a MESA work directory

`inlist` → `inlist_project` — sets physics and numerical parameters for the run

`inlist` → `inlist_pgstar` — parameters for plotting

`LOGS/` → `history.data`

`LOGS/` → `profiles.index`, `profileN.data`

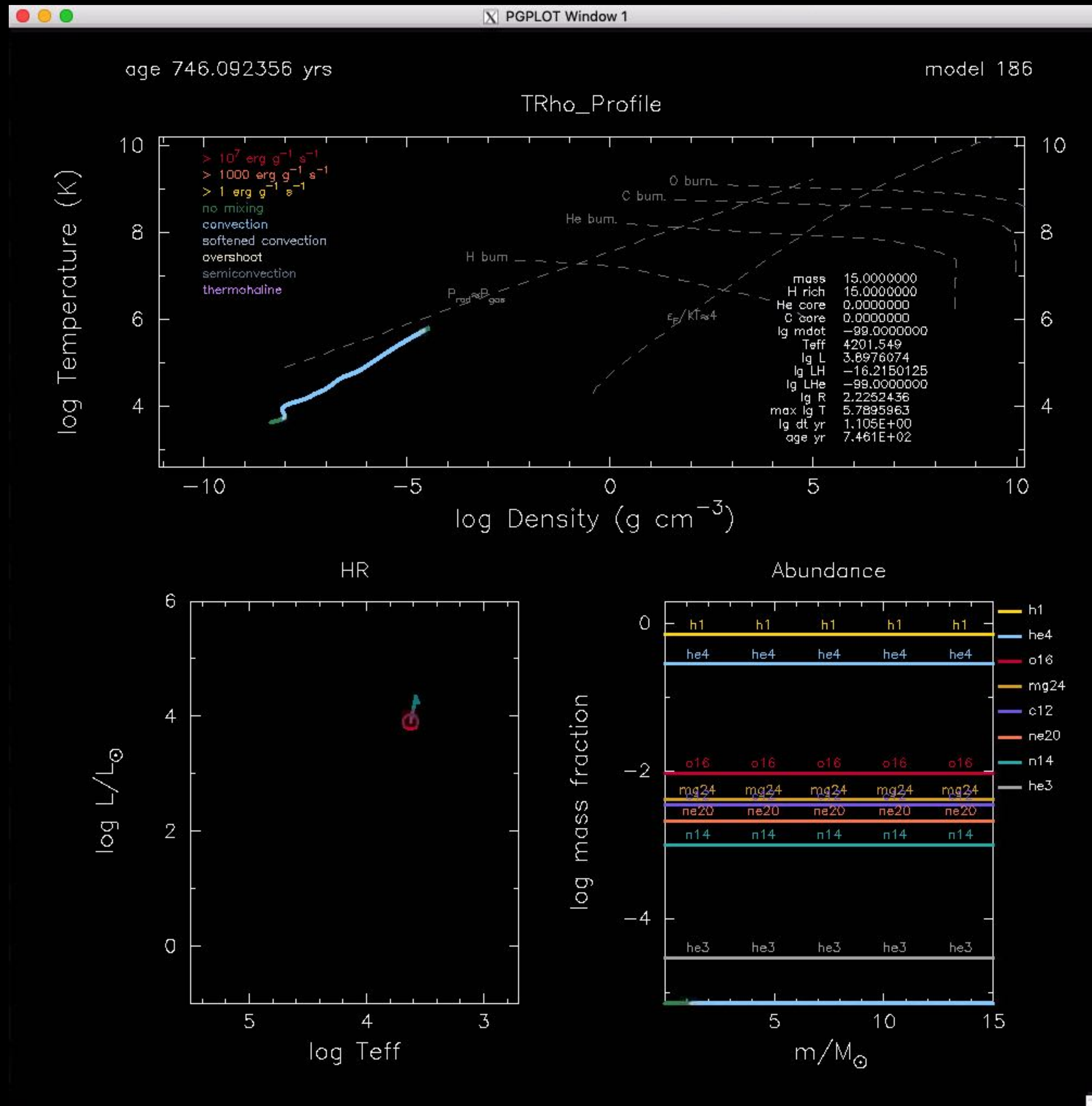
this is where MESA puts output files

`photos/` — model files that you can use to restart MESA, e.g. `./re x050` instead of `./rn`

`src/run_star_extras.f` — where you can extend MESA for more advanced use

`*.mod` — model files that you can use to initialize the run, save at the end, distribute etc.

Copy the `inlist_pgstar` file provided to your work directory and run MESA again



MESA has many built in options for different types of plots

http://mesa.sourceforge.net/pgstar_defaults.html

Now look at `inlist_project`

```
! inlist to evolve a 15 solar mass star

! For the sake of future readers of this file (yourself included),
! ONLY include the controls you are actually using.  DO NOT include
! all of the other controls that simply have their default values.

&star_job

  ! begin with a pre-main sequence model
    create_pre_main_sequence_model = .true.

  ! save a model at the end of the run
    save_model_when_terminate = .false.
    save_model_filename = '15M_at_TAMS.mod'

  ! display on-screen plots
    pgstar_flag = .true.

/ !end of star_job namelist


&controls

  ! starting specifications
    initial_mass = 15 ! in Msun units

  ! options for energy conservation (see MESA V, Section 3)
     use_dedt_form_of_energy_eqn = .true.
     use_gold_tolerances = .true.

  ! stop when the star nears ZAMS (Lnuc/L > 0.99)
    Lnuc_div_L_zams_limit = 0.99d0
    stop_near_zams = .true.

  ! stop when the center mass fraction of h1 drops below this limit
    xa_central_lower_limit_species(1) = 'h1'
    xa_central_lower_limit(1) = 1d-3

/ ! end of controls namelist
```

Now look at `inlist_project`

```
! inlist to evolve a 15 solar mass star

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/ ! end of controls namelist
```

Let's make a model of the Sun

First step is to change `initial_mass` to 1 instead of 15

Now look at `inlist_project`

```
! inlist to evolve a 15 solar mass star

! For the sake of future readers of this file (yourself included),
! ONLY include the controls you are actually using.  DO NOT include
! all of the other controls that simply have their default values.

&star_job

  ! begin with a pre-main sequence model
    create_pre_main_sequence_model = .true.

  ! save a model at the end of the run
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    pgstar_flag = .true.

/ !end of star_job namelist


&controls

  ! starting specifications
    initial_mass = 15 ! in Msun u

  ! options for energy conservati
     use_dedt_form_of_energy_eqn
     use_gold_tolerances = .true.

  ! stop when the star nears ZAMS
    Lnuc_div_L_zams_limit = 0.99d
    stop_near_zams = .true.

  ! stop when the center mass fra
    xa_central_lower_limit_specie
    xa_central_lower_limit(1) = 1

/ ! end of controls namelist
```

Let's make a model of the Sun

First step is to change `initial_mass` to 1 instead of 15

To make things faster for now (less accurate!)

```
! options for energy conservation (see MESA V, Section 3)
  use_dedt_form_of_energy_eqn = .false.
  use_gold_tolerances = .false.

 ! global timestep control (default 1e-4)
varcontrol_target = 1d-3
```

We need a new stopping condition to stop when we get to the age of the Solar System

http://mesa.sourceforge.net/controls_defaults.html#when_to_stop

max_age ¶

Stop when the age of the star exceeds this value (in years). only applies when > 0.

```
max_age = 1d36
```

Add this to your inlist_project in the controls section:

```
!stop at a given age
max_age = 4.5d9
```

and change this one from `.true.` to `.false.` :

```
stop_near_zams = .false.
```

We've set the flag `Grid1_file_flag = .true.` in `inlist_pgstar` so that MESA will output png files. Before running make sure the png directory is empty

```
\rm -r png/*
```

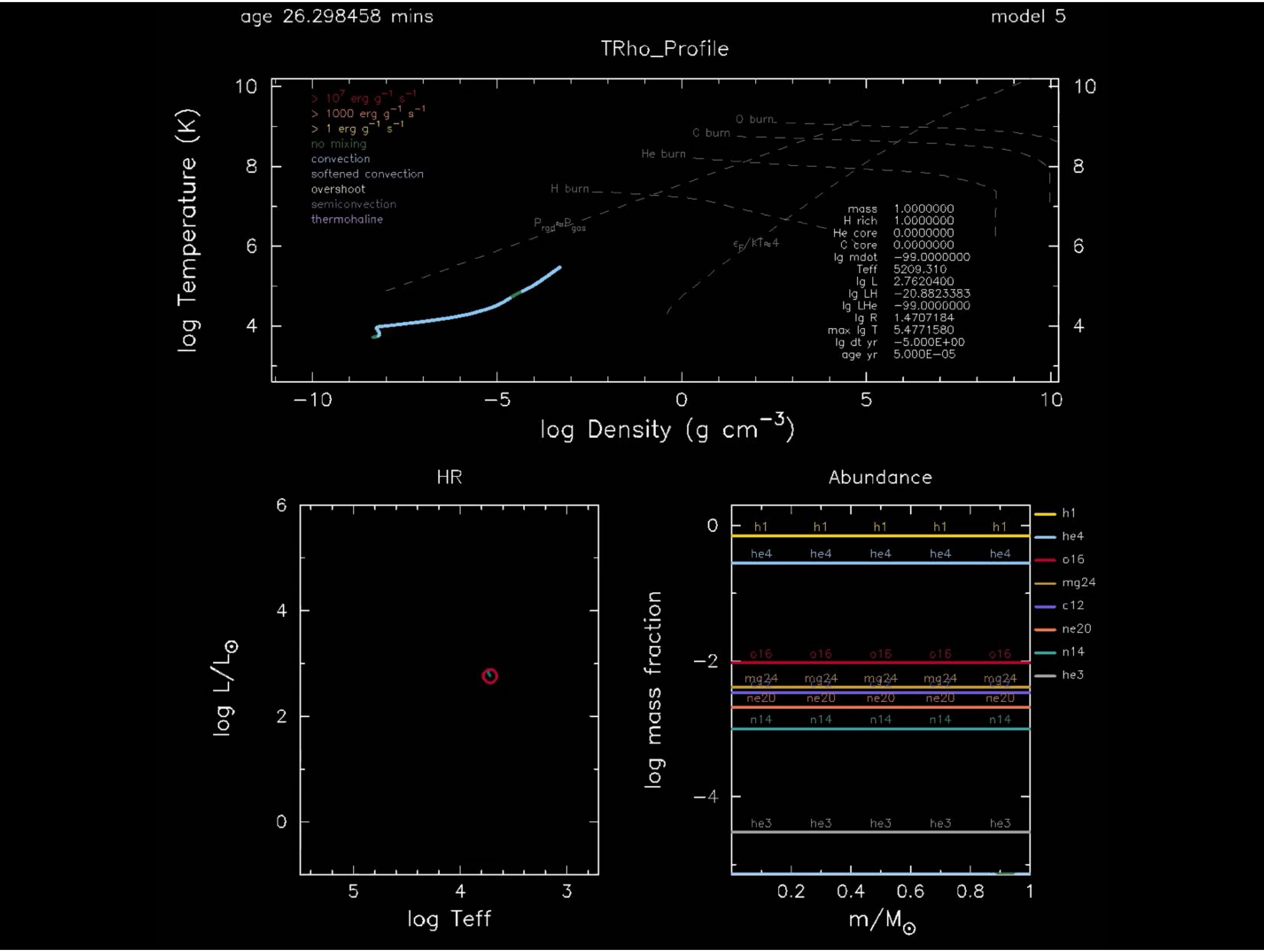
Now run the code!

```
./rn
```

To make a movie use

```
images_to_movie 'png/grid*.png' movie.mp4
```

Take a look at the movie — does the star have the structure you expect for the Sun?



age 26.298458 mins
model 5
TRho_Profile
log Temperature (K)
log Density (g cm^{-3})
> 10^7 erg g^{-1} s^{-1}
> 1000 erg g^{-1} s^{-1}
> 1 erg g^{-1} s^{-1}
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
$P_{rad} \approx P_{gas}$
$\epsilon_F/kT \approx 4$
mass 1.0000000
H rich 1.0000000
He core 0.0000000
C core 0.0000000
lg mdot −99.0000000
Teff 5209.310
lg L 2.7620400
lg LH −20.8823383
lg LHe −99.0000000
lg R 1.4707184
max lg T 5.4771580
lg dt yr −5.000E+00
age yr 5.000E−05
HR
log L/L$_\odot$
log Teff
Abundance
log mass fraction
m/M$_\odot$
h1
he4
o16
mg24
c12
ne20
n14
he3

The equations of stellar structure

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \varrho} \,,$$

mass continuity

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \,,$$

hydrostatic balance

$$\frac{\partial l}{\partial m} = \varepsilon_{\mathrm{n}} - \varepsilon_{\nu} - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\varrho} \frac{\partial P}{\partial t} \,,$$

energy equation

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla \,,$$

heat transport

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composition changes

<u>Equation of state</u>

$P(\rho, T, X_i)$

e.g. ideal gas $P = nk_BT$

radiation $P = \frac{1}{3}aT^4$

degenerate electrons $P = \frac{3}{5}n_eE_F = K\rho^{5/3}$

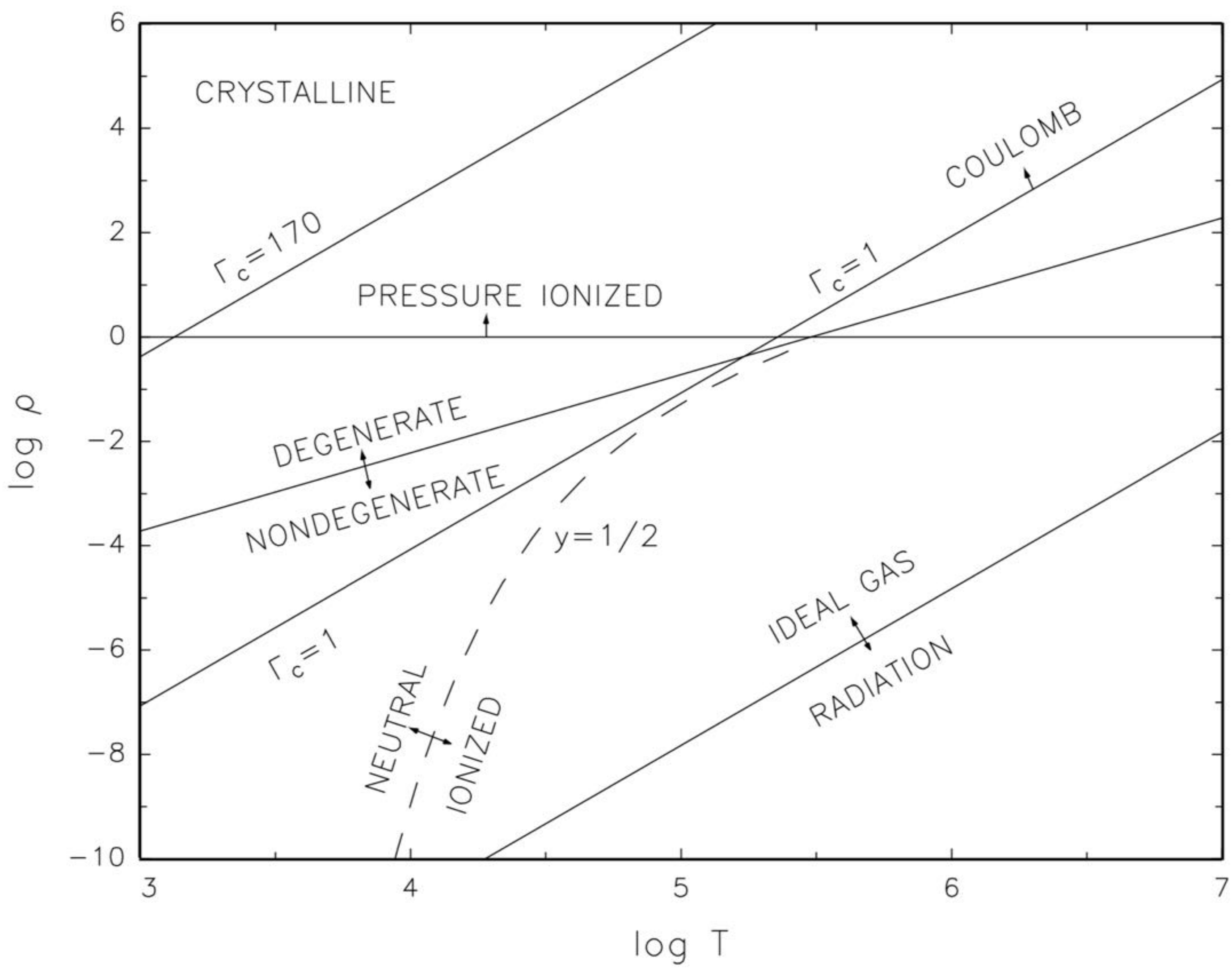
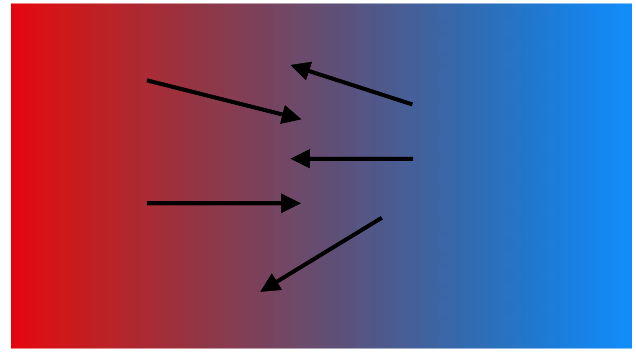


Fig 3.9 from Hansen & Kawaler

Heat transfer I: photon diffusion



mean free path $\ell \approx \dfrac{1}{n_e\sigma} = \dfrac{1}{\rho\kappa}$

Contributions to the opacity:

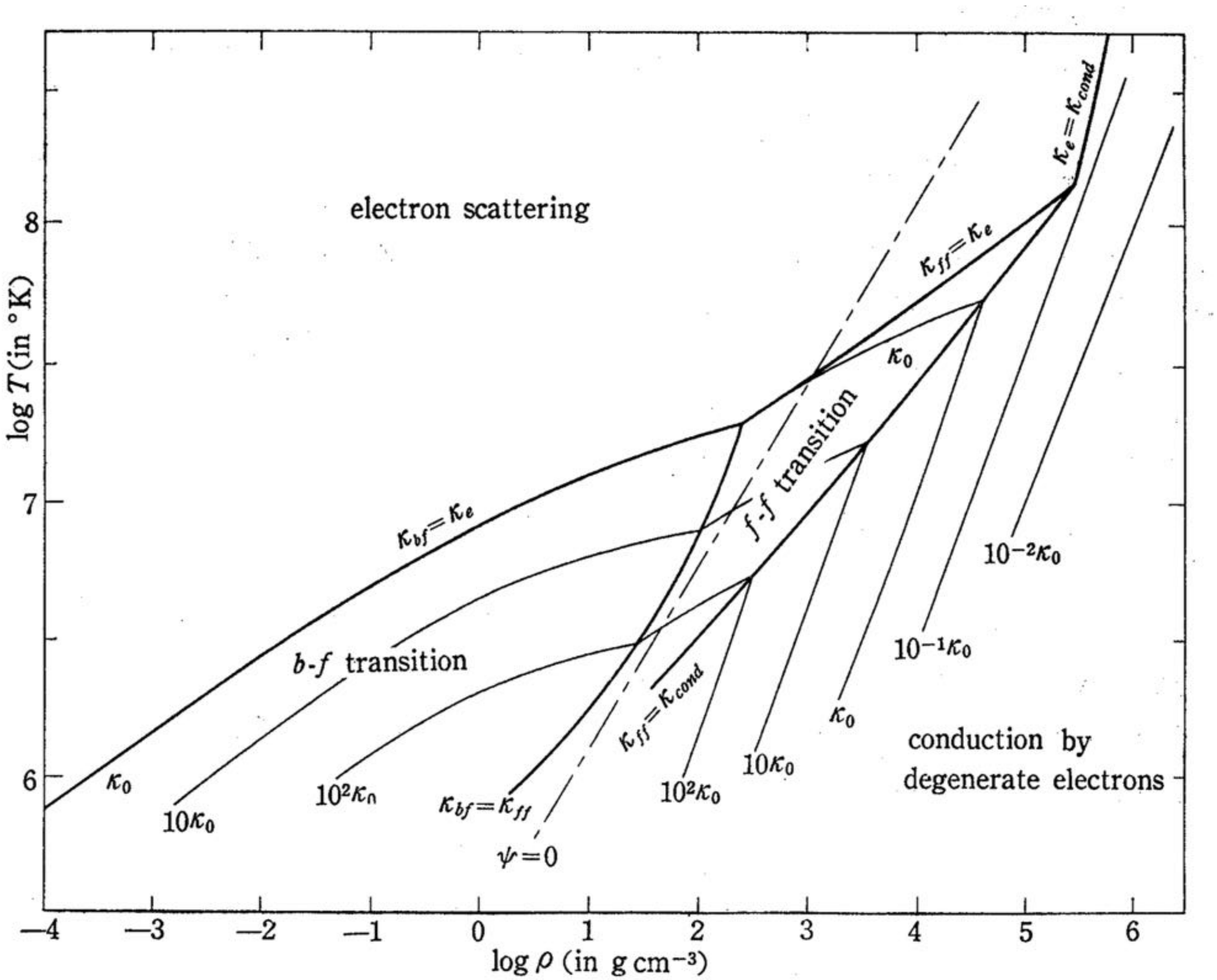
* electron scattering

(Thomson cross-section σ_T)

$$\kappa = 0.2\ \mathrm{cm}^2\ \mathrm{g}^{-1}\ (1+X)$$

* free-free and bound-free absorption

$$\kappa \propto \rho T^{-7/2}$$



Hayashi et al. (1962); reproduced in Fig 4.7 of Hansen & Kawaler

Heat transfer II: convection

* hot fluid elements can become buoyant and rise, transporting energy

* an intrinsically multi-D process!

1D model: mixing length theory

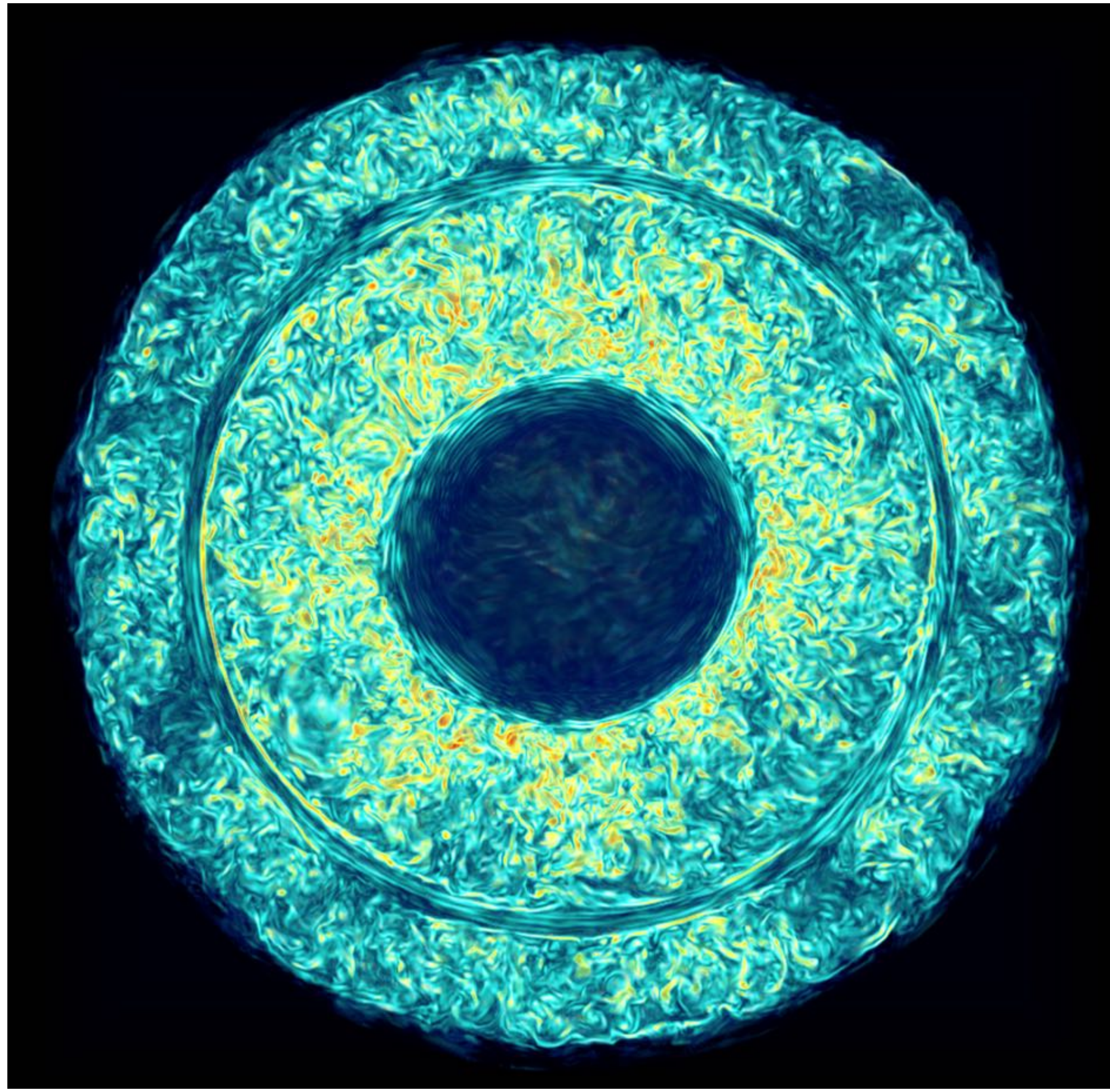
$$F_{\rm conv} \sim \rho v_{\rm conv} c_P T(\nabla - \nabla_{\rm ad})$$
$$v_{\rm conv}^2 \sim g\ell(\nabla - \nabla_{\rm ad})$$

where the mixing length ℓ is a parameter calibrated to observations

Convection turns on when the temperature gradient exceeds the adiabatic temperature gradient

$$\nabla \equiv \frac{d\ln T}{d\ln P} > \nabla_{\rm ad} \equiv \left.\frac{\partial \ln T}{\partial \ln P}\right|_S$$

In convecting region $\nabla \sim \nabla_{\rm ad}$ (effective upper limit on temperature gradient)



Clarkson et al. (2018)

Two ways to burn hydrogen

pp chains

PP–I

$$\begin{aligned}
{}^1\mathrm{H} + {}^1\mathrm{H} &\longrightarrow {}^2\mathrm{H} + \mathrm{e}^+ + \nu_\mathrm{e} \\
{}^2\mathrm{H} + {}^1\mathrm{H} &\longrightarrow {}^3\mathrm{He} + \gamma \\
\Downarrow\ {}^3\mathrm{He} + {}^3\mathrm{He} &\longrightarrow {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H}
\end{aligned}$$

–or–

PP–II

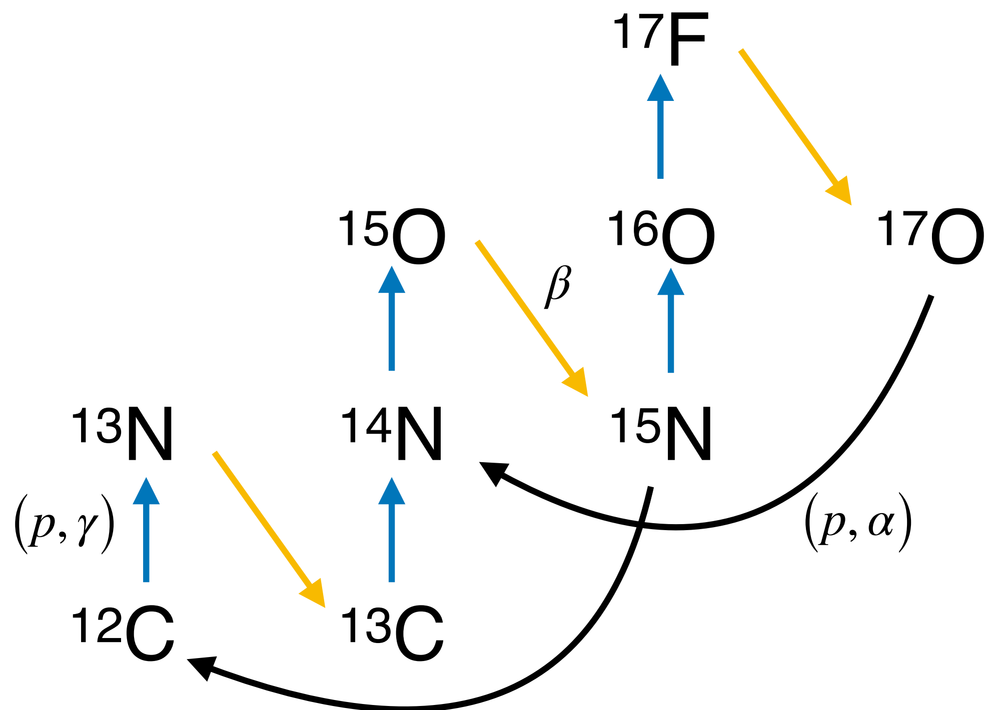
$$\begin{aligned}
{}^3\mathrm{He} + {}^4\mathrm{He} &\longrightarrow {}^7\mathrm{Be} + \gamma \\
\Downarrow\ {}^7\mathrm{Be} + \mathrm{e}^- &\longrightarrow {}^7\mathrm{Li} + \nu_\mathrm{e} (+\gamma) \\
{}^7\mathrm{Li} + {}^1\mathrm{H} &\longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}
\end{aligned}$$

–or–

PP–III

$$\begin{aligned}
{}^7\mathrm{Be} + {}^1\mathrm{H} &\longrightarrow {}^8\mathrm{B} + \gamma \\
{}^8\mathrm{B} &\longrightarrow {}^8\mathrm{Be} + \mathrm{e}^+ + \nu_\mathrm{e} \\
{}^8\mathrm{Be} &\longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}
\end{aligned}$$

CNO cycle



$$\tau\left({}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C}\right) = 863\ \mathrm{s}$$
$$\tau\left({}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N}\right) = 176\ \mathrm{s}$$

* Much easier to fuse low Z nuclei than high Z (Coulomb barrier) => need higher temperatures for the CNO cycle

* In the CNO cycle, p capture on ^{14}N is rate limiting step => C,N,O evolve to ^{14}N

Part 2 : Properties of main sequence stars

We want to compare the properties of main sequence stars with different masses

Define "main sequence" as the time when the central hydrogen abundance reaches X = 0.35 (you can use a stopping condition to stop the model at that point)

Choose a mass for the star, and run MESA until the star reaches X=0.35 at the centre.

Record your results in the google sheet. Fill in as many of the columns as you can: (all evaluated when the star reaches X=0.35)

* The age of the star
* The luminosity (in solar luminosities)
* The central temperature
* The mass of the convective core
* The opacity at the centre
* The ratio of luminosity from pp burning to CNO burning

https://docs.google.com/spreadsheets/d/1glyFBOLwJfaERpJhPoYSuVgFe_-3jwf0doFIAL0sY2Q/edit?usp=sharing

MESA output

`LOGS/history.data`

One line per model with information such as the age, luminosity, mass etc. so you can see how things change over time

`LOGS/profileN.data`

One file per model with the internal structure of each model, e.g. composition profile, temperature profile etc.

Adding columns

Let’s add opacity to the output for each model

First copy the file that tells MESA which columns to output into your work directory:

```
cp $MESA_DIR/star/defaults/profile_columns.list .
```

If you look in the file, you will see many options for things to output. Uncomment this line:

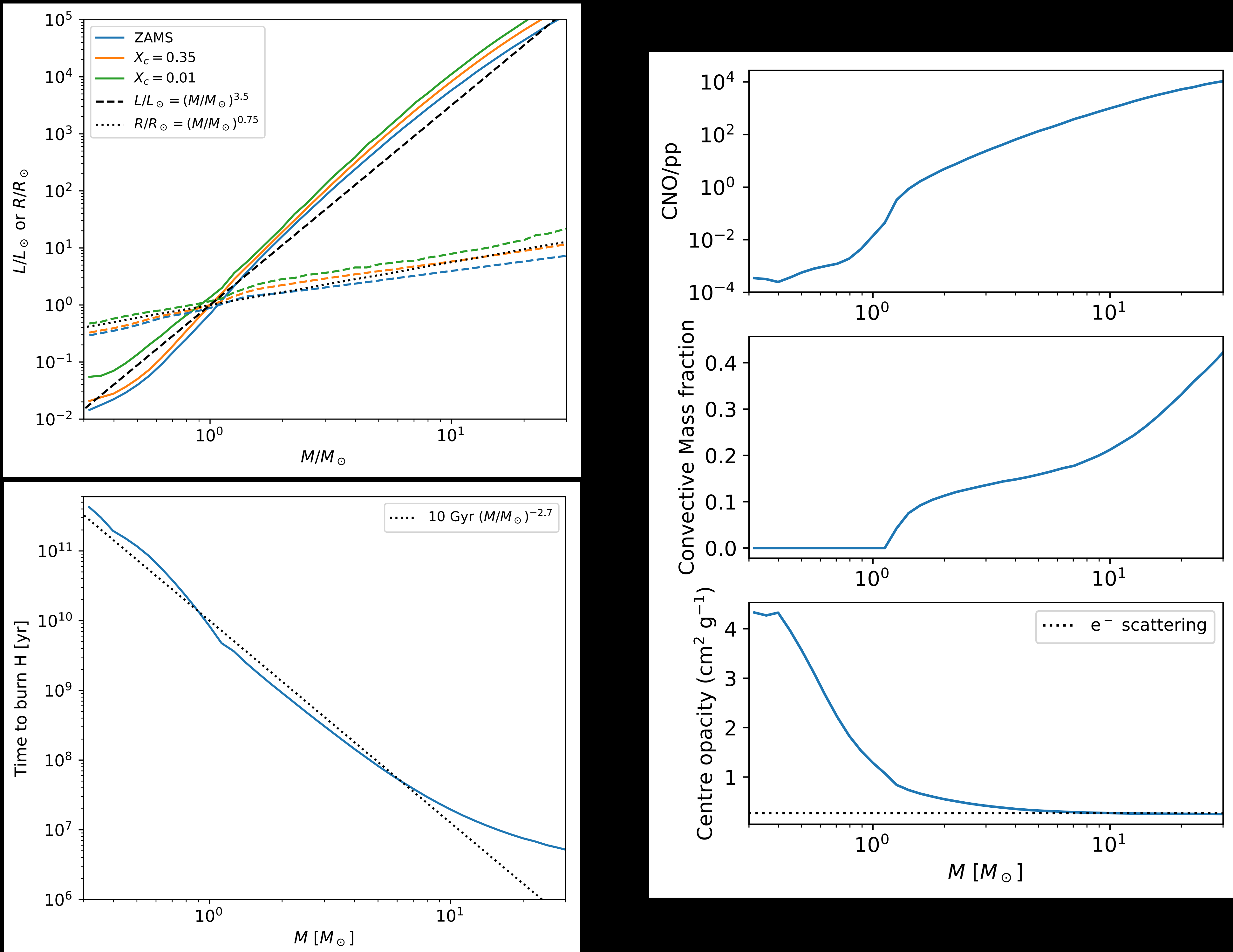
```
opacity ! opacity measured at center of zone
```

Rerun MESA and you should find the profile files have an extra column showing you the opacity at each location in the star

(To add columns to the history, use `history_columns.list`)

You can find my results for different masses here:

http://45.56.103.199/CRAQ_summer_school_2019/


ZAMS
$X_c = 0.35$
$X_c = 0.01$
$L/L_\odot = (M/M_\odot)^{3.5}$
$R/R_\odot = (M/M_\odot)^{0.75}$
$L/L_\odot$ or $R/R_\odot$
$M/M_\odot$
10 Gyr $(M/M_\odot)^{-2.7}$
Time to burn H [yr]
M [$M_\odot$]
CNO/pp
Convective Mass fraction
e^- scattering
Centre opacity (cm^2 g^{-1})
M [$M_\odot$]

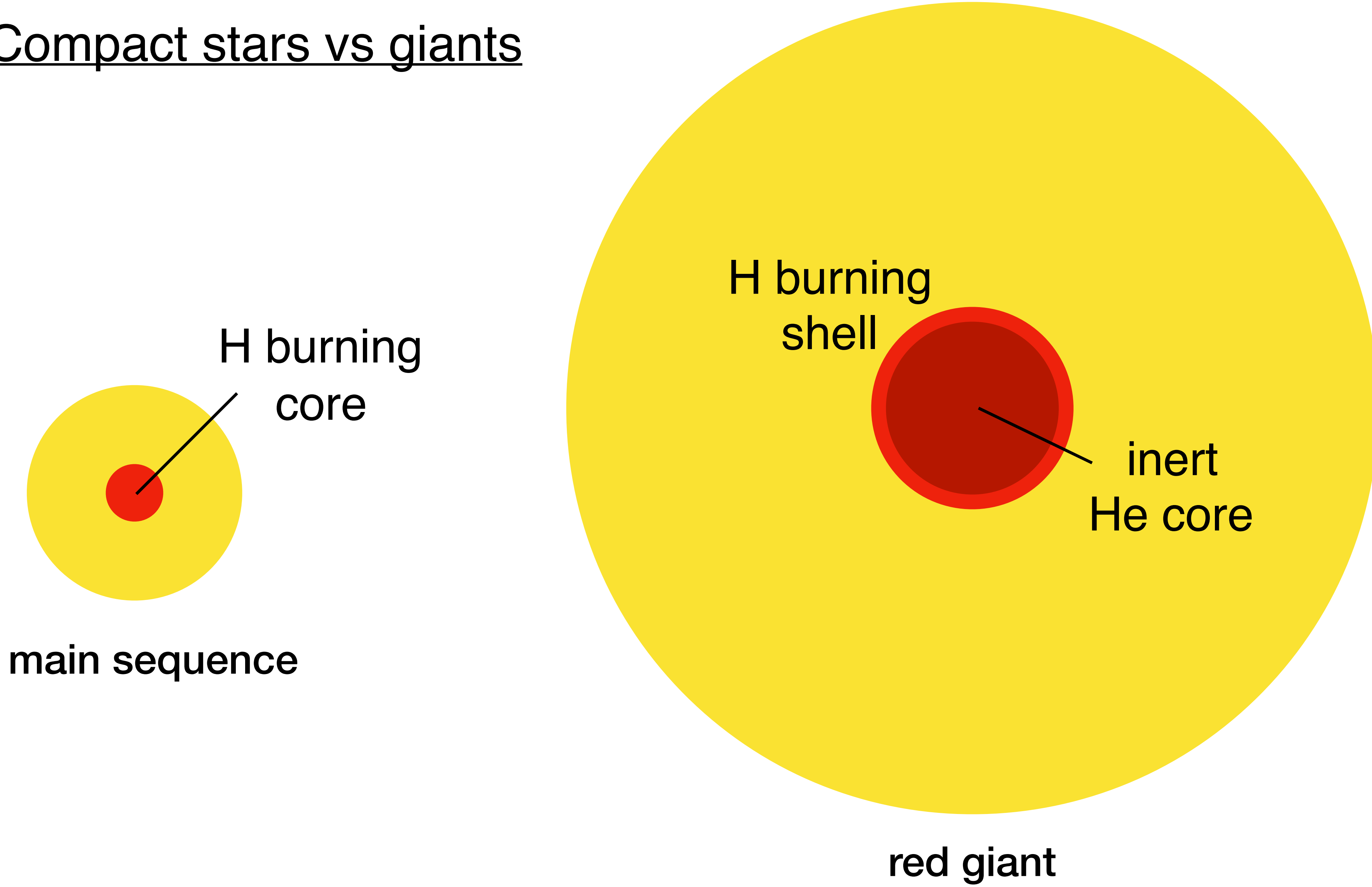
Low mass vs high mass stars

A few different aspects of stellar physics change at about the mass of the Sun:

Low mass (< solar mass)	**High mass (> solar mass)**
free-free opacity	electron scattering opacity
pp chain	CNO cycle
radiative core, surface convection zone	convective core, radiative envelope

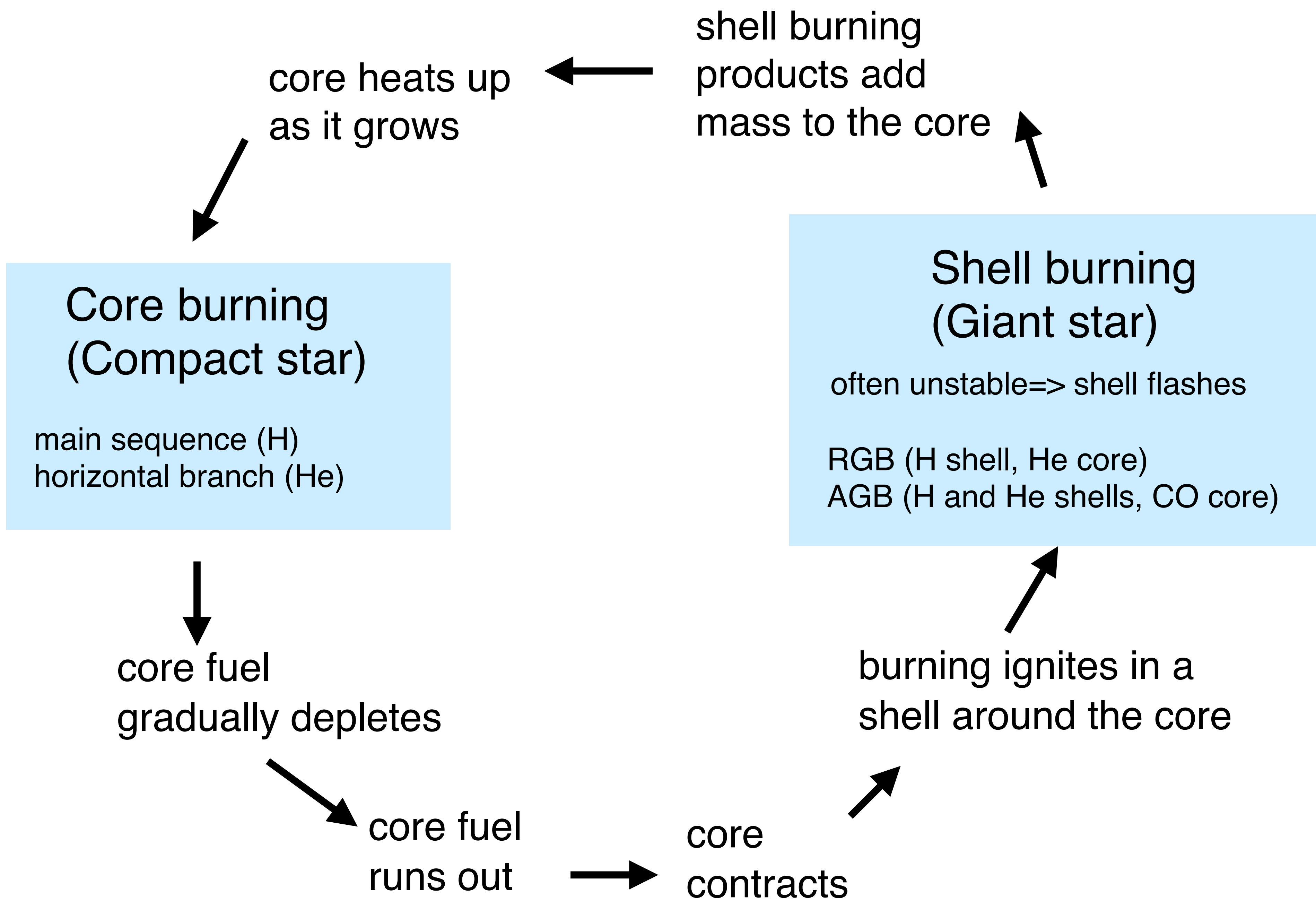
Part 3: Beyond the main sequence

Compact stars vs giants



Moving the burning source off-centre drives the star into a giant configuration

Compact stars vs giants



Helium burning

Once we leave the main sequence, we need to worry about He burning

- He burns by the **triple alpha reaction** $3\alpha \Rightarrow {}^{12}\mathrm{C}$
- Requires temperatures of ~10^8K
- Further alpha captures make ${}^{16}\mathrm{O}, {}^{20}\mathrm{Ne}, {}^{22}\mathrm{Mg}$

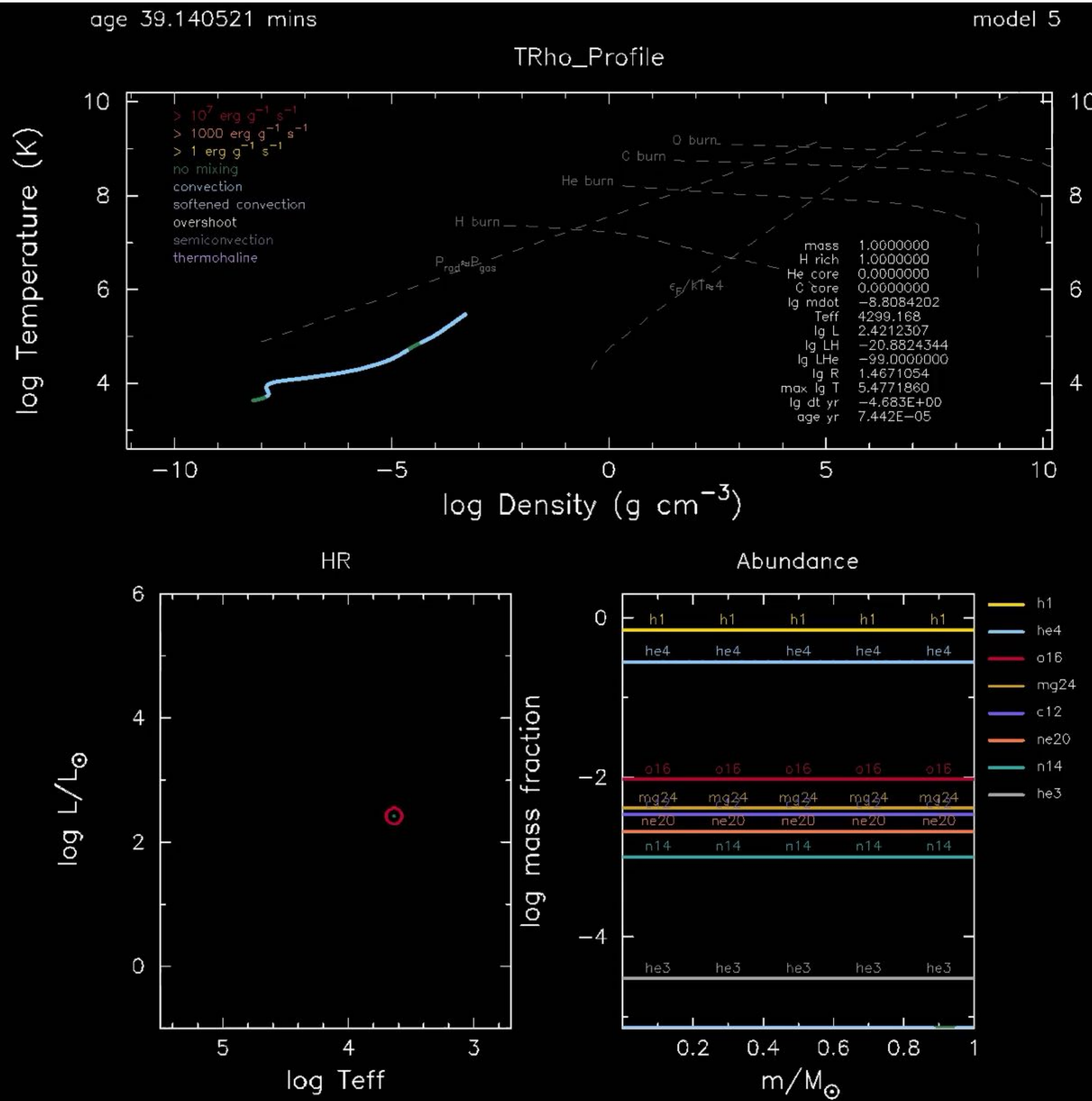
More advanced burning beyond oxygen happens in massive stars (see lectures this afternoon)

To run your model beyond the main sequence, you just need to remove the stopping condition so that it keeps going

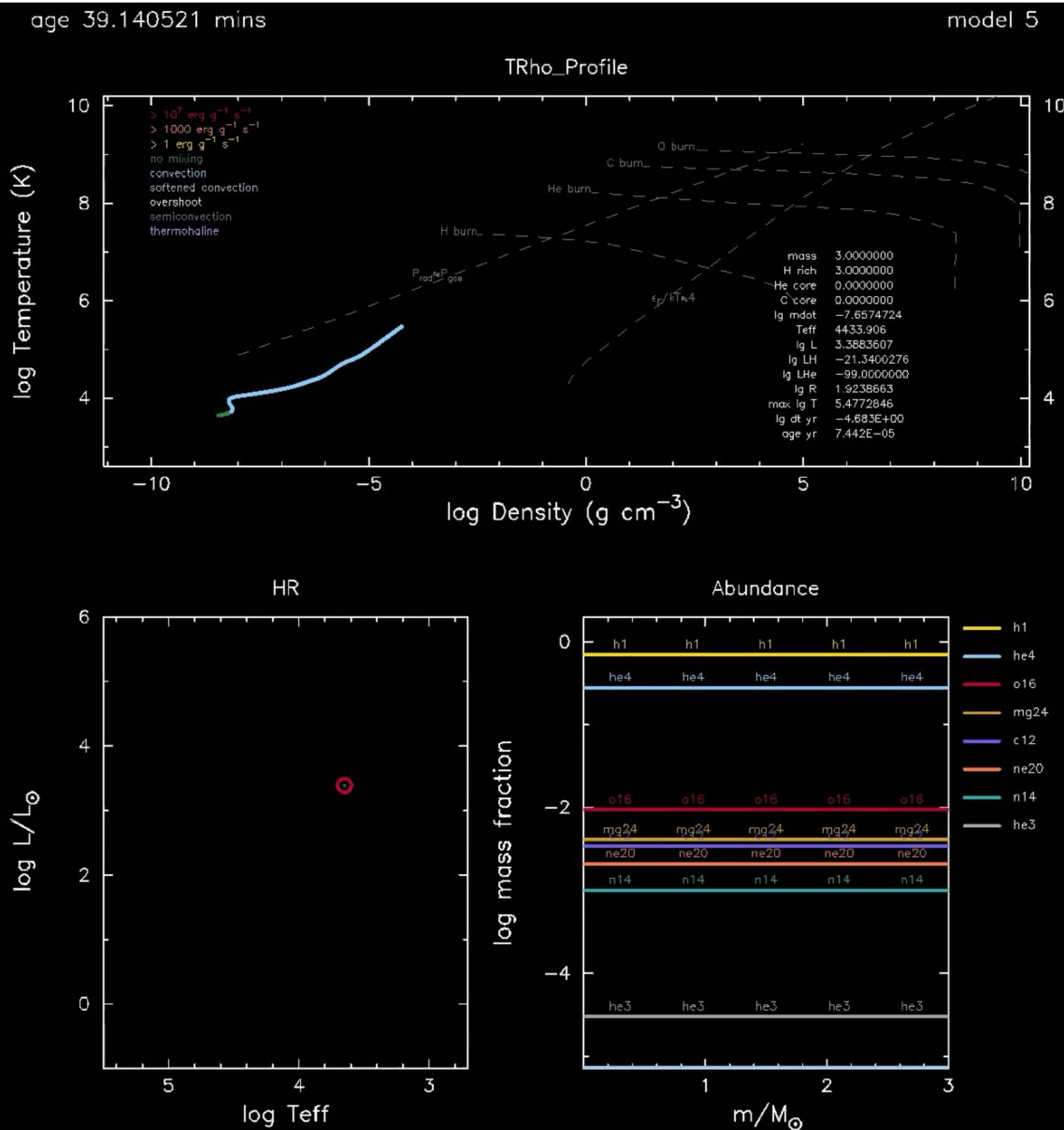
There is one more piece of physics to add to the `inlist_project`, which is mass loss

```
cool_wind_full_on_T = 9.99d9
hot_wind_full_on_T = 1d10
cool_wind_RGB_scheme = 'Reimers'
cool_wind_AGB_scheme = 'Blocker'
RGB_to_AGB_wind_switch = 1d-4
Reimers_scaling_factor = 0.8d0
Blocker_scaling_factor = 0.7d0
```

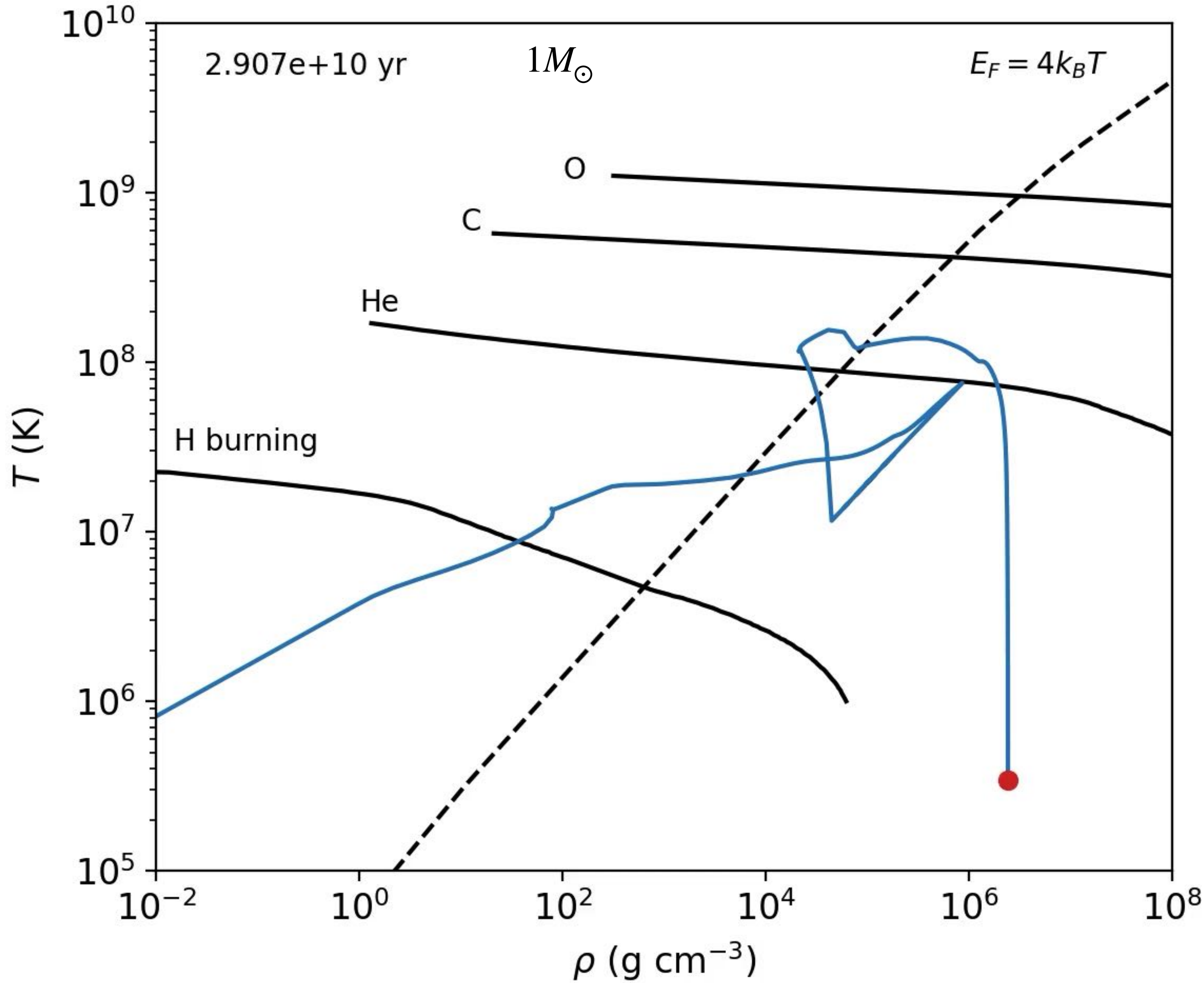
(I copied these from the MESA test_suite `1M_pre_ms_to_wd`)



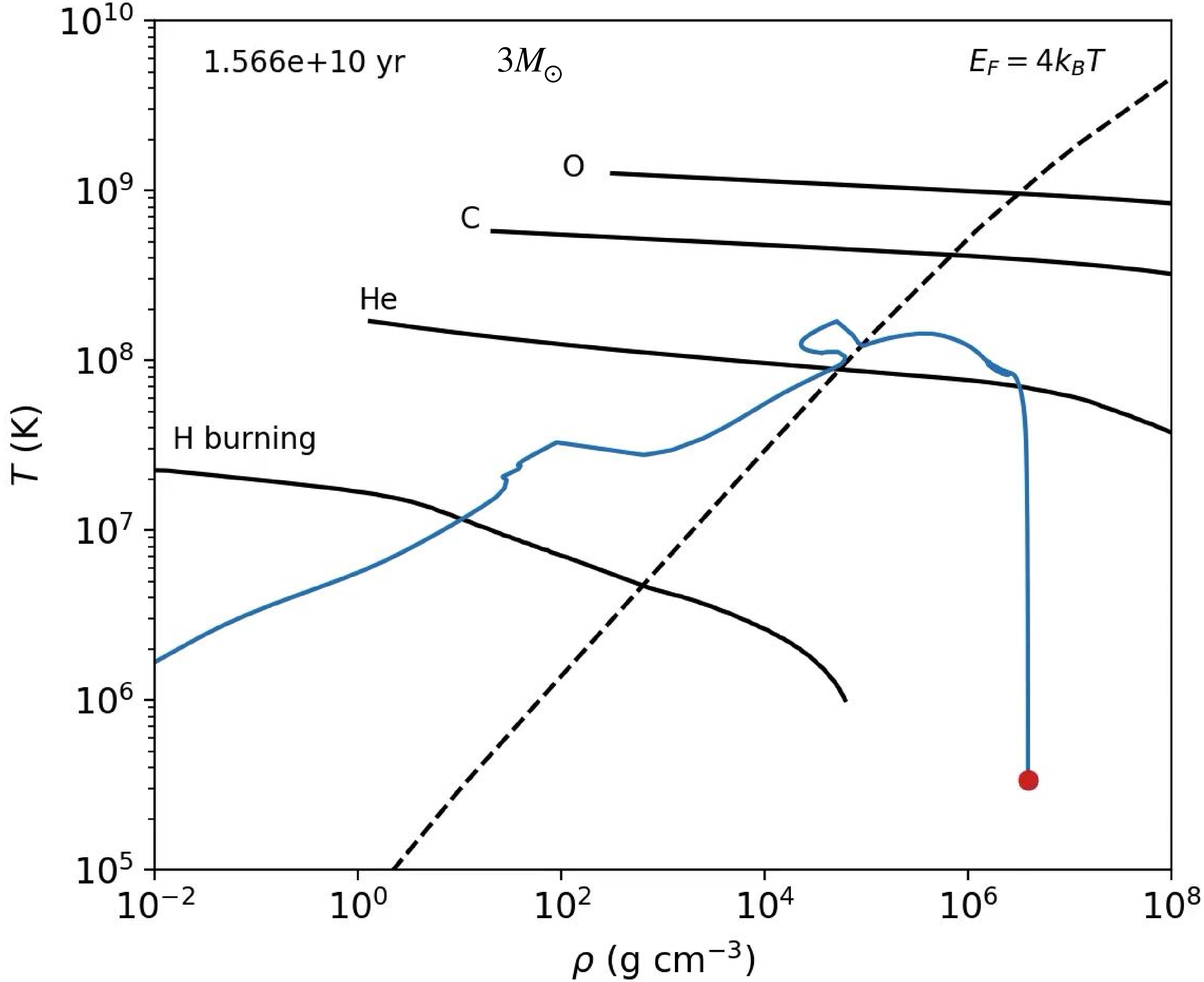
age 39.140521 mins
model 5
TRho_Profile
> 10^7 erg g^-1 s^-1
> 1000 erg g^-1 s^-1
> 1 erg g^-1 s^-1
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
P_rad≈P_gas
ε_F/kT≈4
mass 1.0000000
H rich 1.0000000
He core 0.0000000
C core 0.0000000
lg mdot −8.8084202
Teff 4299.168
lg L 2.4212307
lg LH −20.8824344
lg LHe −99.0000000
lg R 1.4671054
max lg T 5.4771860
lg dt yr −4.683E+00
age yr 7.442E−05
log Temperature (K)
log Density (g cm^−3)
HR
log L/L⊙
log Teff
Abundance
h1
he4
o16
mg24
c12
ne20
n14
he3
log mass fraction
m/M⊙


age 39.140521 mins
model 5
TRho_Profile
log Temperature (K)
log Density (g cm^-3)
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
mass 3.0000000
H rich 3.0000000
He core 0.0000000
C core 0.0000000
lg mdot −7.6574724
Teff 4433.906
lg L 3.3883607
lg LH −21.3400276
lg LHe −99.0000000
lg R 1.9238663
max lg T 5.4772846
lg dt yr −4.683E+00
age yr 7.442E−05
HR
log L/L⊙
log Teff
Abundance
log mass fraction
m/M⊙
h1
he4
o16
mg24
c12
ne20
n14
he3

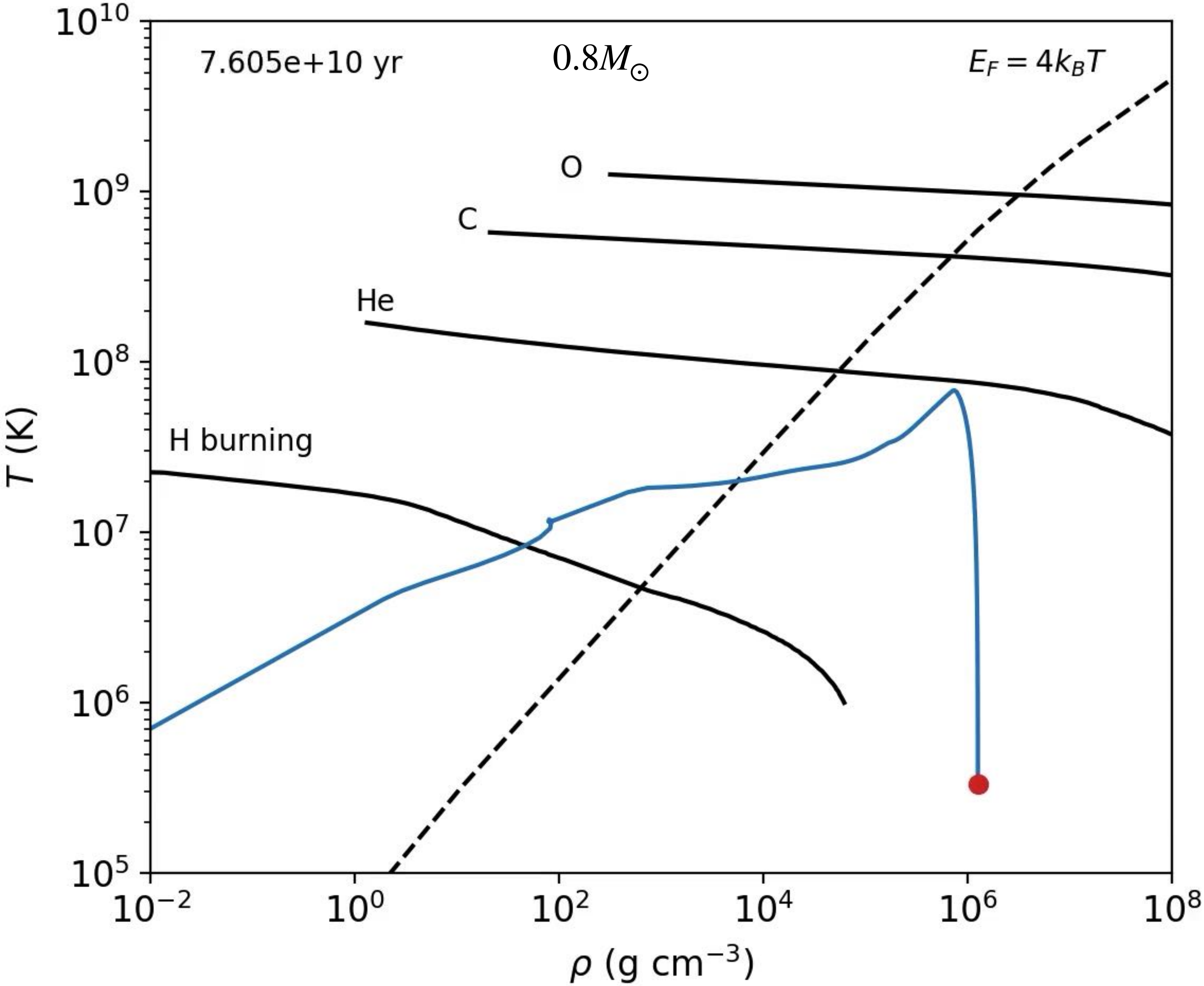
Tracks in central density and temperature



Tracks in central density and temperature



Tracks in central density and temperature



Going further with MESA (what we didn't talk about)

- The test suite as a resource
- Convergence studies
- Things you can change using the `inlist`

  Examples are:
  - the metallicity of the star
  - changing to a different nuclear network, e.g. more isotopes for advanced burning
  - change the atmospheric boundary conditions
  - turn on accretion, rotational mixing, diffusion, hydrodynamics

  The MESA website is a good place to browse the different options

- Using `run_star_extras` to extend MESA

  Examples are:
  - implementing a more precise stopping condition (e.g. you need to stop the code when X=0.35 *precisely*)
  - adding your own opacity, mass loss prescription
  - outputting custom information to the history or profile output files

When in doubt, email the user list! You will get a quick answer..

ls $MESA_DIR/star/test_suite/

```
Andrews-Mac-mini:evolve_test cumming$ ls /Applications/mesa/star/test_suite/
1.3M_ms_high_Z                 custom_rates                   irradiated_planet                          rsp_Type_II_Cepheid
1.4M_ms_op_mono                debugging_stuff_for_inlists    list_tests                                 rsp_check_2nd_crossing
1.5M_with_diffusion            det_riemann                    low_z                                      rsp_gyre
15M_dynamo                     diffusion_smoothness           magnetic_braking                           rsp_save_and_load_file
16M_conv_premix                do1_rsp_test_source            make_brown_dwarf                           sample_he_zams
16M_predictive_mix             do1_test_source                make_co_wd                                 sample_pre_ms
1M_pre_ms_to_wd                each_rsp_test_clean            make_he_wd                                 sample_zams
1M_thermohaline                each_rsp_test_run              make_low_mass_with_uniform_composition     sedov_omega_1
1M_thermohaline_split_mix      each_rsp_test_run_and_diff     make_metals                                semiconvection
25M_pre_ms_to_core_collapse    each_test_clean                make_o_ne_wd                               sewind
25M_z2m2_high_rotation         each_test_do                   make_planets                               simplex_solar_calibration
5M_cepheid_blue_loop           each_test_run                  make_sdb                                   split_burn_20M_si_burn_qp
7M_prems_to_AGB                each_test_run_and_diff         mesa_dir.rb                                split_burn_big_net_30M
8.8M_urca                      each_test_up_final             multimass                                  split_burn_big_net_30M_logT_9.8
README                         envelope_inflation             multiple_stars                             surface_effects
accreted_material_j            example_astero                 neutron_star_envelope                      test_case_template
accretion_with_diffusion       example_ccsn_IIp               noh_riemann                                test_case_template_for_bill
adjust_net                     example_make_pre_ccsn          nova                                       test_memory
agb                            gyre_in_mesa_bcep              ns_c                                       test_memory2
agb_to_wd                      gyre_in_mesa_envelope          ns_h                                       timing
astero_adipls                  gyre_in_mesa_ms                ns_he                                      very_low_mass
astero_gyre                    gyre_in_mesa_rsg               other_physics_hooks                        wd
axion_cooling                  gyre_in_mesa_spb               ppisn                                      wd2
black_hole                     gyre_in_mesa_wd                pre_zahb                                   wd3
brown_dwarf                    hb_2M                          profile_mesa                               wd_acc_small_dm
build_and_run                  he_core_flash                  radiative_levitation                       wd_aic
c13_pocket                     high_mass                      relax_composition_j_entropy                wd_cool
cburn_inward                   high_rot_darkening             report                                     wd_cool_0.6M
conductive_flame               high_z                         rsp_BEP                                    wd_diffusion
conserve_angular_momentum      hot_cool_wind                  rsp_BLAP                                   wd_ignite
count_tests                    hse_riemann                    rsp_Cepheid                                wd_surf_at_tau_1m4
create_zams                    hydro_Ttau_evolve              rsp_Delta_Scuti
custom_colors                  hydro_Ttau_solar               rsp_RR_Lyrae
```

MESA resources

MESA webpage: **http://mesa.sourceforge.net/index.html**

MESA Marketplace: **http://cococubed.asu.edu/mesa_market/**

MESA Summer schools:

http://cococubed.asu.edu/mesa_summer_school_2019/index.html

"Beyond inlists"

https://jschwab.github.io/mesa-2018/

The mailing list `mesa-users@lists.mesastar.org`

The MESA "instrument papers"
Paxton et al. (2011,2013,2015,2018,2019)

https://ui.adsabs.harvard.edu/abs/2019arXiv190301426P/abstract