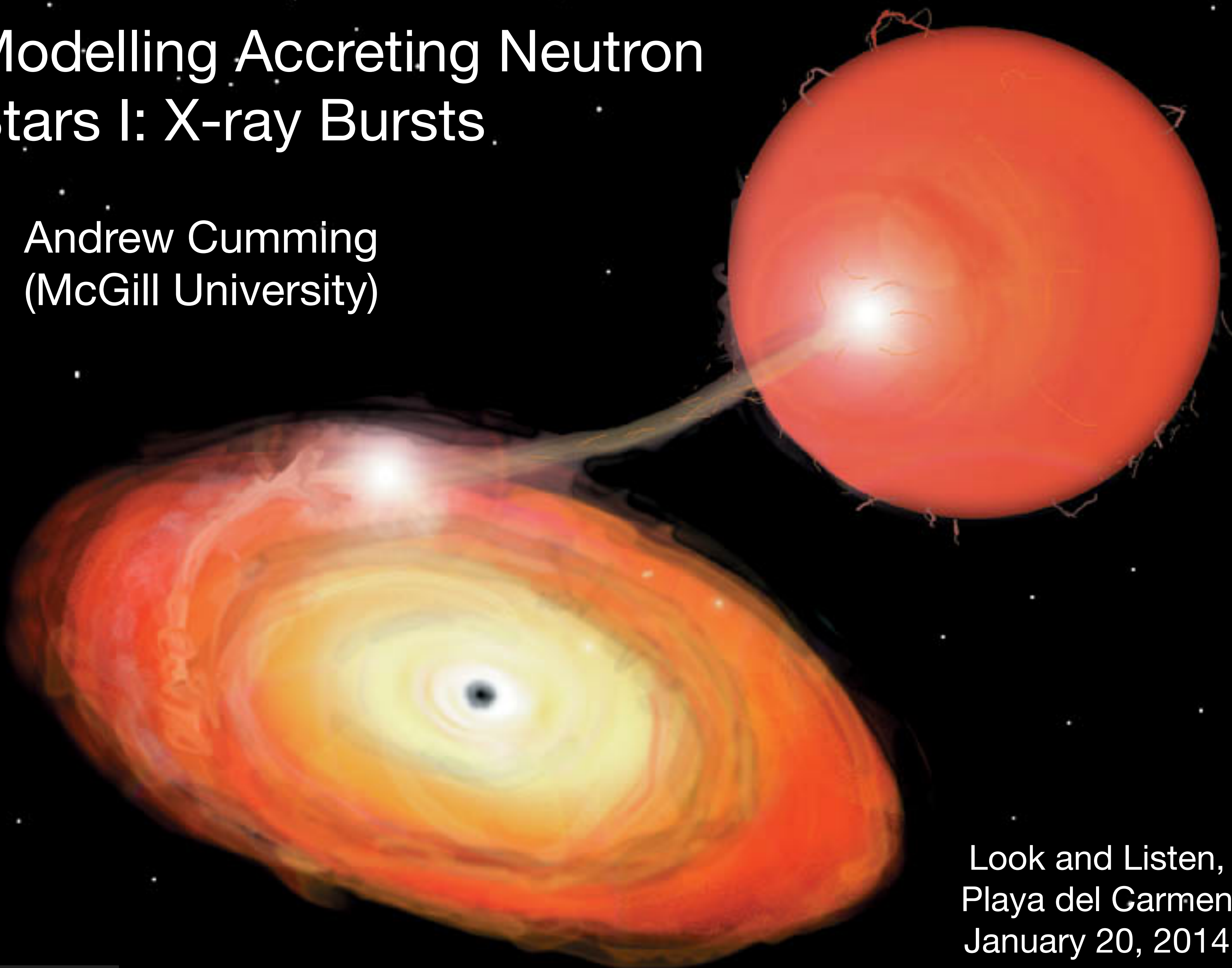
Modelling Accreting Neutron Stars I: X-ray Bursts

Andrew Cumming
(McGill University)



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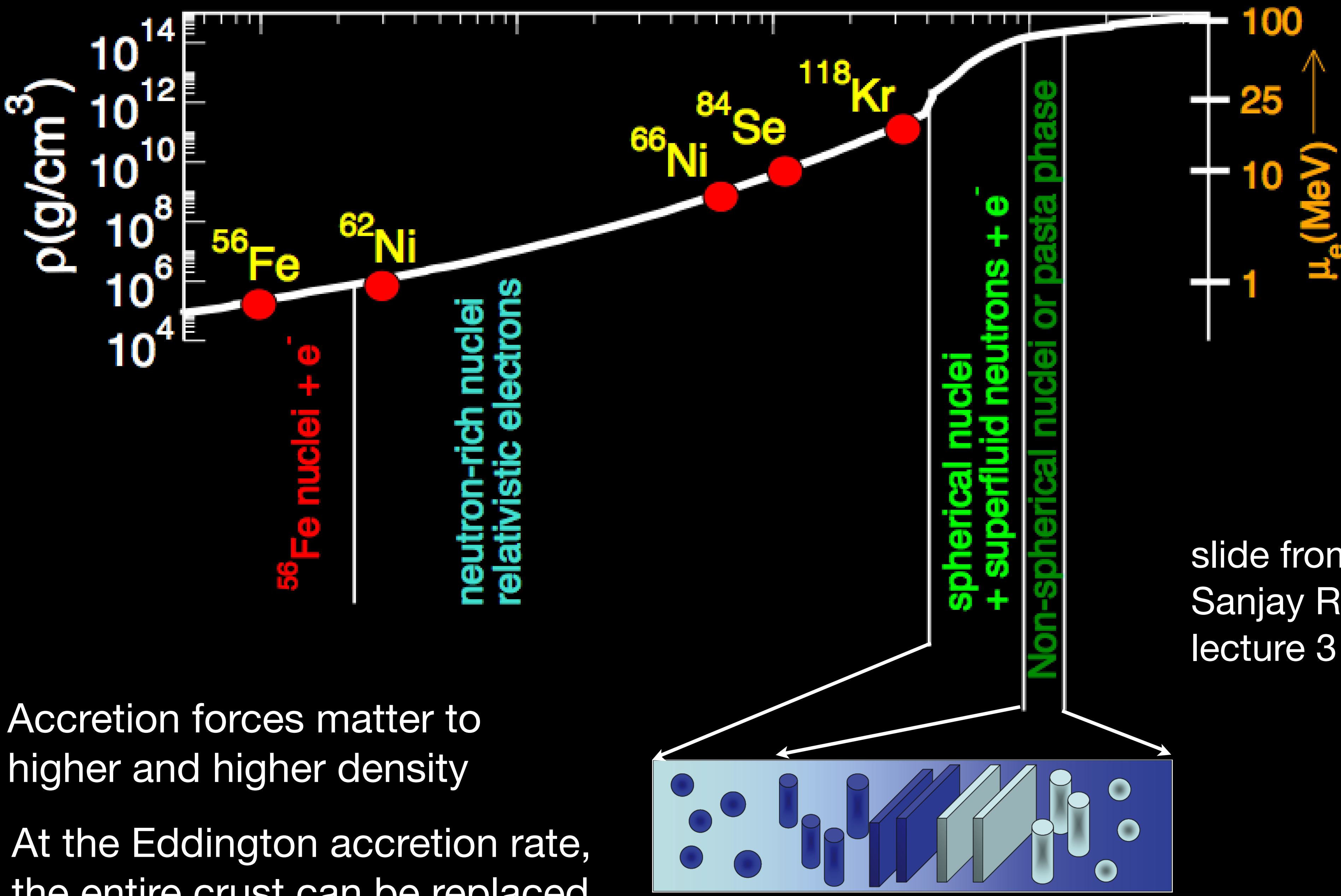
Look and Listen,
Playa del Carmen
January 20, 2014

Transformations of matter at high density



slide from
Sanjay Reddy's
lecture 3

Accretion forces matter to higher and higher density

At the Eddington accretion rate, the entire crust can be replaced in 1 Myr $(10^{-8}\ M_{\odot}\ yr^{-1})$

Why is it interesting to study accreting neutron stars?

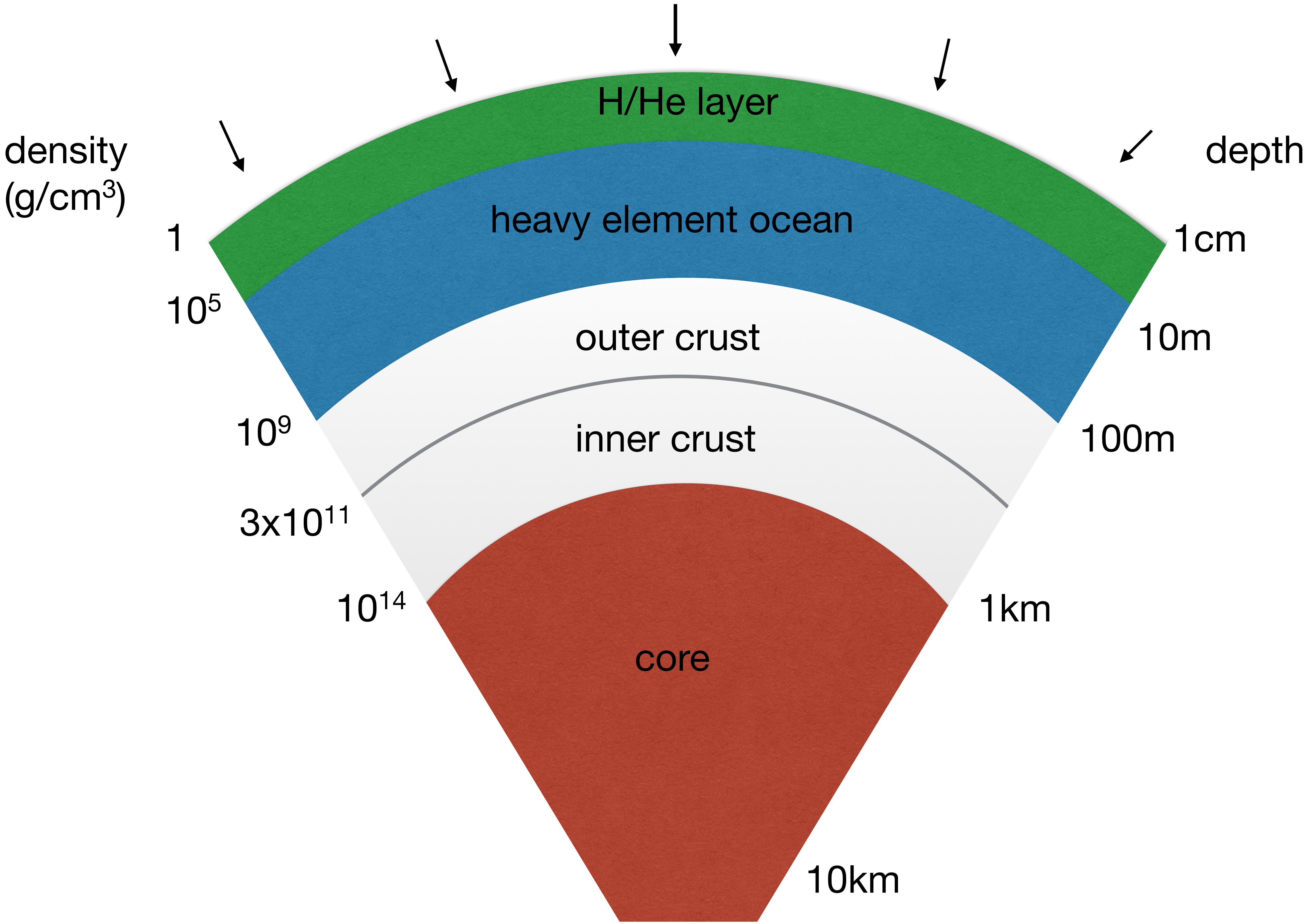
- They can be studied! X-ray binaries provide opportunities to study the neutron star directly: either during thermonuclear flashes, or in quiescence

- Matter arrives on the surface as light elements (H/He) and is slowly compressed to nuclear density. In doing so, it covers the entire nuclear chart from the proton drip line to the neutron drip line.

- Many stellar physics processes are accessible on observable timescales. These include: shell flashes, angular momentum transport and meridional circulation, shell flashes, solidification and chemical separation, burning front propagation

- Studying what happens to matter that hits the surface can tell us about the binary, e.g. the composition of the transferred material, or the geometry of the accretion flow

The journey of a fluid element



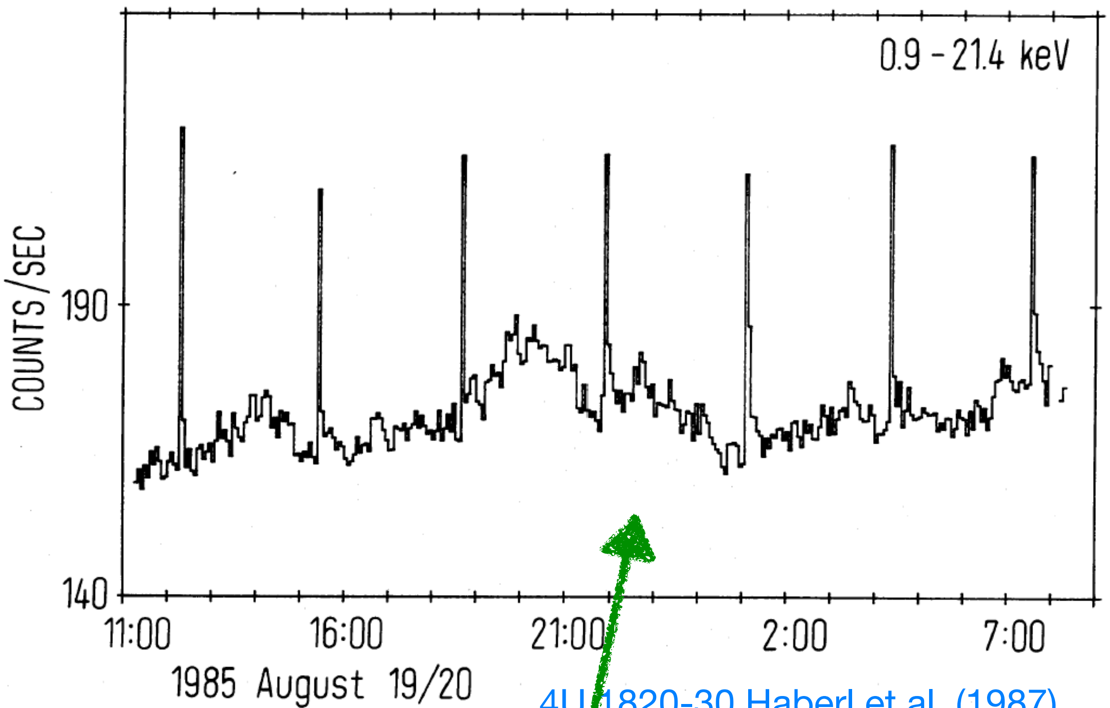
Today’s lecture

- Discuss the basic physics underlying Type I X-ray bursts
- Build a simple time-dependent model

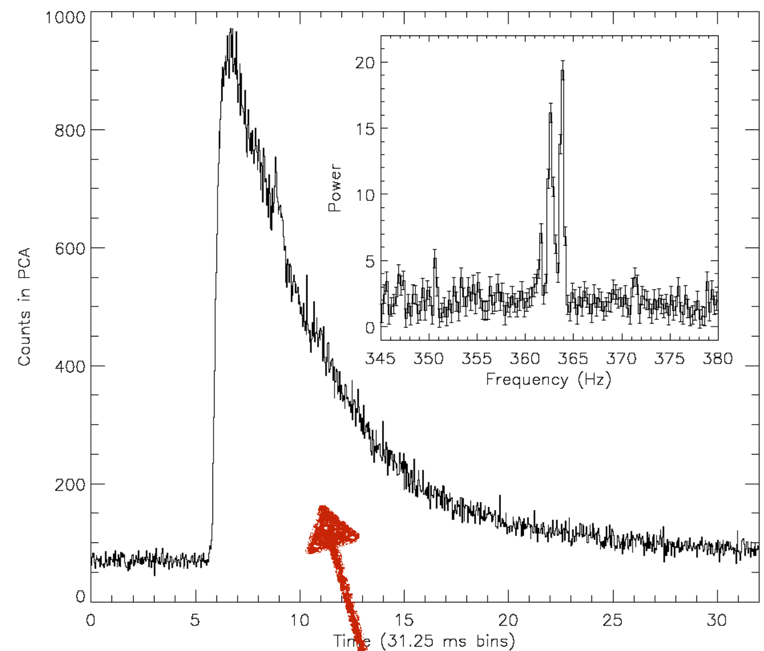
Next time we will go deeper into the star, continuing our journey into the crust

In all of these lectures we will revisit ideas from last week

Basic energetics of bursts



4U 1820-30 Haberl et al. (1987)



4U 1728-34 Strohmayer et al. (1996)

gravitational energy release

$$\frac{GM}{R} \approx 200 \text{ MeV per nucleon}$$

nuclear energy release

$$Q_{\rm nuc} \approx (1-5) \text{ MeV per nucleon}$$

Their ratio is $\alpha \equiv \dfrac{\int F_p\, dt}{\int F_b\, dt} \approx \dfrac{GM/R}{Q_{\rm nuc}} \approx (40-100)$

Wait a minute, Type I X-ray bursts were discovered 40 years ago - why are they interesting to study now?

- New types of burning discovered only in the last decade or so
 e.g. mHz quasi-periodic oscillations
superbursts, intermediate duration bursts

- Large catalogs of bursts
e.g. RXTE catalog >1100 bursts Galloway et al.

- Interface with nuclear experiment, specifically radioactive ion beams
e.g. FRIB facility being built at MSU

- Lots of work in recent years trying to use X-ray bursts to measure neutron star radius

- They determine the composition of the heavy ashes and therefore the outer crust - important for crust heating and cooling

- We don’t really understand them — major puzzles remain

What is a thin shell flash?

In a thin layer, the pressure is fixed by the weight of overlying layers

$$P = g\Sigma$$

pressure

gravity $\dfrac{GM}{R^2}$

column depth
(g cm^2)

(an example of
hydrostatic balance)

Exercise: check this works for Earth's atmosphere by estimating the column of air above your head

What is a thin shell flash?

The entropy equation is then

no dP term (constant pressure)

$$TdS = dU + PdV = c_P dT$$

$$c_P \frac{dT}{dt} = \epsilon_{\rm heat} - \epsilon_{\rm cool}$$

e.g. ideal gas

$$c_P = \frac{5}{2}\frac{k_B}{\mu m_p}$$

heating by nuclear reactions (erg g^{-1} s^{-1})

cooling of the layer (heat transport to the surface)

What is a thin shell flash?

Perturb the entropy equation $T \rightarrow T + \delta T$ (with pressure held constant)

$$c_P \frac{\partial \delta T}{\partial t} = \delta T \left[\frac{d\epsilon_{\rm heat}}{dT} - \frac{d\epsilon_{\rm cool}}{dT} \right]$$

We will get a thermal runaway if

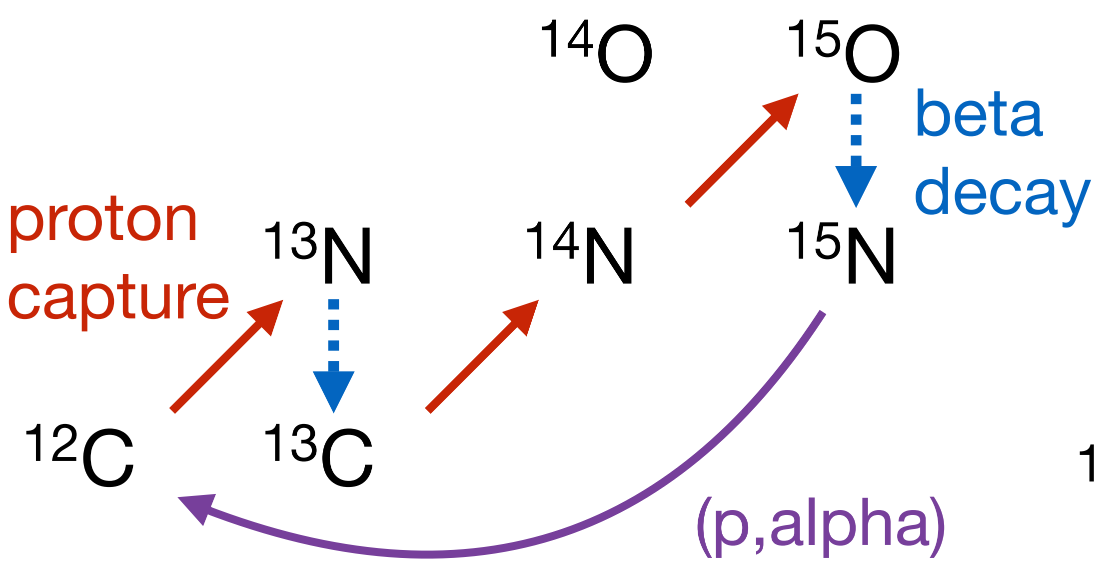
$$\boxed{\frac{d\epsilon_{\rm heat}}{dT} > \frac{d\epsilon_{\rm cool}}{dT}}$$

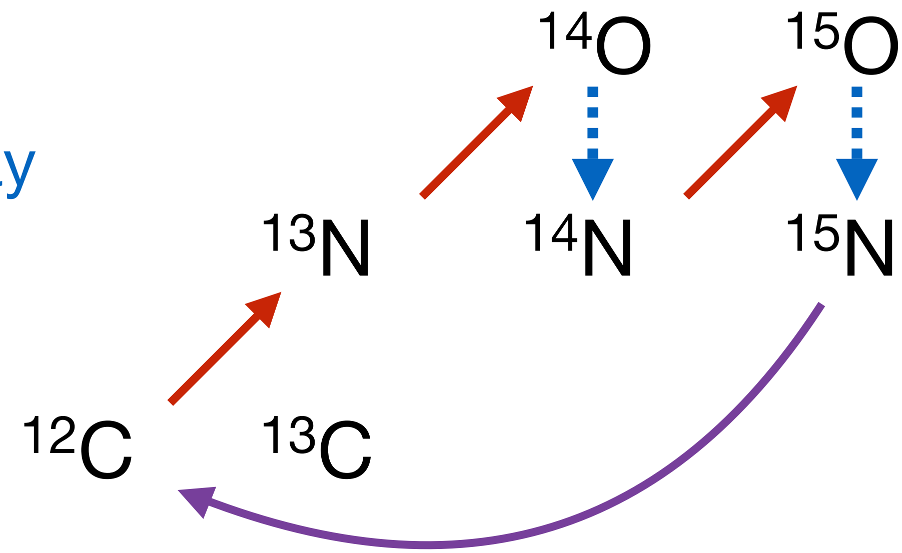
in general this is satisfied because nuclear reactions are extremely temperature sensitive!

Question: Is the Sun thermally stable? Could a whole star become thermally unstable? What conditions would be required?

Examples: Type I X-ray bursts; classical novae; helium shell flashes in AGB stars

How hydrogen burns on an accreting neutron star





main sequence stars

$T < 8 \times 10^7$ K

(cold) CNO cycle

accreting neutron stars

$T > 8 \times 10^7$ K

hot CNO cycle

(^{14}O and ^{15}O beta decay half-lives are 71 and 122s respectively)

$$\epsilon_H = 5.8 \times 10^{13} \left(\frac{Z_{\rm CNO}}{0.01} \right) \ {\rm ergs\ g^{-1}\ s^{-1}}$$

Helium burning by the triple alpha reaction

$$3\alpha \rightarrow^{12} \mathrm{C}$$

energy generation rate

$$\epsilon_{3\alpha} = 5.3 \times 10^{21} \text{ ergs g}^{-1} \text{ s}^{-1} f \frac{\rho_5^2 Y^3}{T_8^3} \exp\left(\frac{-44}{T_8}\right)$$

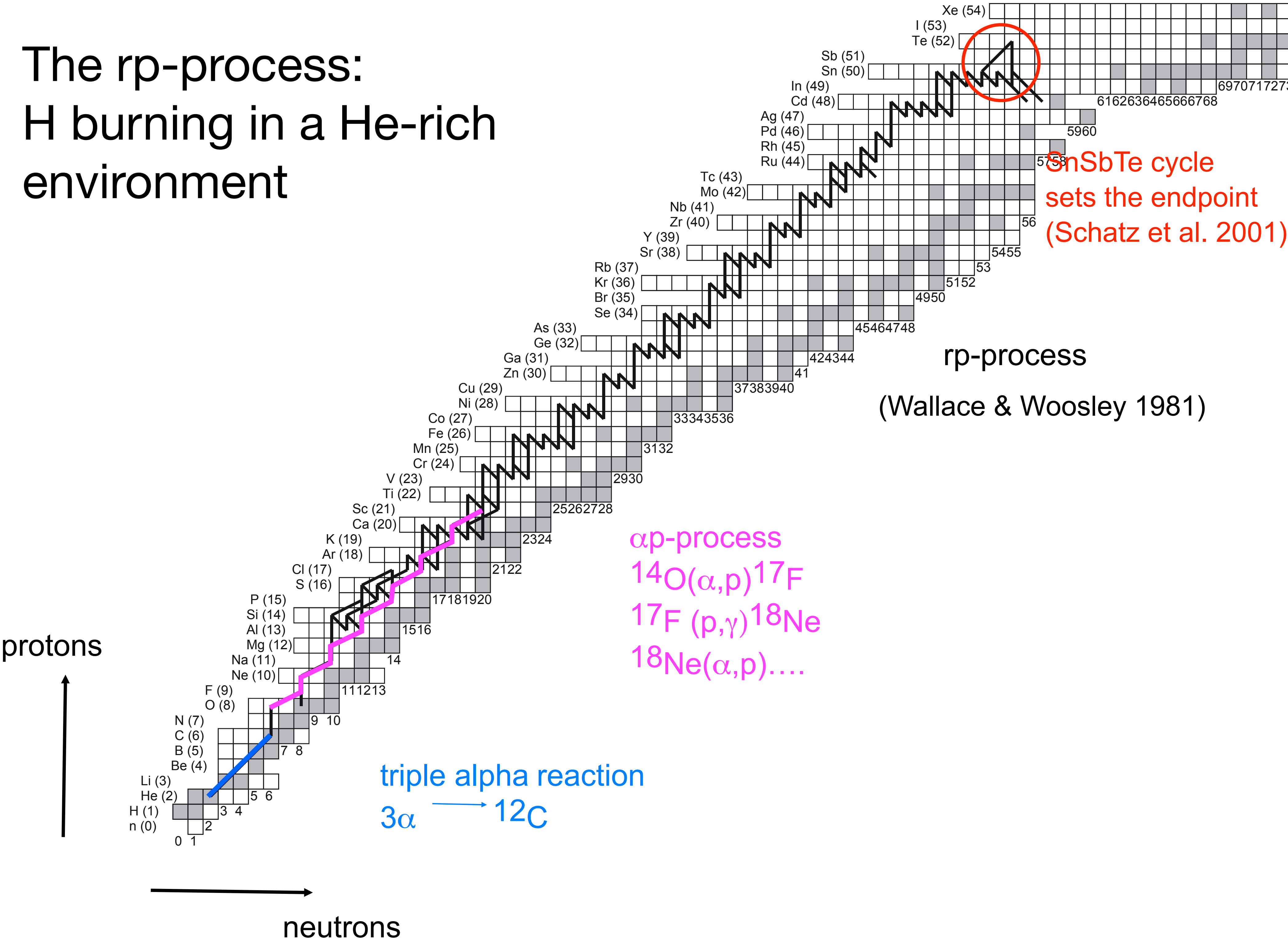
Fushiki & Lamb 1987

temperature sensitivity

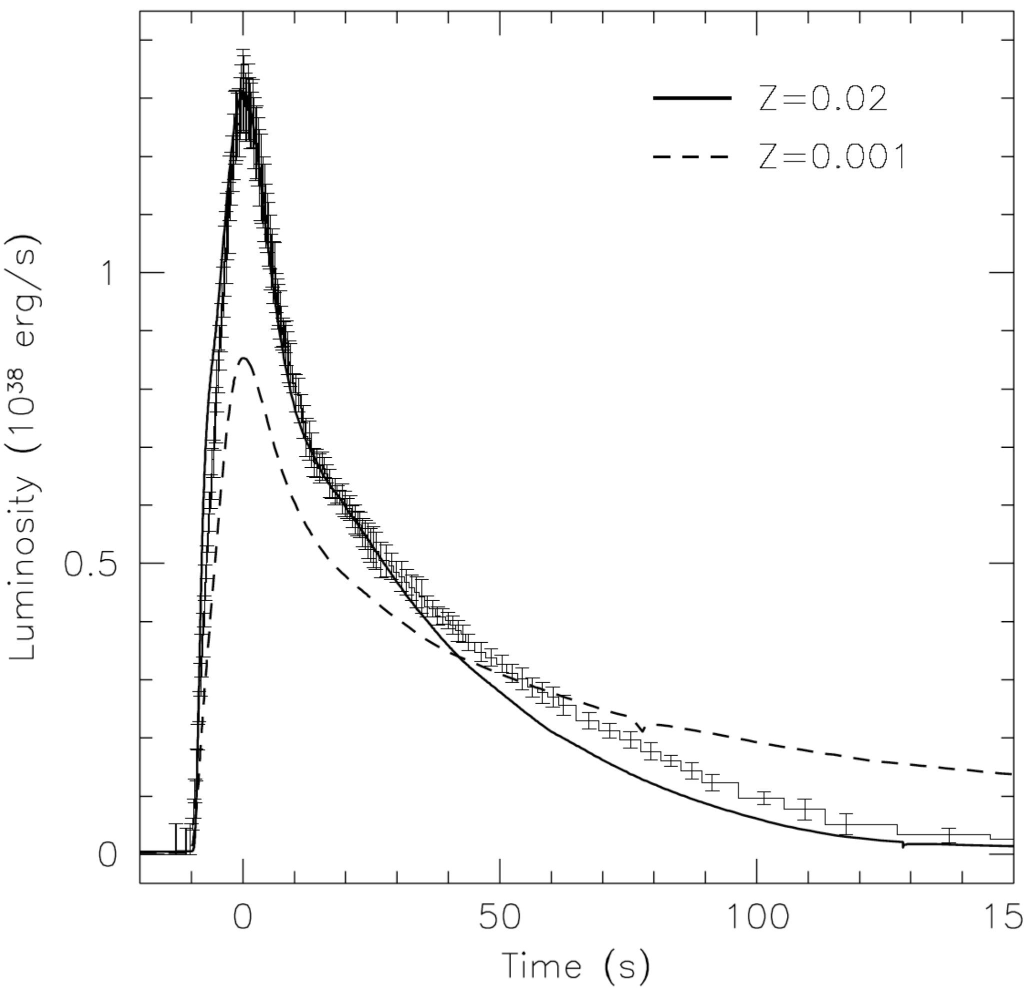
$$\epsilon \propto T^\nu \qquad \nu = \frac{44}{T_8} - 3$$

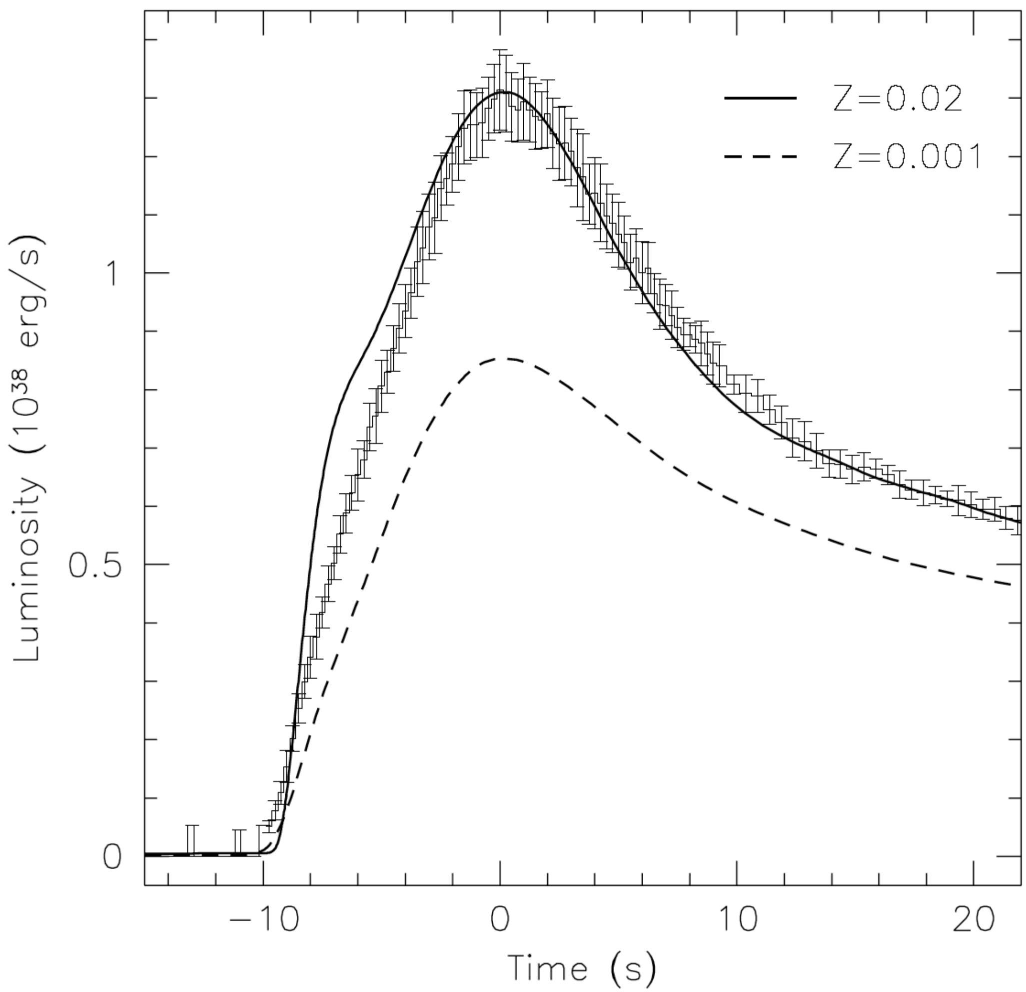
Exercise: given that CNO burning accounts for about 1% of the Sun's luminosity and that the transition from hot to cold CNO cycle occurs at 8x107K, estimate the temperature sensitivity of the CNO proton captures

The rp-process:
H burning in a He-rich environment



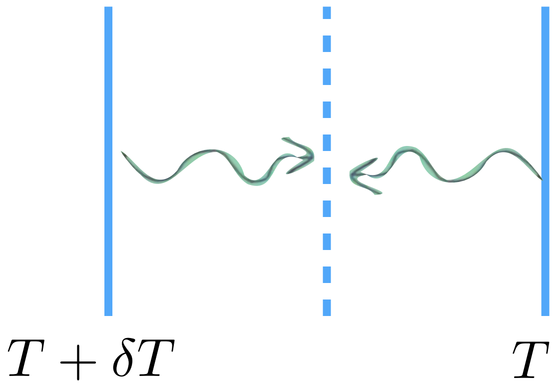
Comparison with observations: GS 1826-24





Heger et al. (2007)

The cooling is set by radiative diffusion



heat flux $$F = \frac{1}{3} c\lambda \frac{d}{dr}\left(aT^4\right)$$

mean free path $$\lambda = \frac{1}{n\sigma} = \frac{1}{\rho\kappa}$$

The cooling rate is

$$\epsilon_{\rm cool} = \frac{1}{\rho}\frac{dF}{dr}$$

(energy per gram per second)

The cooling is set by radiative diffusion

opacity (cross section per gram)

electron scattering $\kappa_{es} \approx \frac{\sigma_T}{\mu m_p} = \frac{0.4}{\mu}\ \mathrm{cm}^2\ \mathrm{g}^{-1}$

(Thomson with degeneracy and relativistic corrections)

free-free absorption

$$\kappa_{\mathrm{ff}} = 0.753\,\frac{\mathrm{cm}^2}{\mathrm{g}}\,\frac{\rho_5}{\mu_e T_8^{7/2}}\sum \frac{Z_i^2 X_i}{A_i}\, g_{\mathrm{ff}}(Z_i,\, T,\, n_e)$$

[for references, see Schatz et al. 1999 ApJ appendix]

A useful coordinate

The entropy equation becomes

$$c_P \frac{\partial T}{\partial t} = \epsilon_{\rm nuc} - \frac{1}{\rho}\frac{\partial F}{\partial r} \qquad\qquad F = -\frac{4acT^3}{3\kappa\rho}\frac{\partial T}{\partial r}$$

For a layer with constant gravity, a useful coordinate is the column depth y

$$dy = -\rho dr = \frac{dP}{g} \Rightarrow P = gy$$

$$y = \frac{\Delta M}{4\pi R^2}$$

column depth y tells you the pressure and the mass

Equation of state

see Pacynski 1983 for useful interpolation formulae
or Frank Timmes codes at http://cococubed.asu.edu

1. Ions (nuclei) are an ideal gas

$$P_i = n_i k_B T$$

2. Non-degenerate or degenerate electrons (assume non-relativistic for these shallow layers)

$$P_e = n_e k_B T \qquad\qquad P_e = \frac{2}{5} n_e E_F$$

3. Radiation

$$P_{\rm rad} = \frac{1}{3} a T^4$$

Exercise: for a nuclear energy release of 1 MeV/nucleon, and a typical density of 10^5 g/cm^3, show that radiation pressure limits the temperature of the layer to ~ 2x10^9 K

One zone model

1. The entropy equation becomes

$$c_P \frac{\partial T}{\partial t} = \epsilon_{\rm nuc} + \frac{\partial F}{\partial y} \qquad\qquad F = \frac{4acT^3}{3\kappa}\frac{\partial T}{\partial y}$$

Now treat the layer as "one zone": $\partial T/\partial y \sim T/y$ etc.

We have to solve

$$c_P \frac{dT}{dt} = \epsilon_{\rm nuc} - \frac{acT^4}{3\kappa y^2} + \frac{F_b}{y}$$

(include a base flux to simulate a hot underlying neutron star)

Exercise: estimate the typical luminosities you expect to see using $L = 4\pi R^2 y \epsilon_{\rm cool}$

One zone model

2. Track the composition. A nice way to do it is to follow the column depth of the fuel layer (Pacyznski 1986; Heger et al. 2007)

$$\frac{dy}{dt} = \dot{m} - \frac{12\epsilon_{3\alpha}}{Q_{3\alpha}} y$$

(where the energy release from the triple alpha reaction is

$$Q_{3\alpha} = 7.275 \text{ MeV}$$)

3. At each time-step, we need to invert the equation of state to find the density corresponding to the pressure P=gy. (The opacity and energy generation rate depend on density).

[I will run a simple one zone code in the lecture]

Eddington accretion rate (a useful unit for accretion rate)

The local Eddington flux is $$F_{\rm Edd} = \frac{cg}{\kappa}$$

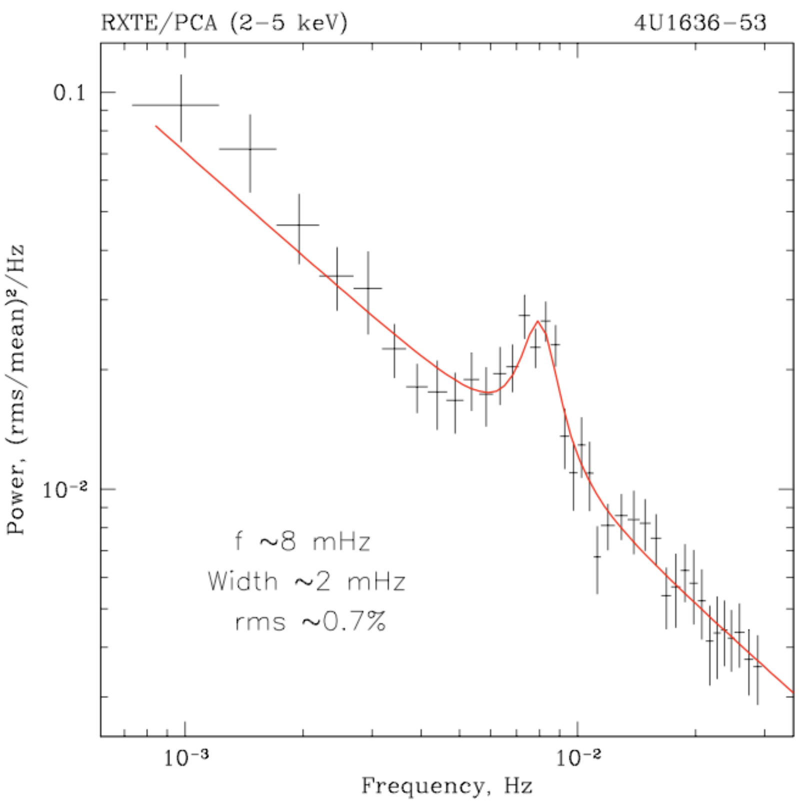
giving
$$L_{\rm Edd} = \frac{4\pi GMc}{\kappa} = \frac{GM}{R}\dot{M}_{\rm Edd}$$

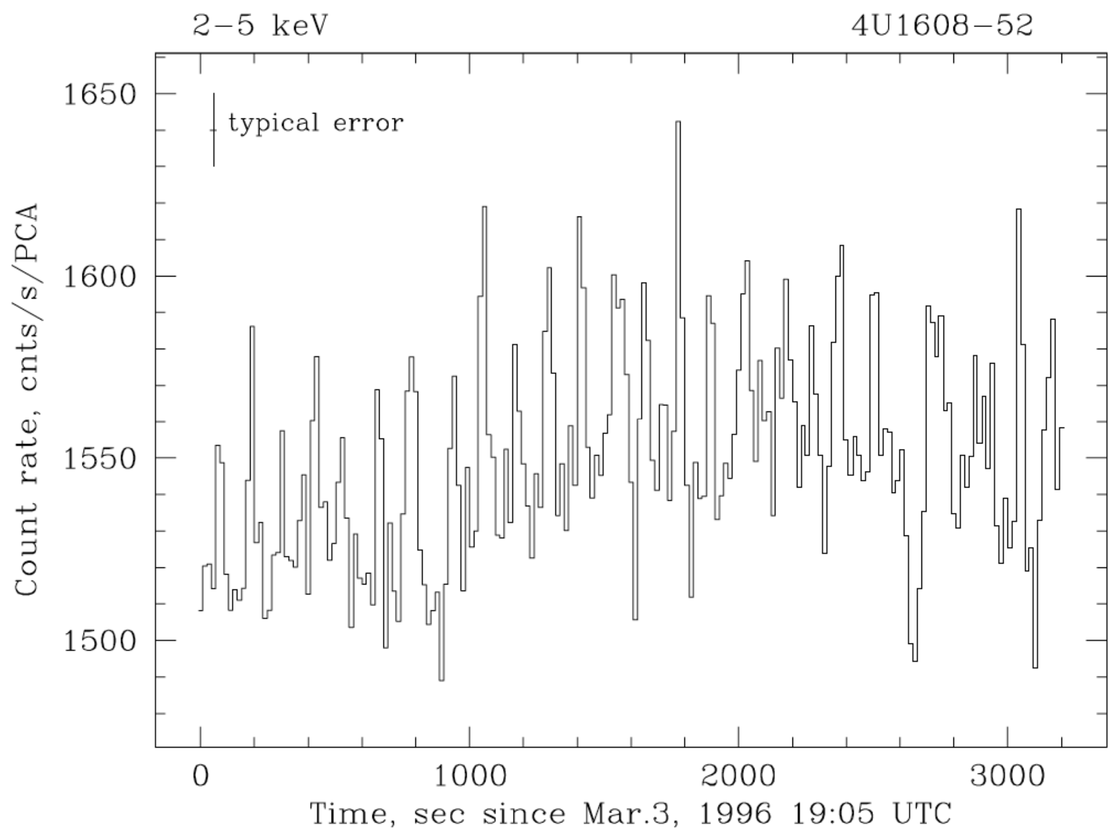
$$L_{\rm Edd} = 2.1\times 10^{38}\ {\rm erg\ s^{-1}}\ \left(\frac{M}{1.4\ M_\odot}\right)\left(\frac{1.7}{1+X}\right)$$
$$\dot{M}_{\rm Edd} \approx 10^{18}\ {\rm g\ s^{-1}} \approx 2\times 10^{-8}\ M_\odot\ {\rm yr^{-1}}$$
$$\dot{m}_{\rm Edd} = \frac{\dot{M}_{\rm Edd}}{4\pi R^2} \approx 10^5\ {\rm g\ cm^{-1}\ s^{-1}}$$

Exercise: confirm these numbers and include the redshift factors for observers at infinity

Observations of mHz QPOs

- discovered from Atoll sources 4U 1608-52, 4U 1636-53, Aql X-1 by Revnitsev et al. (2001) with frequencies (7-9) mHz
- flux variations at ~few percent level





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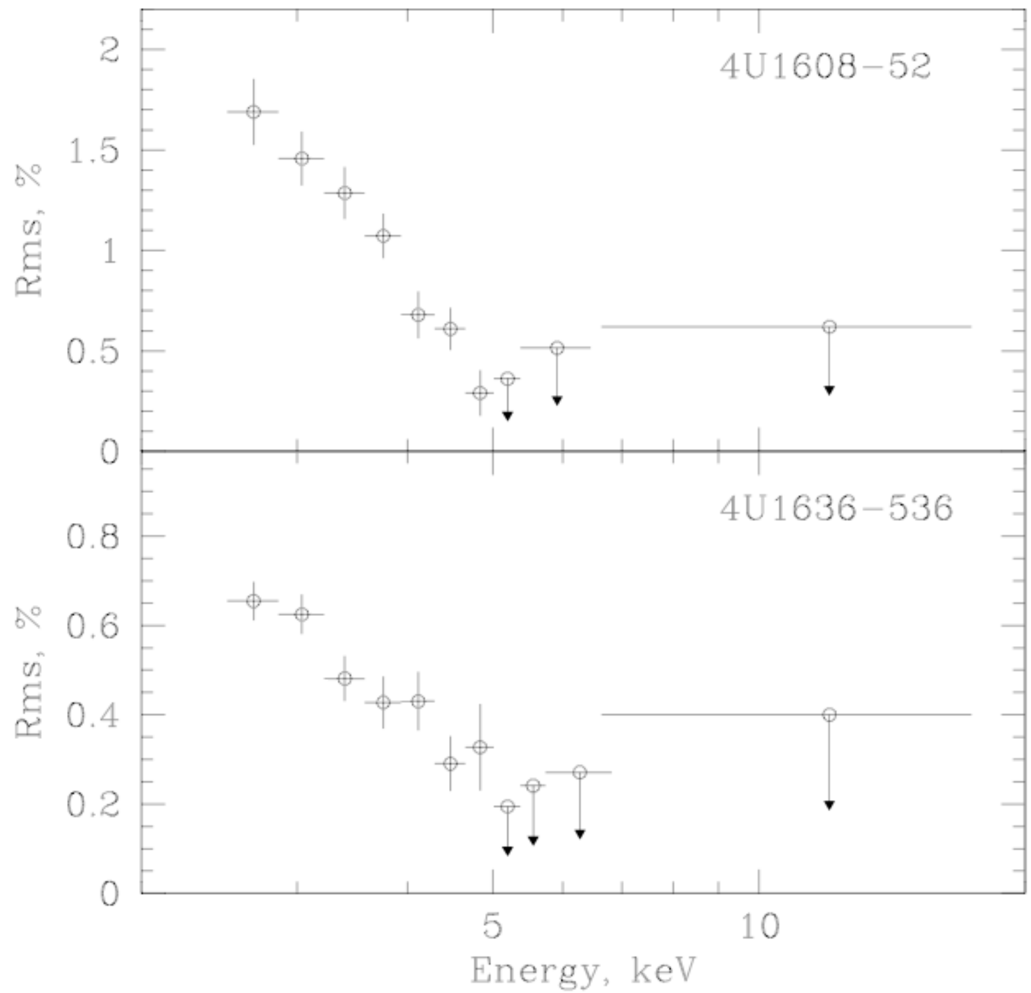

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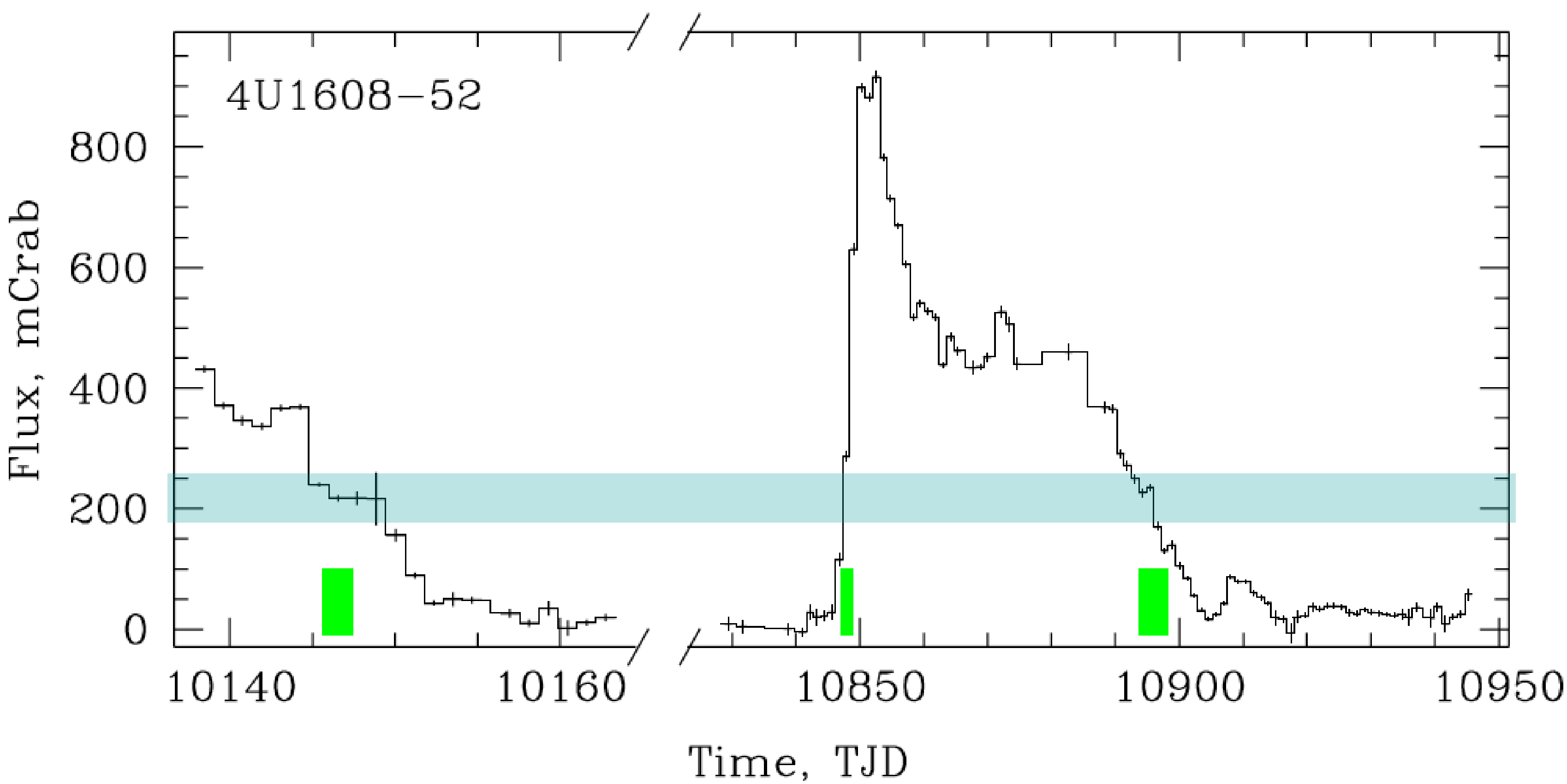
- flux variations at ~few percent level
- unusually for a QPO, they are *soft* (<5 keV)
- they occur in a narrow range of luminosity (0.5-1.5) x 10^{37} erg/s



A new mode of nuclear burning?

• Revnitsev et al. (2001) suggested that we are seeing a **new mode of nuclear burning**

• Importance: first QPO identified with NS surface rather than the accretion flow

• Open questions:

what sets the oscillation period ~ 2 mins?
(stable over many years)

why the narrow luminosity range?

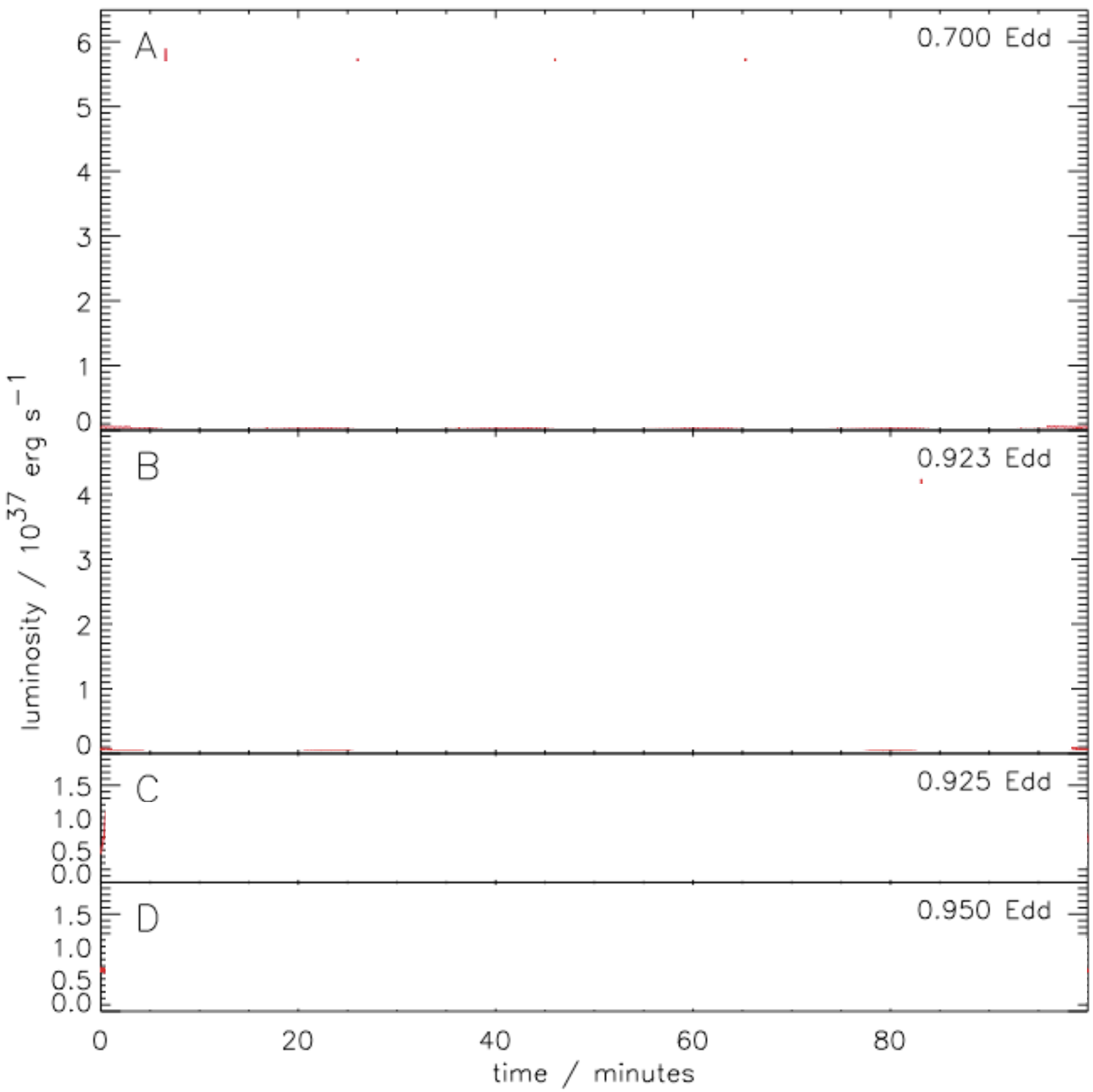
• Marginally stable nuclear burning answers these questions, but brings back an old puzzle!

Calculations of the transition to stable burning

- Extensions of the Woosley et al. 2003 ApJS calculations to higher accretion rates

- Kepler code, follow >1000 nuclei at each depth

- At the boundary between unstable and stable burning see oscillations with periods of 3 minutes



Heger, Cumming, & Woosley (2007)`

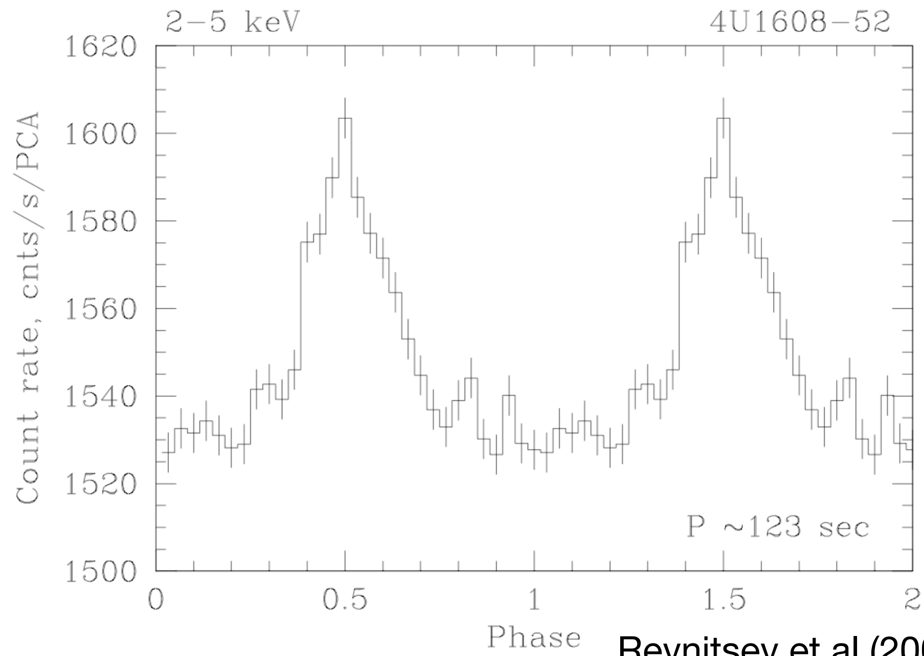
Calculations of the transition to stable burning

• Extensions of the Woosley et al. 2003 ApJS calculations to higher accretion rates

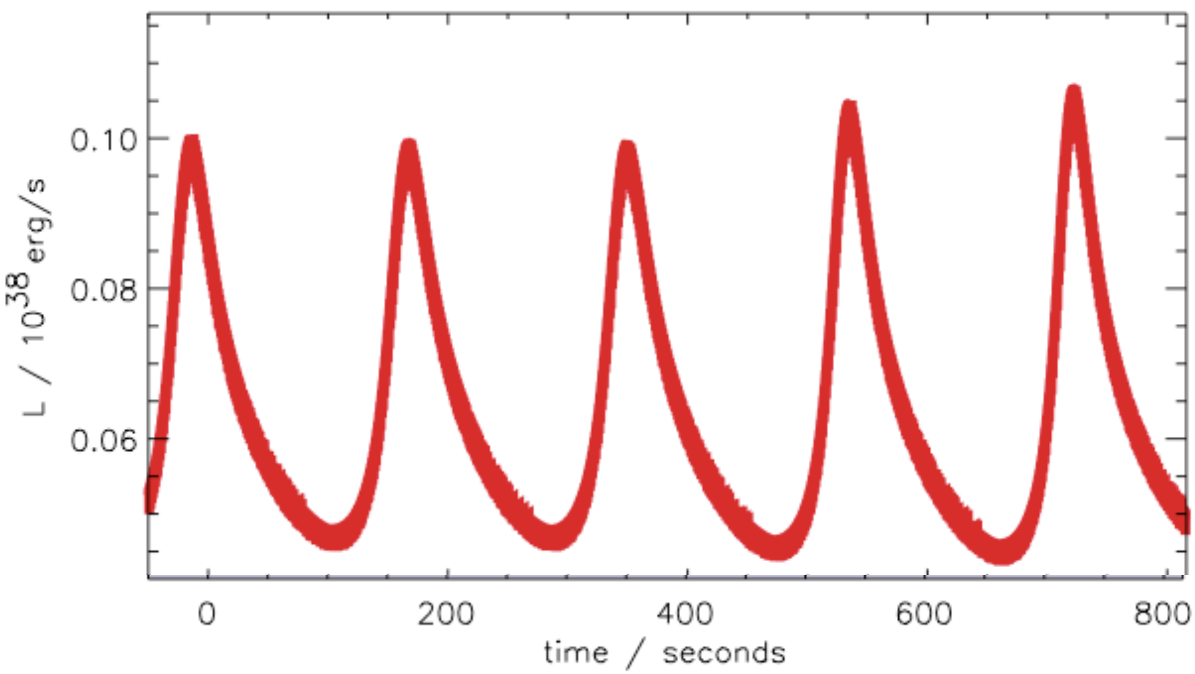
• Kepler code, follow >1000 nuclei at each depth

• At the boundary between unstable and stable burning see oscillations with periods of 3 minutes

• Amplitude and shape of the oscillation similar to the observed mHz QPOs



Revnitsev et al (2001)



Heger, Cumming, & Woosley (2007)`

The physics of the oscillation is in our one-zone model!

- In fact, Paczynski (1981) suggested that oscillations should be present at the boundary between unstable and stable burning. We can get it from our one-zone model, see Heger et al. (2007):

- Simple one-zone model

$$c_P \frac{dT}{dt} = \epsilon - \frac{F}{y}$$

$$\frac{dy}{dt} = \dot{m} - \frac{\epsilon}{E_\star} y.$$

$t_{therm} \sim 10s$

$t_{accr} \sim 1000s$

- Linear perturbations

$$\frac{\partial^2 f}{\partial t^2} + \left(\frac{4-\alpha}{t_{\rm therm}} - \frac{1}{t_{\rm accr}} \right) \frac{\partial f}{\partial t} + \frac{2\alpha}{t_{\rm accr} t_{\rm therm}} f = 0.$$

Usually thermal time dominates with strong driving or damping

$\boxed{\text{Oscillation period } \approx (t_{\rm therm} t_{\rm accr})^{1/2}}$

- A clock on the NS surface that depends on g, X … no mdot uncertainty!

To think about for next time

- How much mass would you expect to be ejected into the ISM in a typical Type I X-ray burst? Will it significantly affect abundances in the galaxy?

- How would you expect burst properties (recurrence time, duration, peak luminosity, lightcurve shape) to change with accretion rate for accretion of either pure helium or a solar mixture of H/He ?

Modelling Accreting Neutron Stars II: More on bursts; Crust heating

Andrew Cumming
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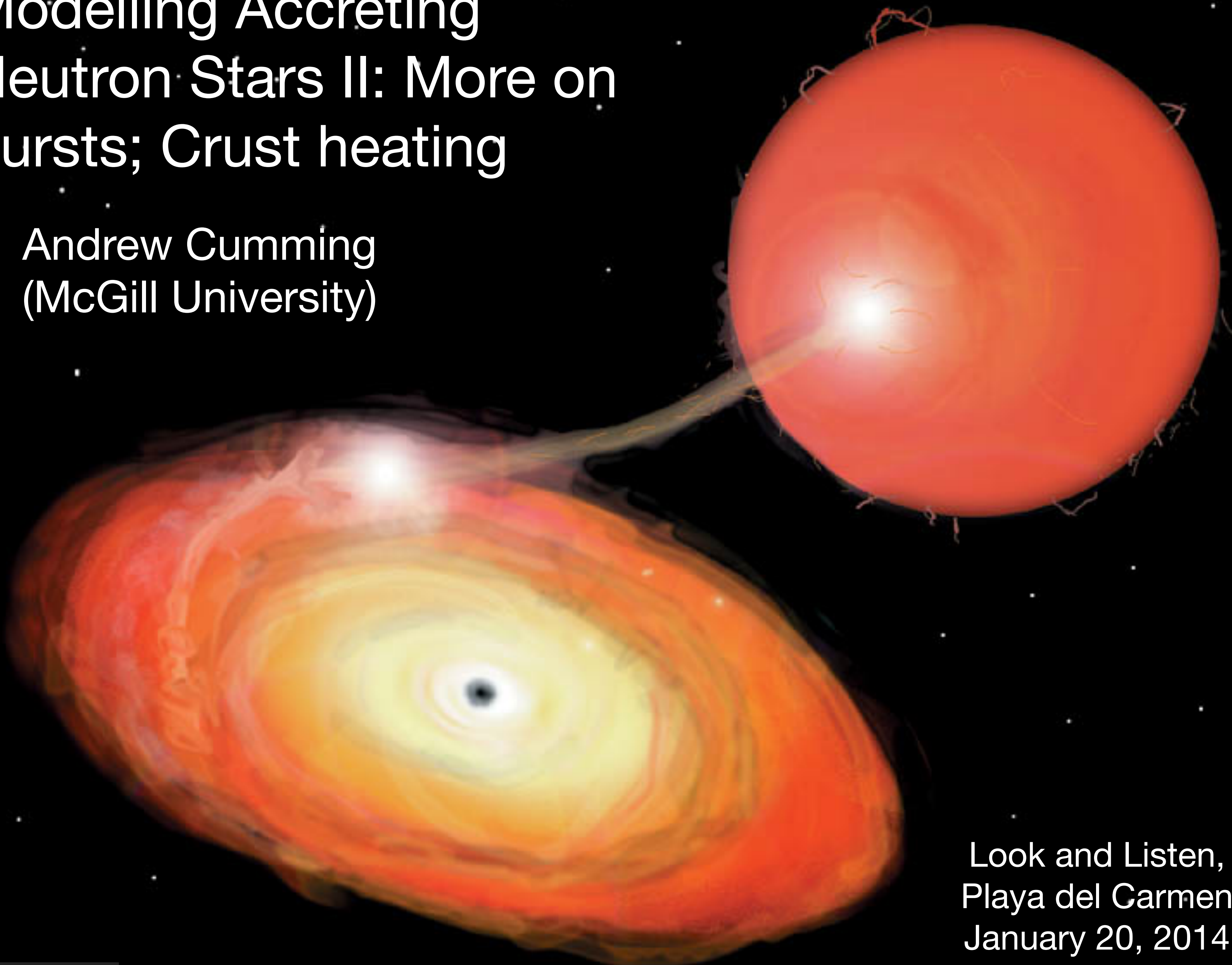

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Last time

- Physics of the thin shell flash
- Nuclear burning: beta-limited hot CNO cycle; triple alpha; rp-process
- A one-zone model of X-ray bursts

$$c_P \frac{dT}{dt} = \epsilon_{\rm nuc} - \frac{acT^4}{3\kappa y^2} + \frac{F_b}{y}$$

$$\frac{dy}{dt} = \dot{m} - \frac{12\epsilon_{3\alpha}}{Q_{3\alpha}} y$$

Multi-zone models

Assume spherical symmetry. Follow temperature and the detailed composition as a function of depth in the layer.

$$c_P \frac{dT}{dt} = \epsilon_{\rm nuc} - \frac{acT^4}{3\kappa y^2} + \frac{F_b}{y} \quad \longrightarrow \quad ?$$

$$\frac{dy}{dt} = \dot{m} - \frac{12\epsilon_{3\alpha}}{Q_{3\alpha}} y$$

Most comprehensive simulations are by Woosley, Heger et al. (Kepler code) - see Woosley et al. (2003) ApJS

The open source code MESA (Paxton et al. 2011,2013) is being developed as a tool for studying accreting neutron stars. I will run an example of helium accretion with MESA in the lecture.

Convection: mixing length theory

convective flux

$$F_{\mathrm{conv}} \approx \rho v_{\mathrm{conv}} c_P T \left(\nabla - \nabla_{\mathrm{ad}}\right)$$

convective velocity

$$v_{\mathrm{conv}}^2 \approx g l \left(\nabla - \nabla_{\mathrm{ad}}\right)$$

efficient convection $\quad v_{\mathrm{conv}}^2 \ll c_s^2 \qquad \nabla \approx \nabla_{\mathrm{ad}}$

Exercise: show that $F_{\mathrm{conv}} \sim \rho v_{\mathrm{conv}}^3$ (useful for estimates)

For F~10^{25} erg/cm^2/s, v_{conv}~50 km/s; c_s~3x10^8 cm/s
The convective turnover time is

H/v_{conv}~1000 cm/5x10^6 cm/s ~ 0.2 ms

MESA for Type I X-ray bursts: Open questions

Are the radiative opacities correct for the ashes mixture of heavy elements?

Is the treatment of convection okay (check against expectations)

Outer boundary condition when at Eddington luminosity

Handling rp-process requires a large reaction network

There is interesting data to compare against even for pure He accretion

Question

How would you expect the following properties of Type I bursts to change with accretion rate?

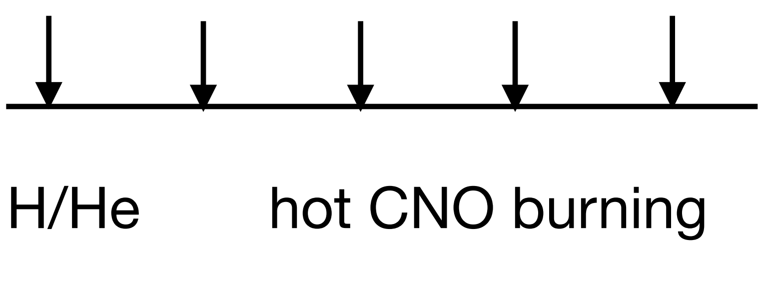
- burst recurrence time
- burst duration
- peak luminosity
- rise time
- shape

We will take a look at the Galloway et al. (2008) catalog of bursts observed with RXTE to test these predictions!

Different burning regimes

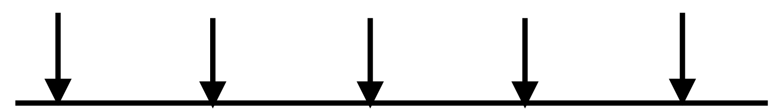
Time to burn the hydrogen at the hot CNO rate $t_H \approx 22 \text{ hr} \left(\frac{0.01}{Z_{CNO}}\right)\left(\frac{X_0}{0.71}\right)$

$t_{recur} > 22$ hrs



pure He

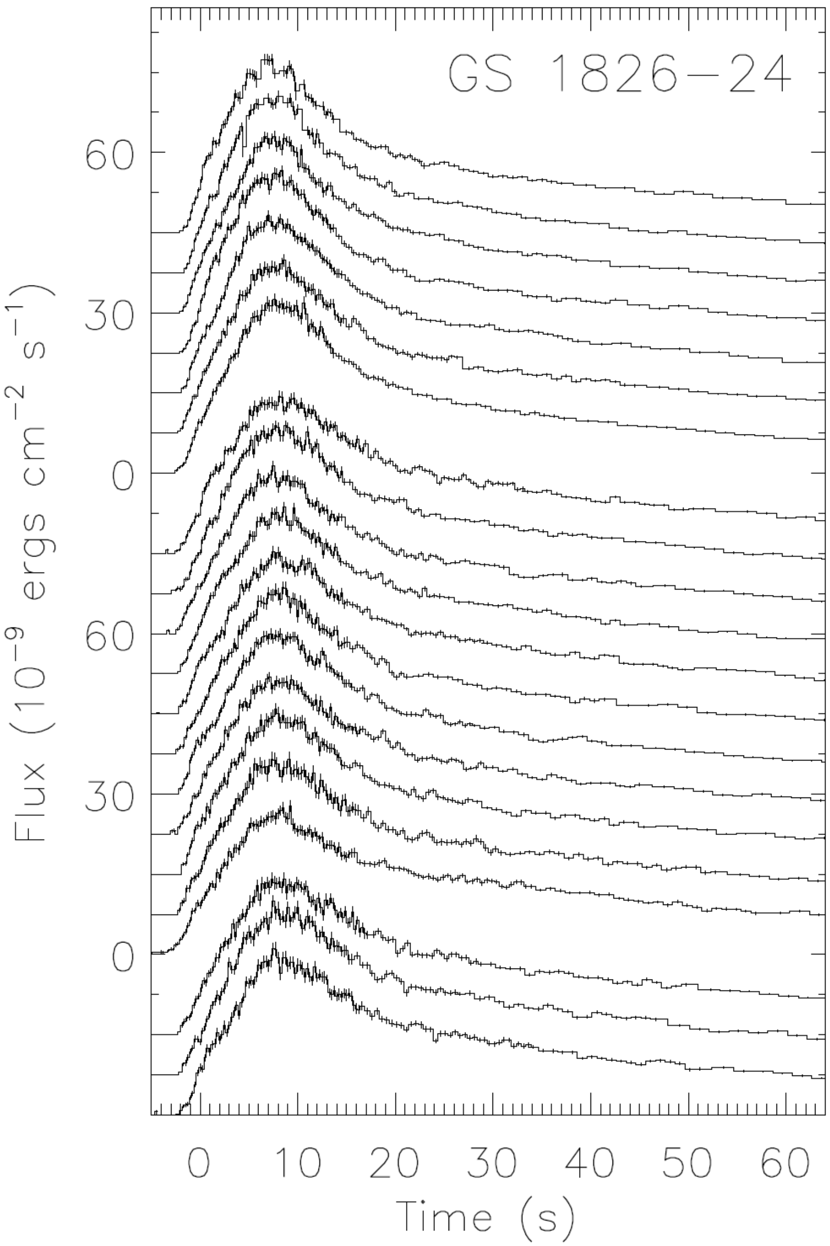
$t_{recur} < 22$ hrs



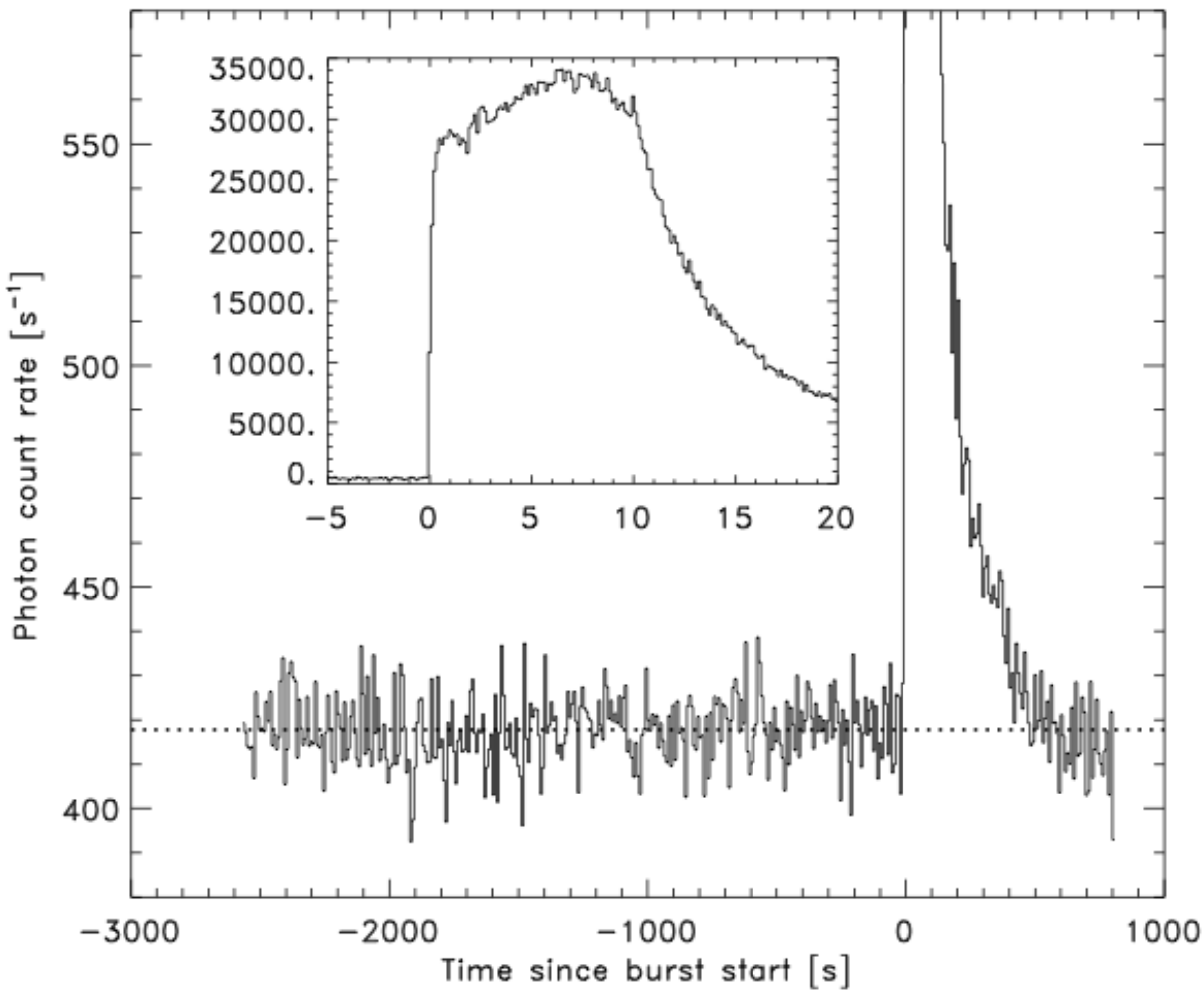
hot CNO burning
throughout the layer

mixed H/He

Different burst shapes

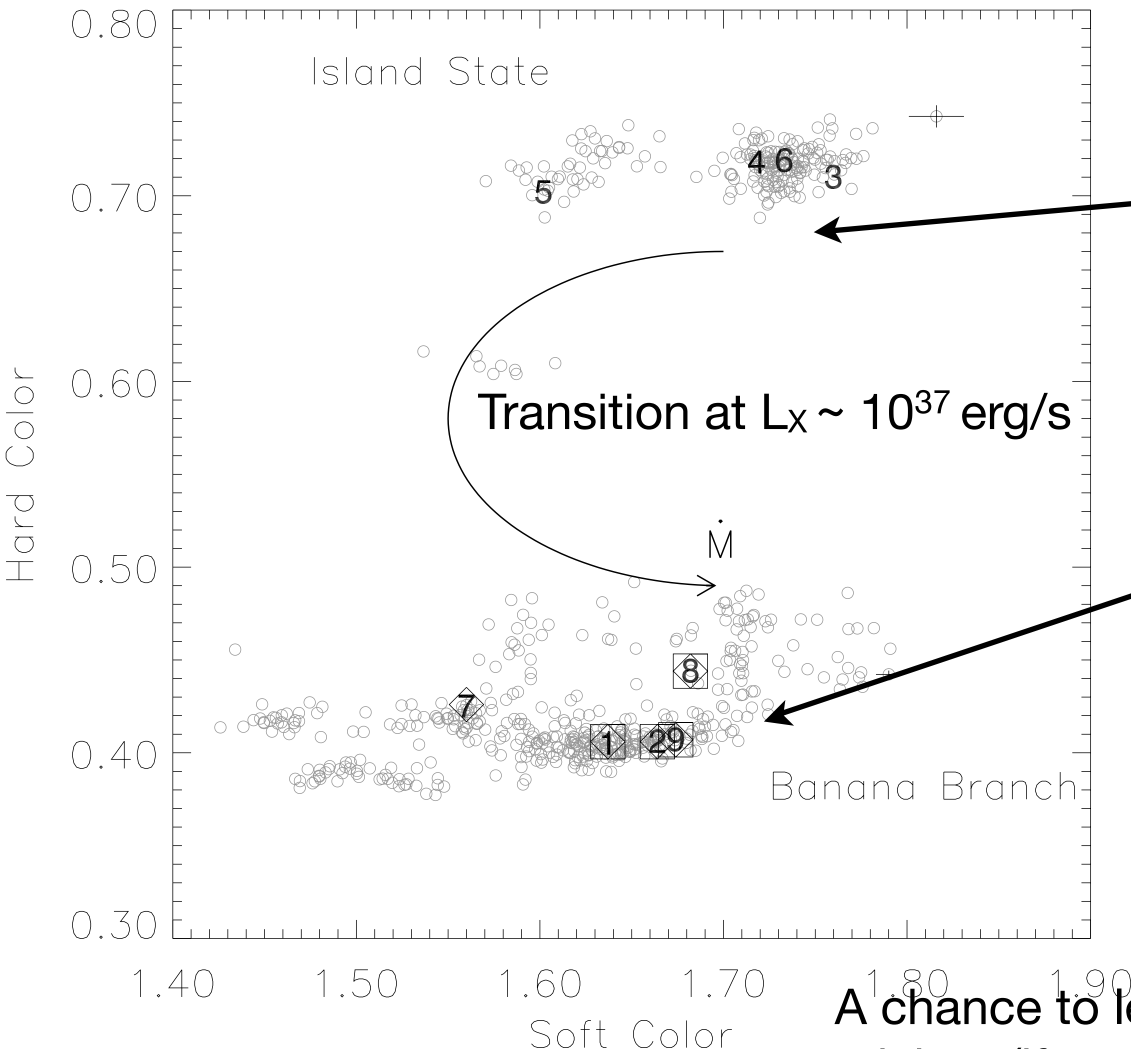


Galloway et al. (2004)



in ’t Zand et al. (2013)

Change in burst behavior with accretion rate: onset of stable burning at accretion rates several times smaller than predicted; interaction between mHz QPOs and bursts

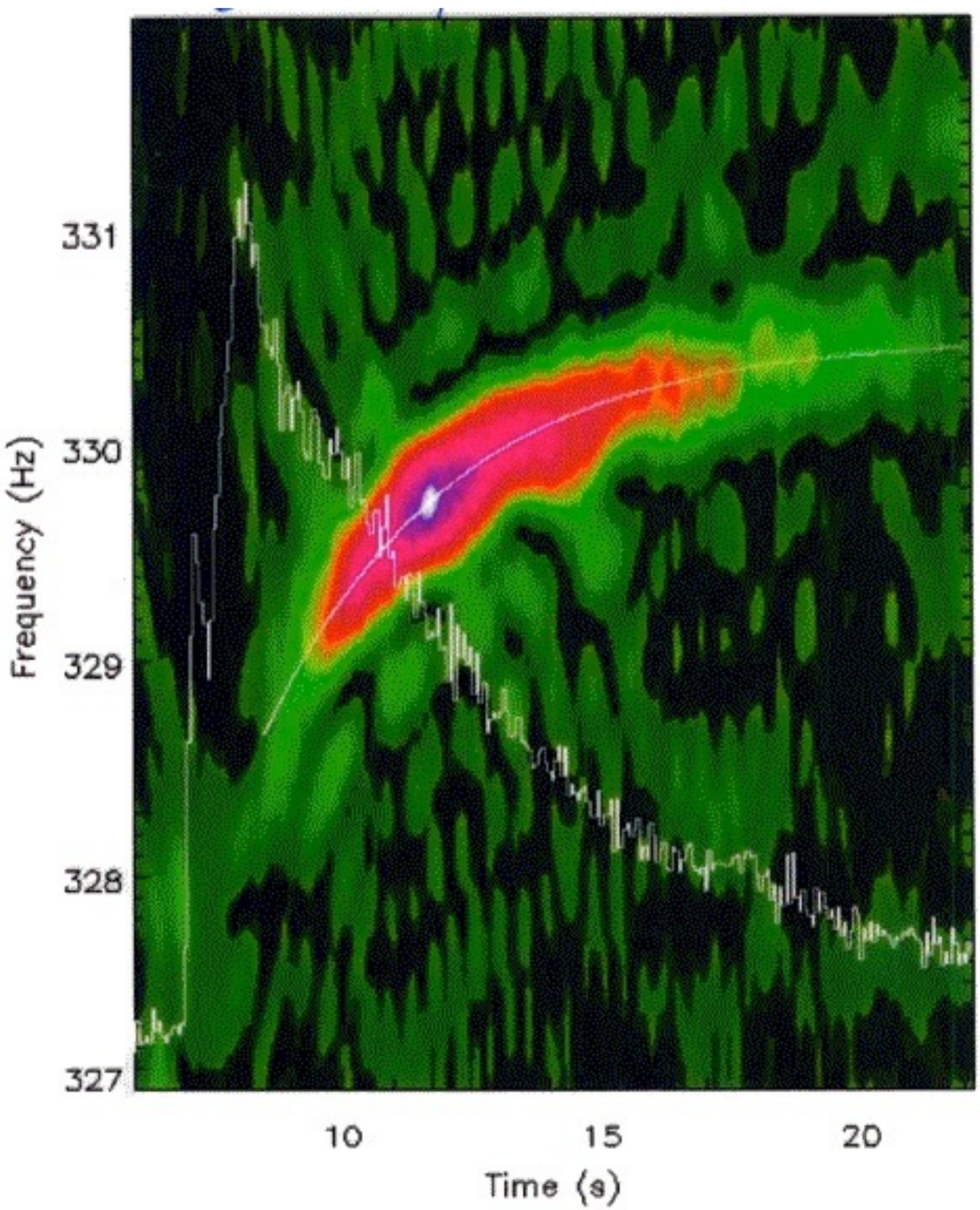


<u>Island state (low accretion rate)</u>
Regular Type I bursts
Long duration, energetics consistent with all fuel burning in bursts

<u>Banana state (high accretion rate)</u>
Short, irregular bursts
Some stable burning
Burst oscillations
Superbursts
mHz QPOs

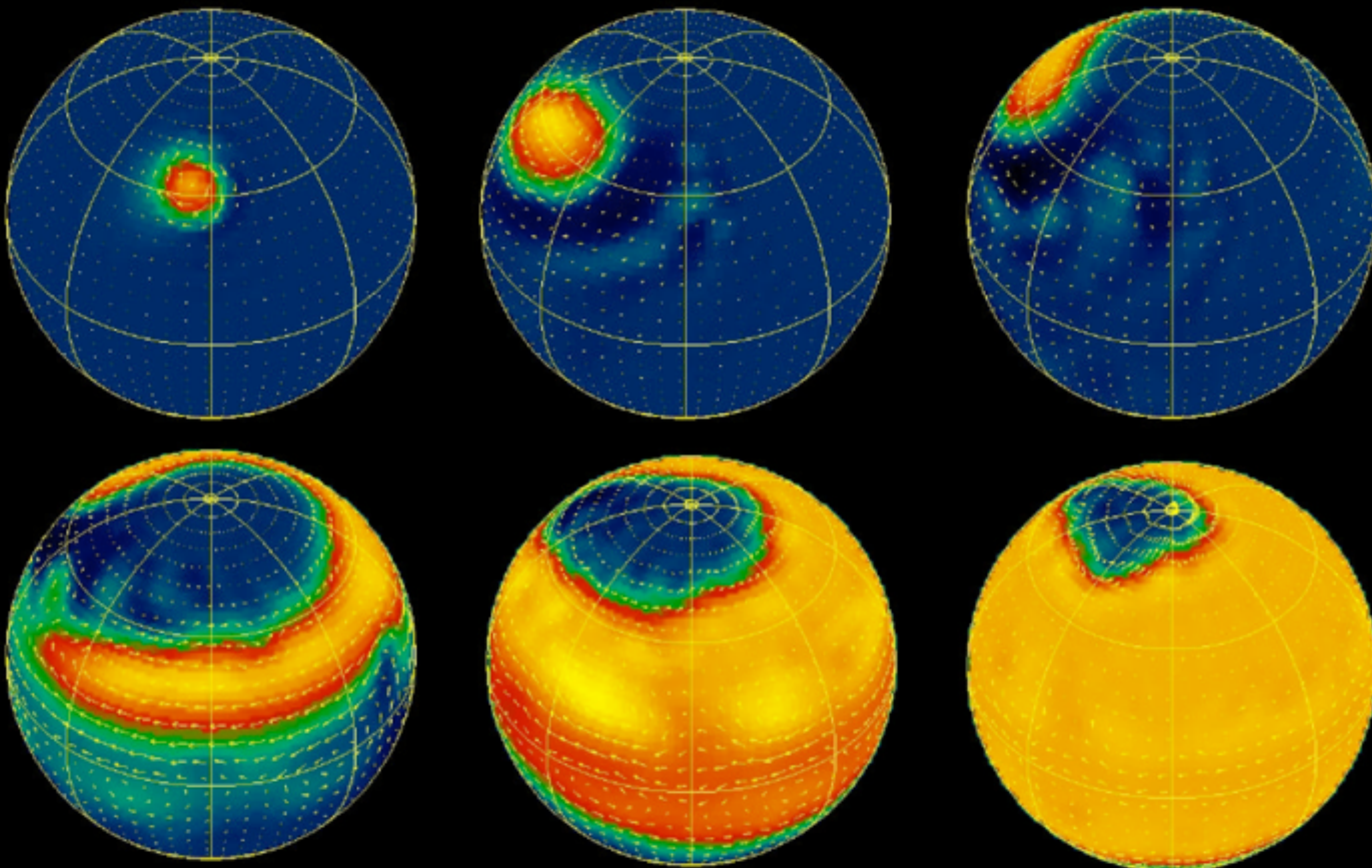
A chance to learn something about accretion/mixing (if we understand the nuclear physics part)

Burst oscillations



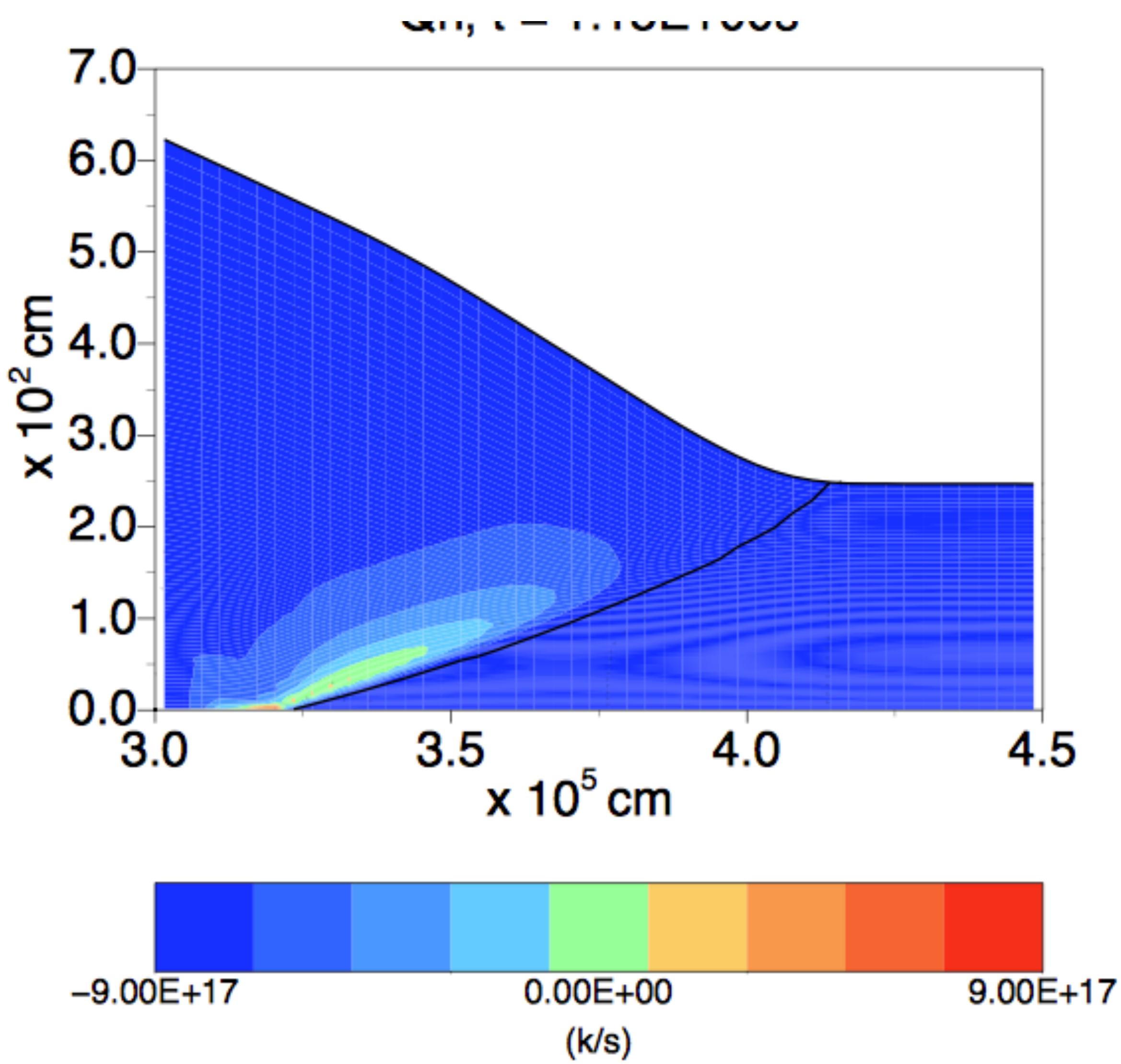
Ignition and spreading

2D shallow water model on a sphere
ν=300 Hz; 200 revolutions



Spitkovsky, Levin, & Ushomirsky (2002)

Burning front propagation on a rotating star

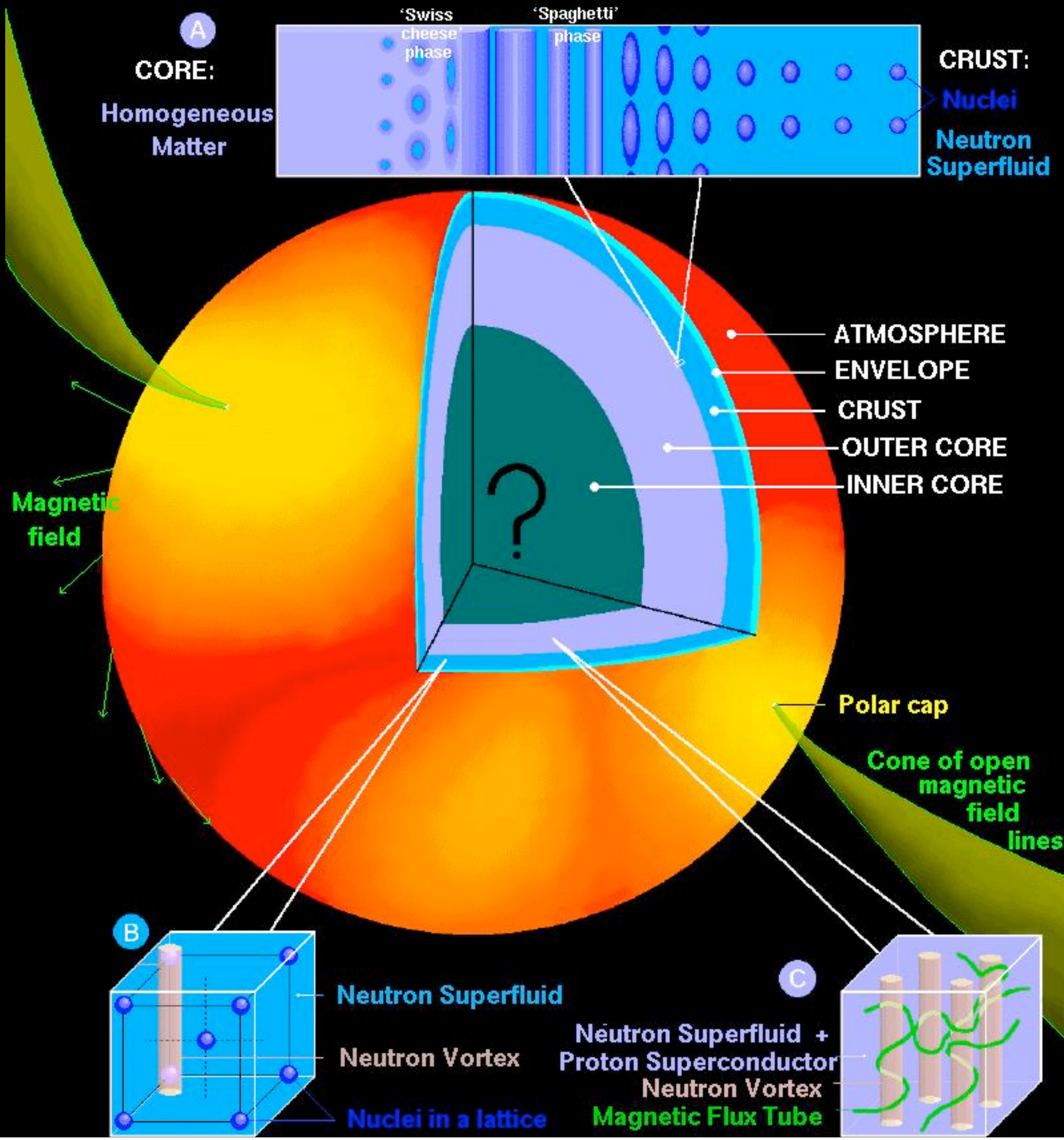


Cavecchi et al. (2013)

Part 2: Crust heating and thermal relaxation

- Why the crust is solid
- How the heating works (crust nuclear reactions)
- How to calculate thermal relaxation

A NEUTRON STAR: SURFACE and INTERIOR



Where does solid form?

In the outer layers, the matter consists of bare nuclei embedded in a smooth background of degenerate electrons

The key parameter that determines the arrangement of the nuclei is

$$\Gamma = \frac{Z^2 e^2}{a k_B T}$$

$$\left(\frac{4\pi}{3} a^3 n_i = 1 \right)$$

< 1	gas
$\gtrsim 1$	liquid
$\gtrsim 175$	solid

The density at the top of the crust is

$$\rho_{\rm top} \approx 10^8 \ {\rm g\ cm^{-3}}\ T_8^3 \left(\frac{Z}{26}\right)^{-6} \left(\frac{A}{56}\right)$$

Modelling Accreting Neutron Stars III: Mapping the crust

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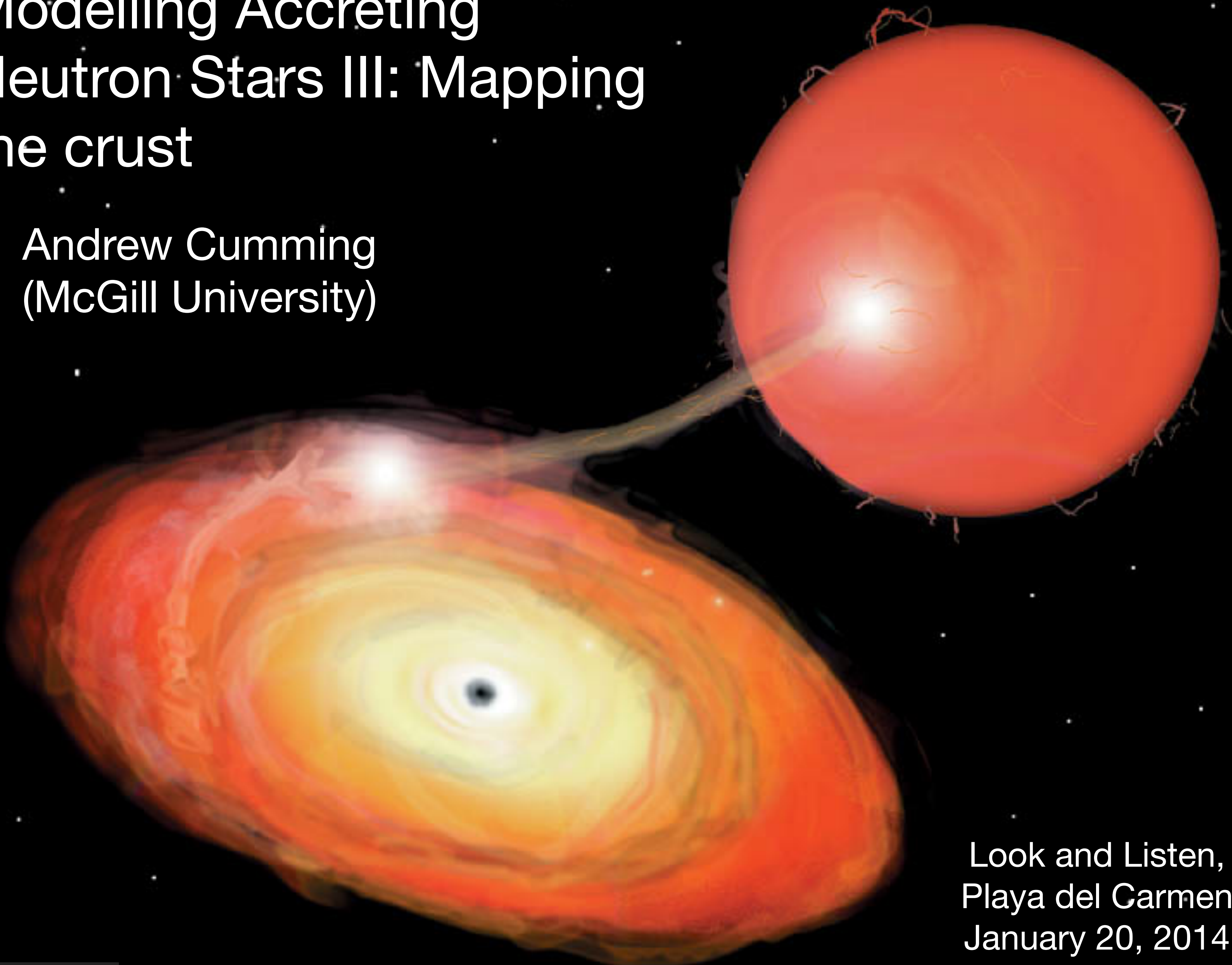

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Nuclear evolution in the outer crust

The increasing electron Fermi energy drives electron capture reactions that reduce the number of protons in the nuclei.

A simple model for the outer crust composition is to minimize the Gibb's free energy per nucleon

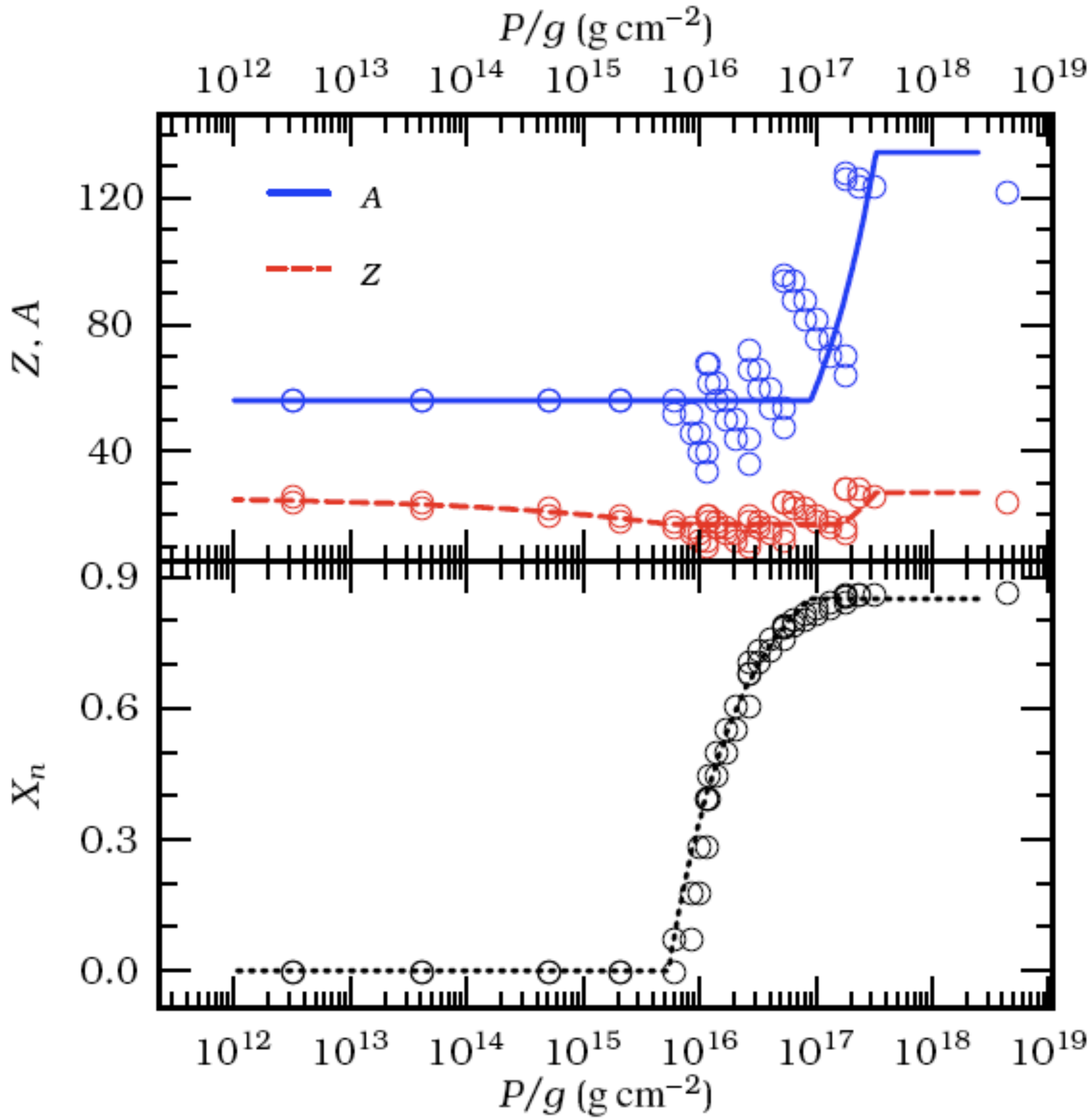
$$G = \frac{Z}{A} E_F - \frac{B(Z,A)}{A}$$

treating Z as a continuous variable, where we get the nuclear binding energy from a liquid drop model

$$B(A,Z) = a_V A - a_S A^{2/3} - a_A \frac{(N-Z)^2}{A} - a_C \frac{Z^2}{A^{1/3}}$$

Setting $\partial G/\partial Z|_A = 0$ gives Z as a function of electron Fermi energy

$$\frac{Z}{A} \approx \left(\frac{1}{2} - \frac{E_F}{8a_A}\right)\left(1 + \frac{a_C A^{2/3}}{4a_A}\right)^{-1}$$



Composition as a function of depth in the crust

Brown and Cumming (2009) following Sato (1979), Haensel & Zdunik (1990)

Reactions in the crust

Example reaction sequences

Outer crust
$^{56}\mathrm{Fe} + e^- \rightarrow {}^{56}\mathrm{Mn} + \nu_e$
$^{56}\mathrm{Mn} + e^- \rightarrow {}^{56}\mathrm{Cr} + \nu_e$

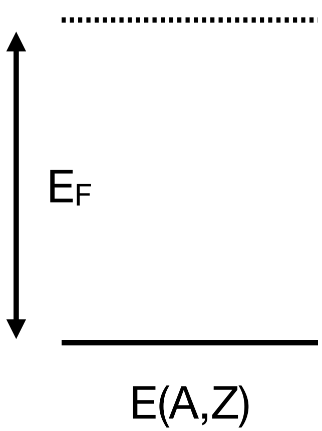
Inner crust
$^{56}\mathrm{Ar} + e^- \rightarrow {}^{56}\mathrm{Cl} + \nu_e$
$^{56}\mathrm{Cl} \rightarrow {}^{55}\mathrm{Cl} + n$
$^{55}\mathrm{Cl} + e^- \rightarrow {}^{55}\mathrm{S} + \nu_e$
$^{55}\mathrm{S} \rightarrow {}^{54}\mathrm{S} + n$
$^{54}\mathrm{S} \rightarrow {}^{52}\mathrm{S} + 2n$

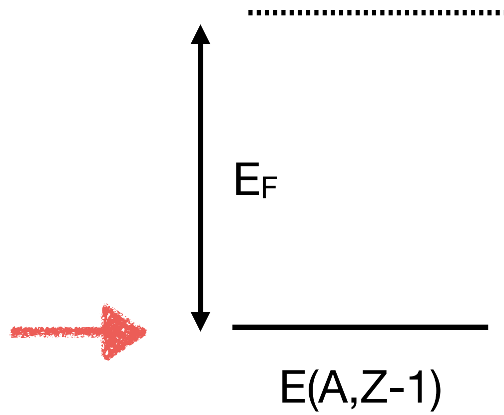
Question: why do these reactions heat the crust?

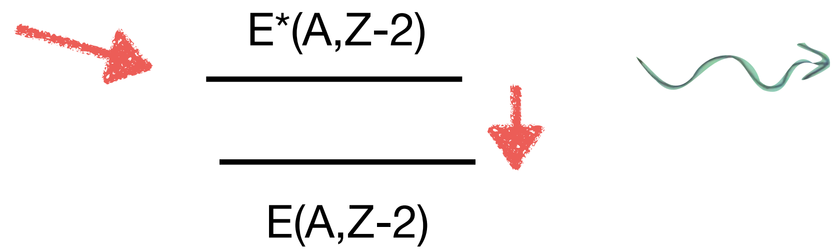
Haensel & Zdunik (1990, 2008)

How heating works in the outer crust

1. The first electron capture happens at threshold, doesn't heat the gas







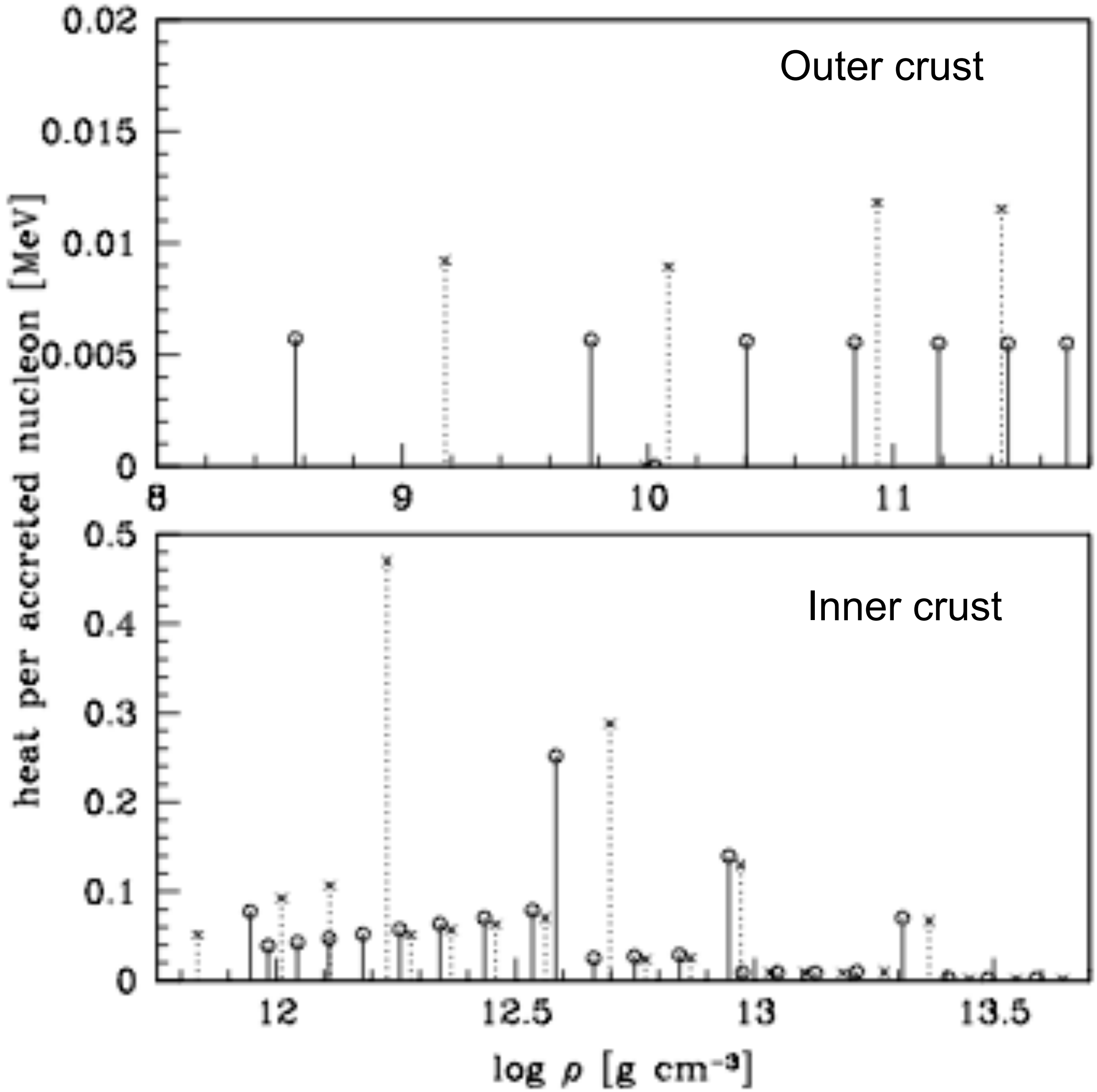
3. The Z-2 nucleus de-excites, releasing a gamma-ray that heats the gas

2. Because even nuclei are lower in energy than odd nuclei, the Z-1 nucleus can immediately capture to form the Z-2 nucleus in an excited state.

Heating in the crust

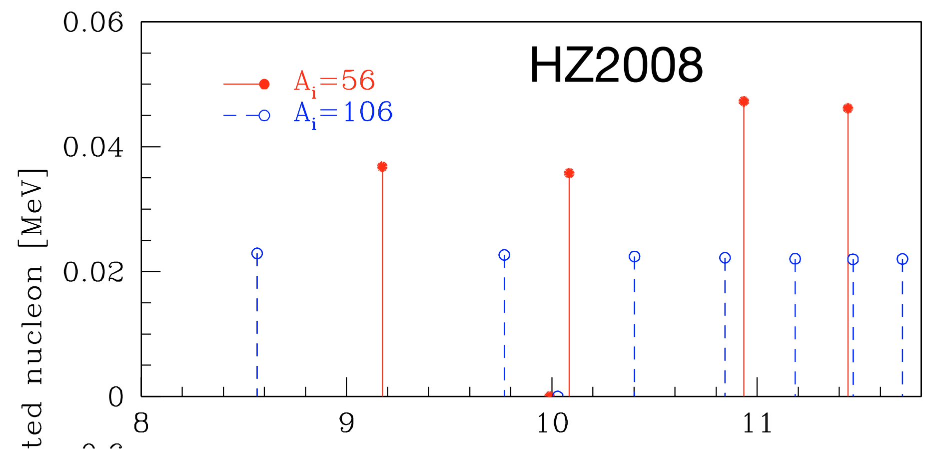
Haensel & Zdunik 2003

The total heat released is between 1 and 2 MeV/nucleon



Question: using the liquid drop model, can you understand the spacing of the reactions in the outer crust, and why they give the same amount of energy?

A simple model for outer crust heating



Beta-equilibrium tells us Z as function of depth(neglect Coulomb term)

$$\frac{2Z}{A} = 1 - \frac{\mu_e}{4a_A}$$

Putting in mu_e=27 MeV for neutron drip, the change in Z in the outer crust is Z=0.14 A

[does this agree with Sanjay's lecture?]

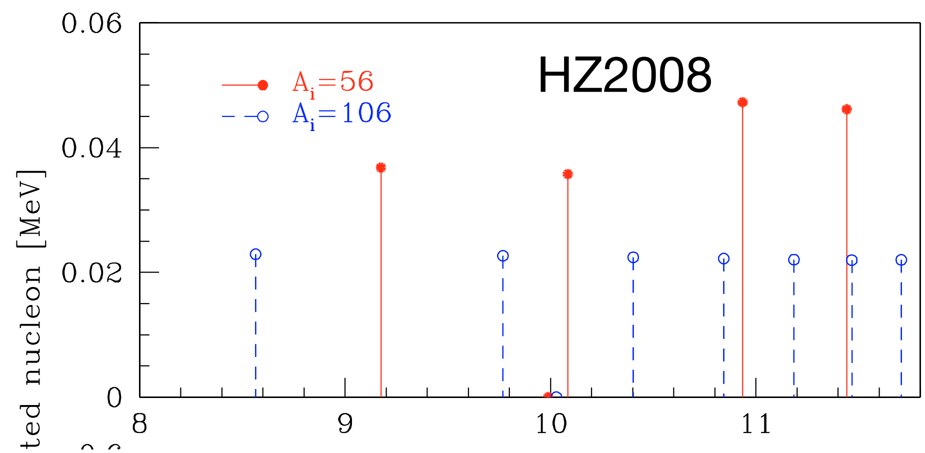
The electron captures occur in pairs, so the number of electron captures is

$$N_{\rm steps} = \frac{A\Delta\mu_e}{16a_A} = 0.07A = 3.9\left(\frac{A}{56}\right)$$

A simple model for outer crust heating II

The first capture occurs at threshold

$$\mu_e + M(Z) = M(Z-1)$$



The heating is determined by the second electron capture

$$Q = M(Z-1) + \mu_e - M(Z-2) = \; 2M(Z-1) - M(Z) - M(Z-2)$$

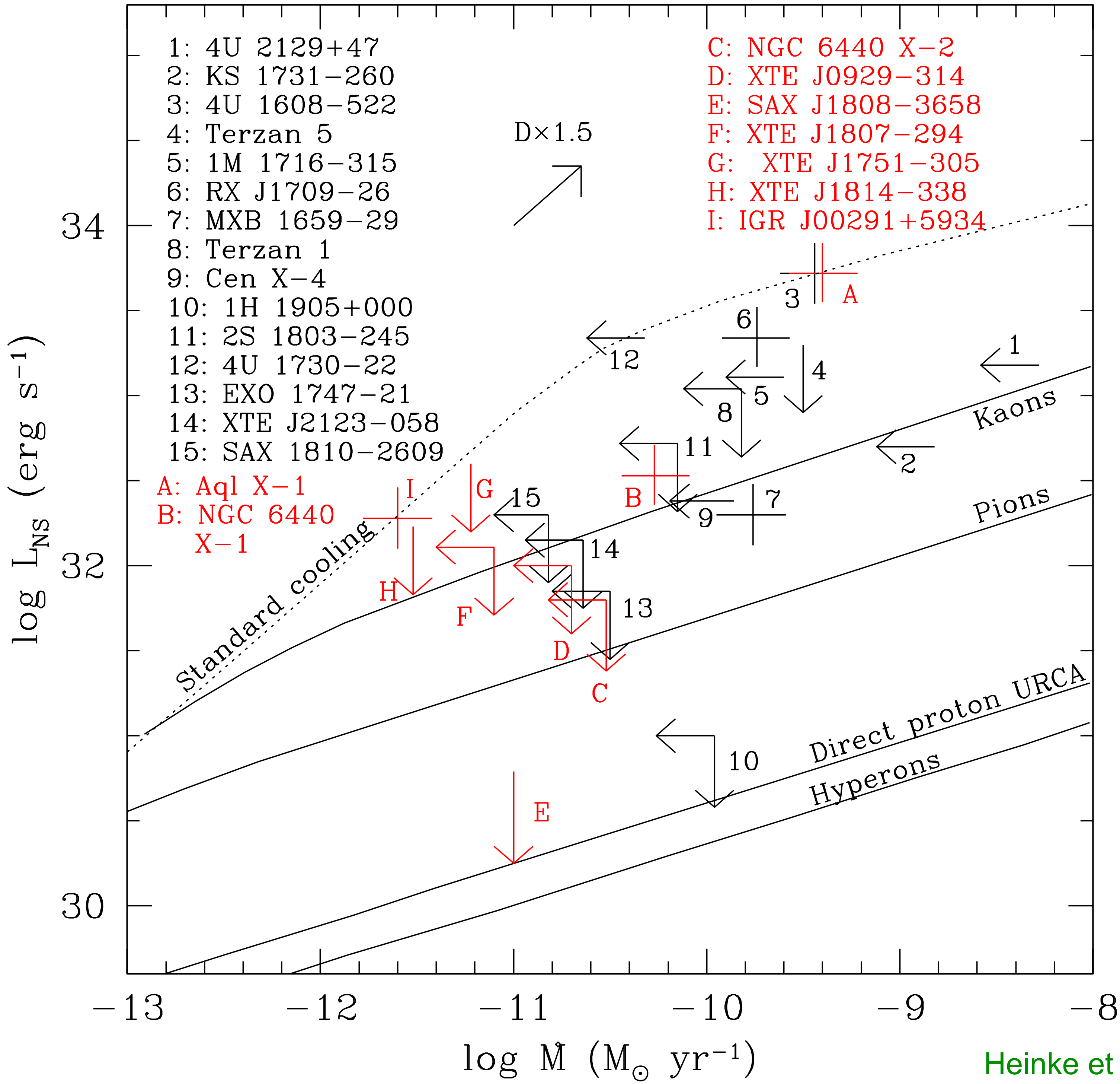
Simple liquid drop model gives

$$Q = \frac{4a_P}{A^{3/2}} - \frac{8a_A}{A^2} - \frac{2a_C}{A^{4/3}} = \frac{4a_P}{A^{3/2}}\left(1 - \frac{2a_A}{a_P A^{1/2}} - \frac{a_C A^{1/6}}{2a_P}\right)$$

$$= 0.11 \text{ MeV } \left(\frac{A}{56}\right)^{-3/2}\left[1 - 0.569\left(\frac{A}{56}\right)^{-1/2} - 0.064\left(\frac{A}{56}\right)^{1/6}\right]$$

Total heating: $$Q_{\text{outer}} = 0.43 \text{ MeV } \left(\frac{A}{56}\right)^{-1/2}\left[1 - 0.569\left(\frac{A}{56}\right)^{-1/2} - 0.064\left(\frac{A}{56}\right)^{1/6}\right]$$

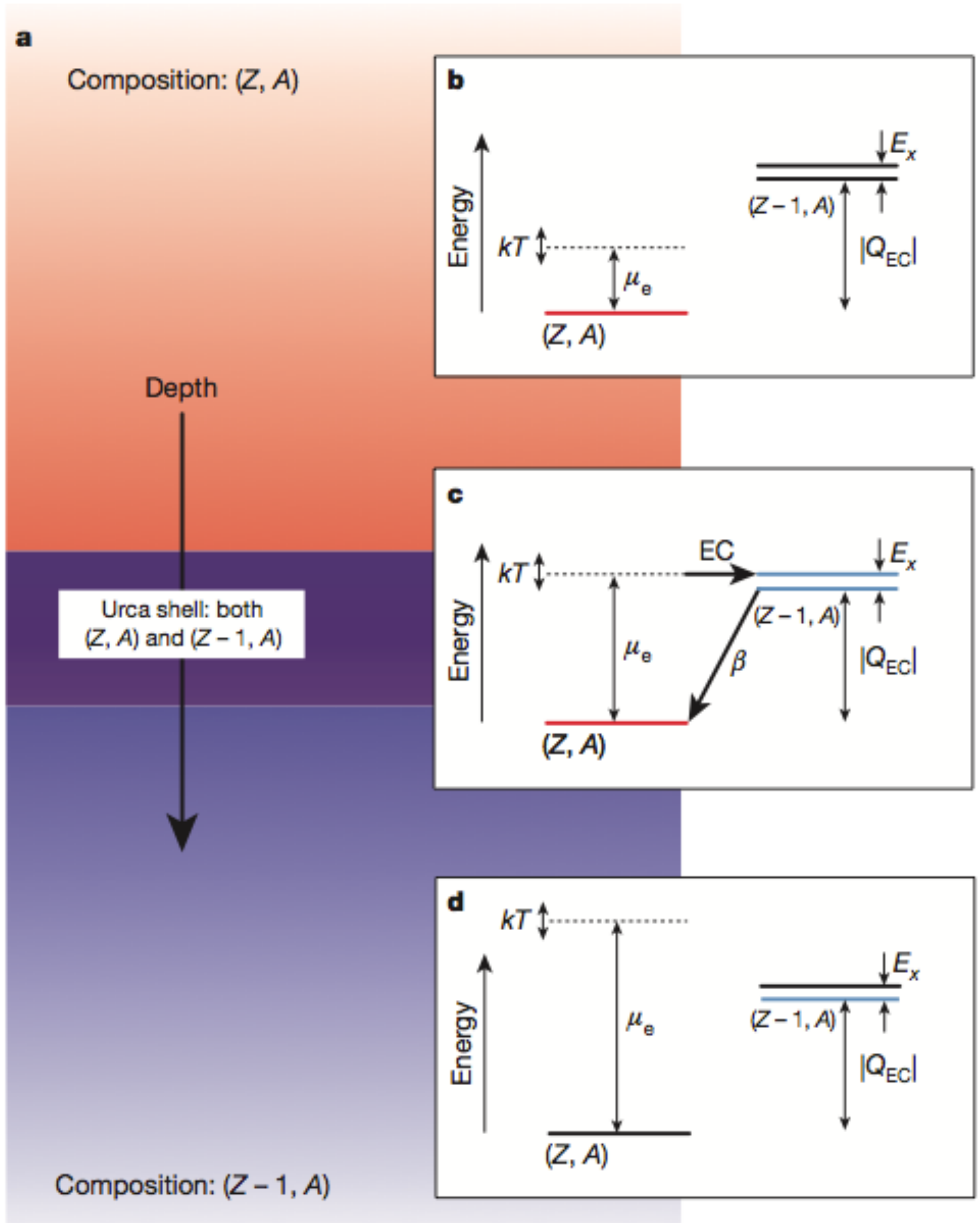
Quiescent luminosity of transiently accreting neutron stars



Heinke et al. (2010)

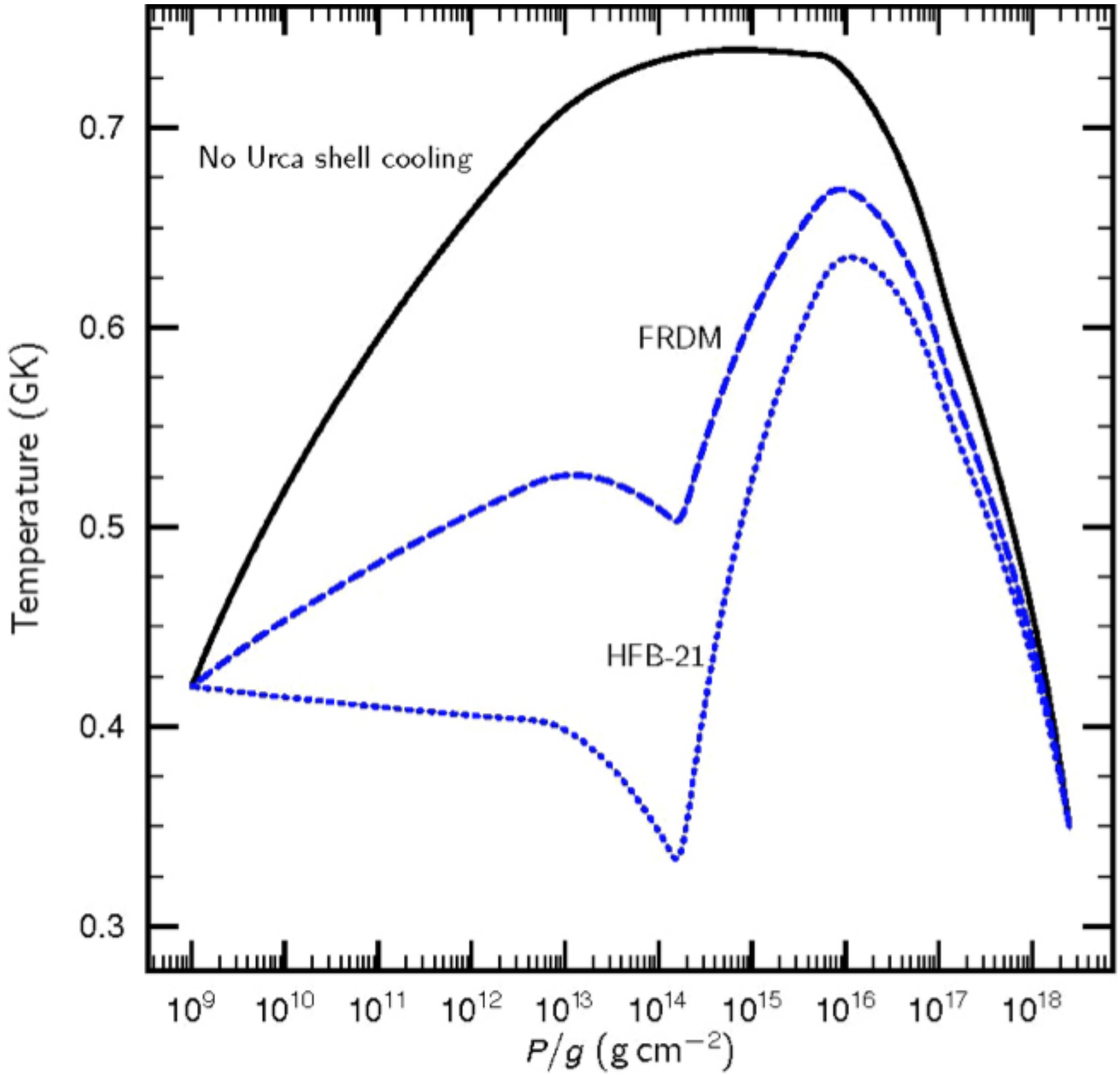
URCA in the outer crust

Schatz et al. 2013, Nature

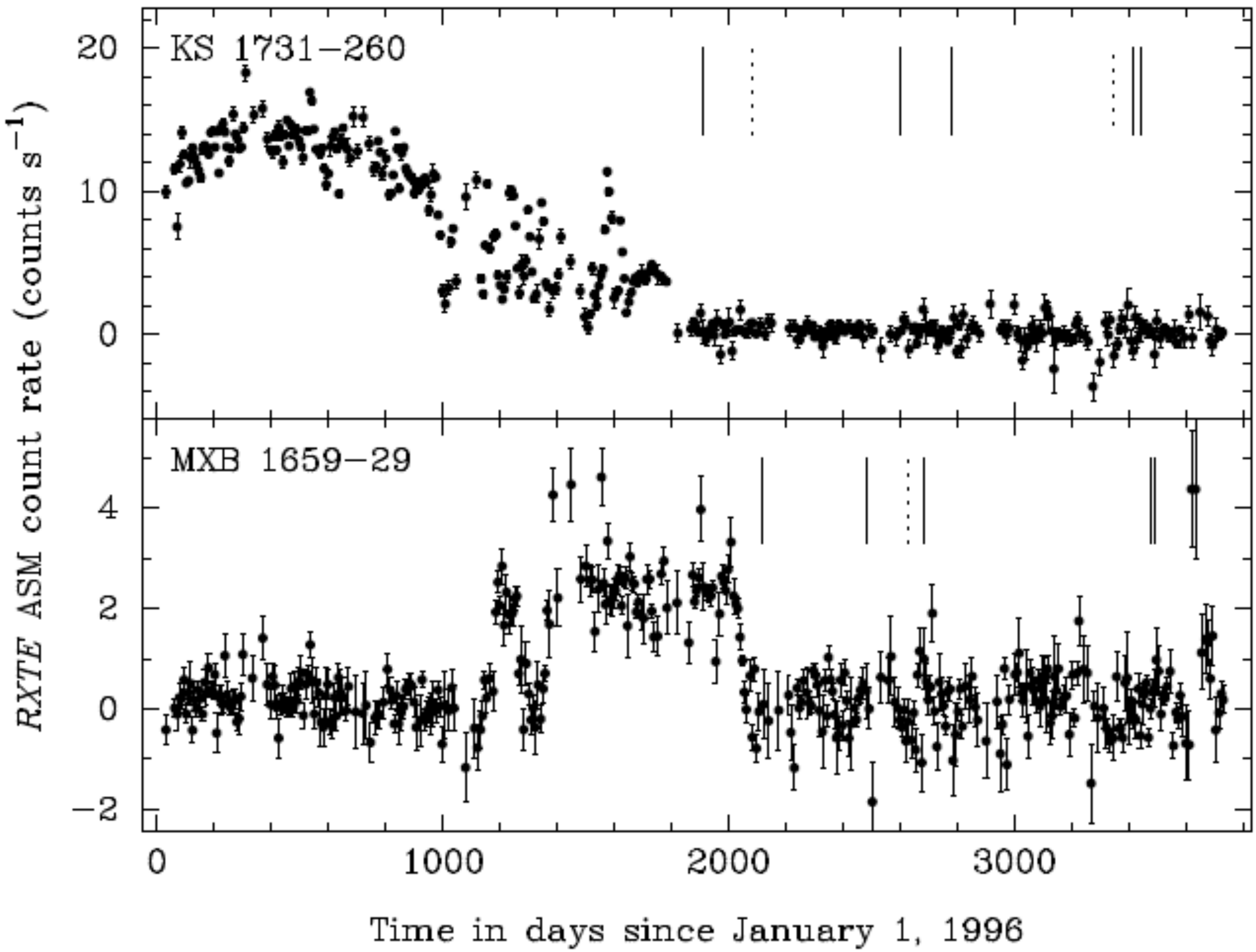


URCA in the outer crust

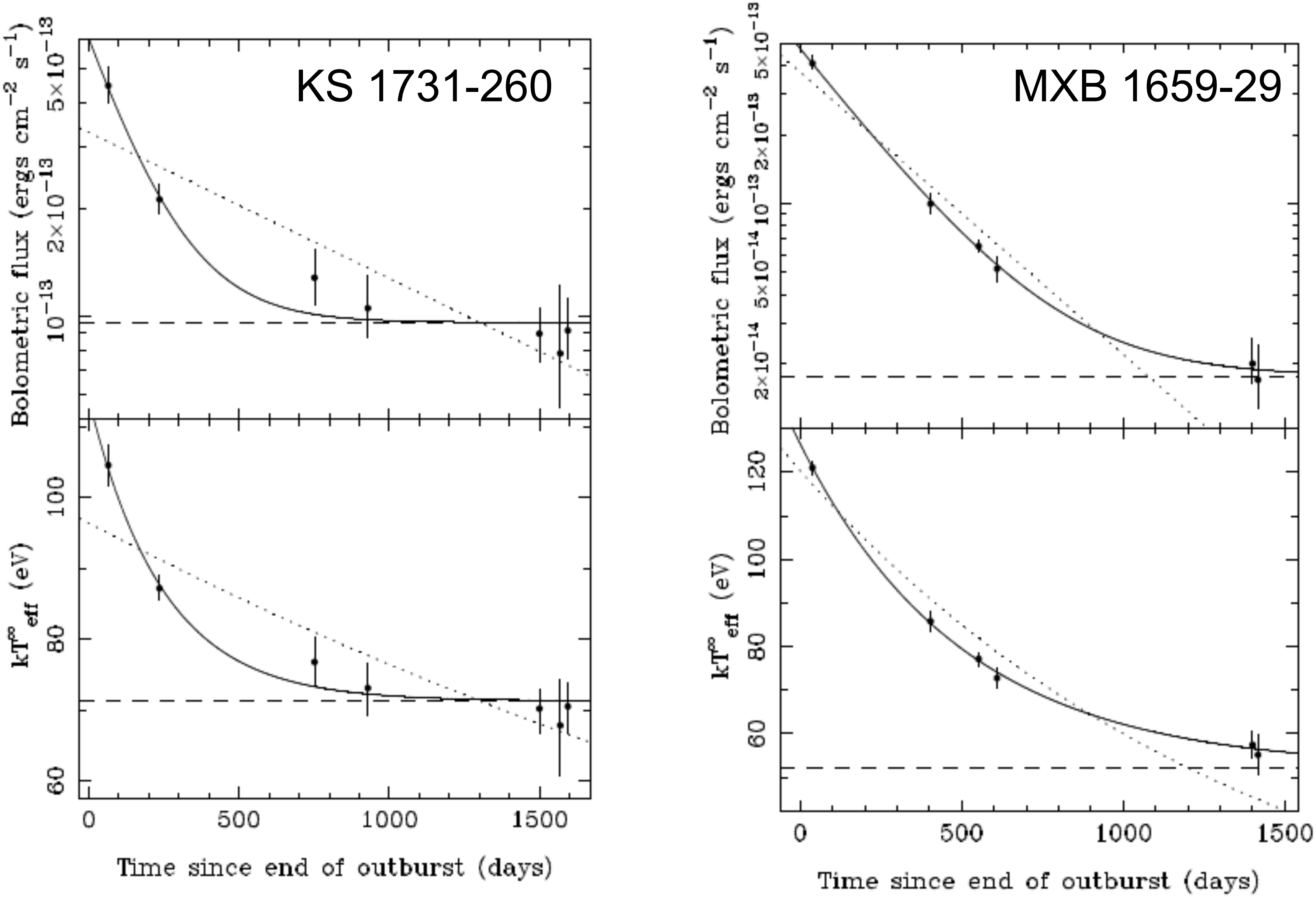
Schatz et al. 2013, Nature



Two transients with long duration outbursts

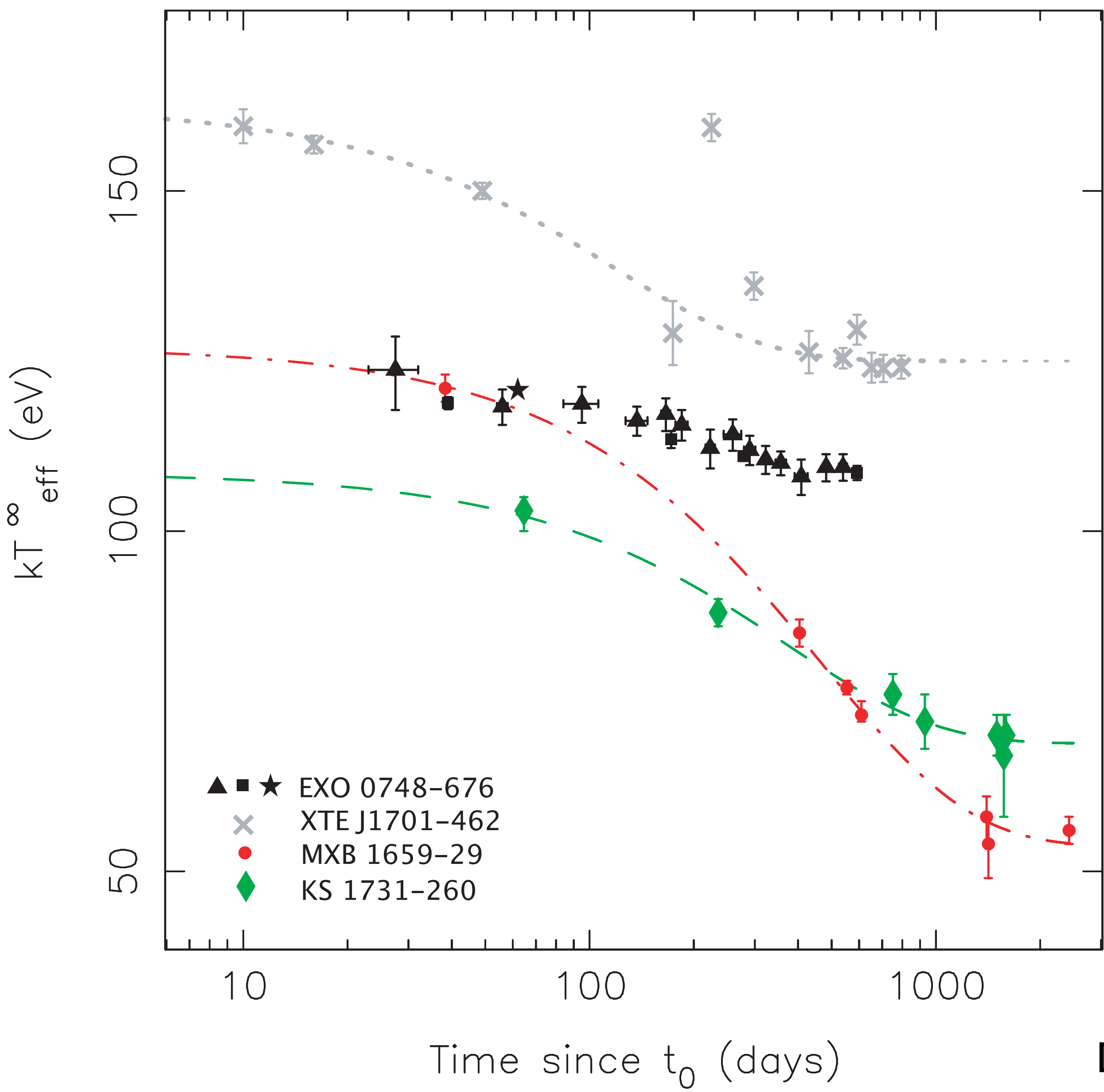


Cackett et al. (2006)



Cackett et al. (2006)

Cooling in accreting neutron star transient LMXBs



Degenaar et al. (2010)
see also Cackett et al. (2006 ...)

Calculation of crust cooling

- The hydrostatic structure of the crust doesn't depend on temperature, so solve separately.
- Follow the thermal evolution in time

$$\frac{\partial}{\partial t}\left(T e^{\phi/c^2}\right) = e^{2\phi/c^2}\frac{\epsilon_{\rm nuc}-\epsilon_\nu}{C} - \frac{1}{4\pi r^2 \rho C(1+z)}\frac{\partial}{\partial r}\left(L e^{2\phi/c^2}\right)$$

$$L e^{2\phi/c^2} = -\frac{4\pi r^2 K e^{\phi/c^2}}{1+z}\frac{\partial}{\partial r}\left(T e^{\phi/c^2}\right)$$

- Need to understand
  - heating $\epsilon_{\rm nuc}$
  - heat capacity

    dominated by the lattice through most of the crust

$$\begin{aligned} C &\approx 3Nk_B & T \gg \Theta_D \\ &\sim 3Nk_B\left(\frac{T}{\Theta_D}\right)^3 & T \ll \Theta_D \end{aligned}$$

  - thermal conductivity

    conduction by electrons which scatter from **phonons** and **impurities**

$$Q_{\rm imp} \equiv n_{\rm ion}^{-1}\sum_i n_i (Z_i - \langle Z\rangle)^2$$

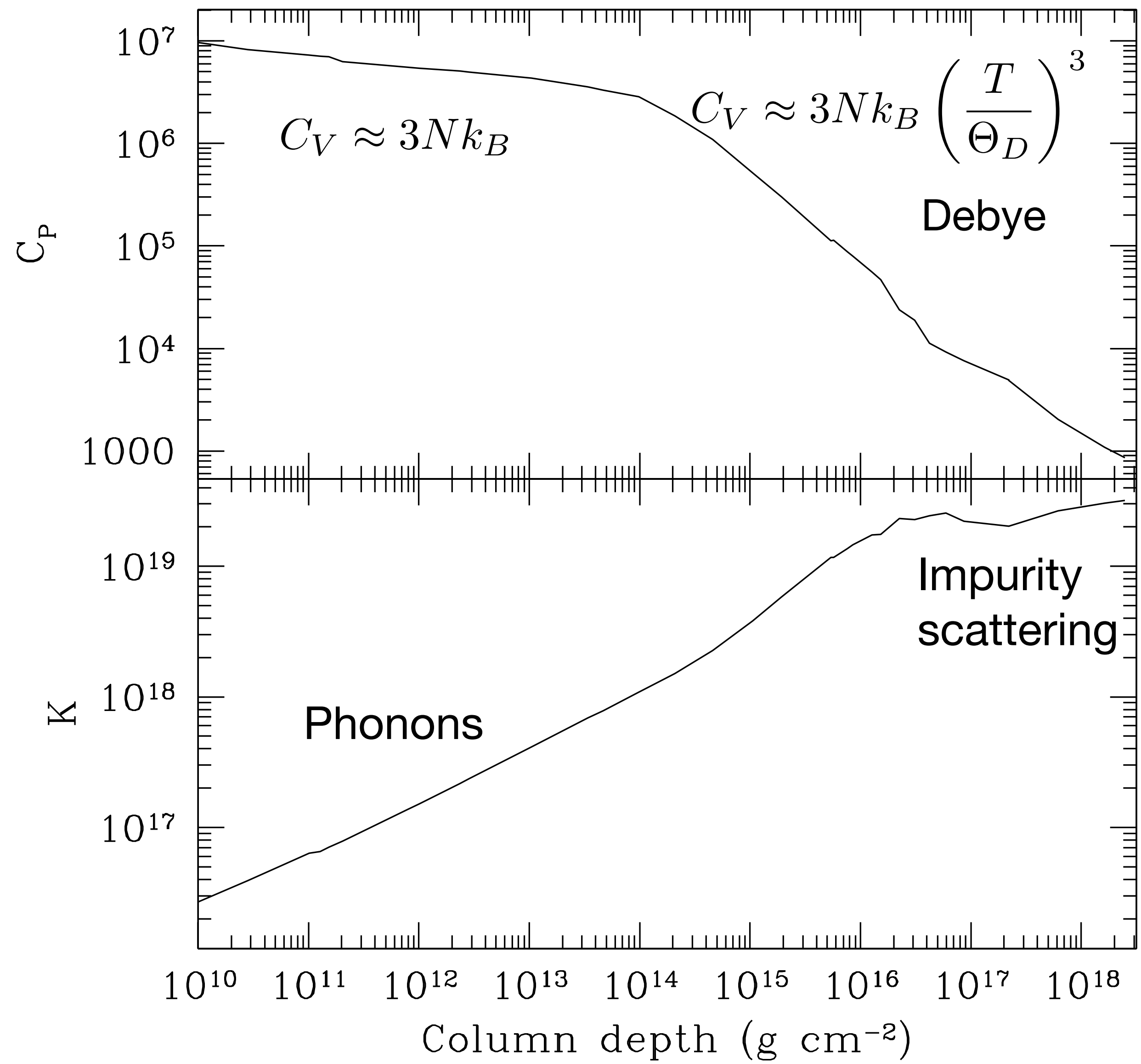
Heat capacity and thermal conductivity in the crust

thermal time

$$\tau = \left[\int_0^z \left(\frac{\rho C_P}{4K} \right)^{1/2} dz \right]^2$$

thermal conductivity

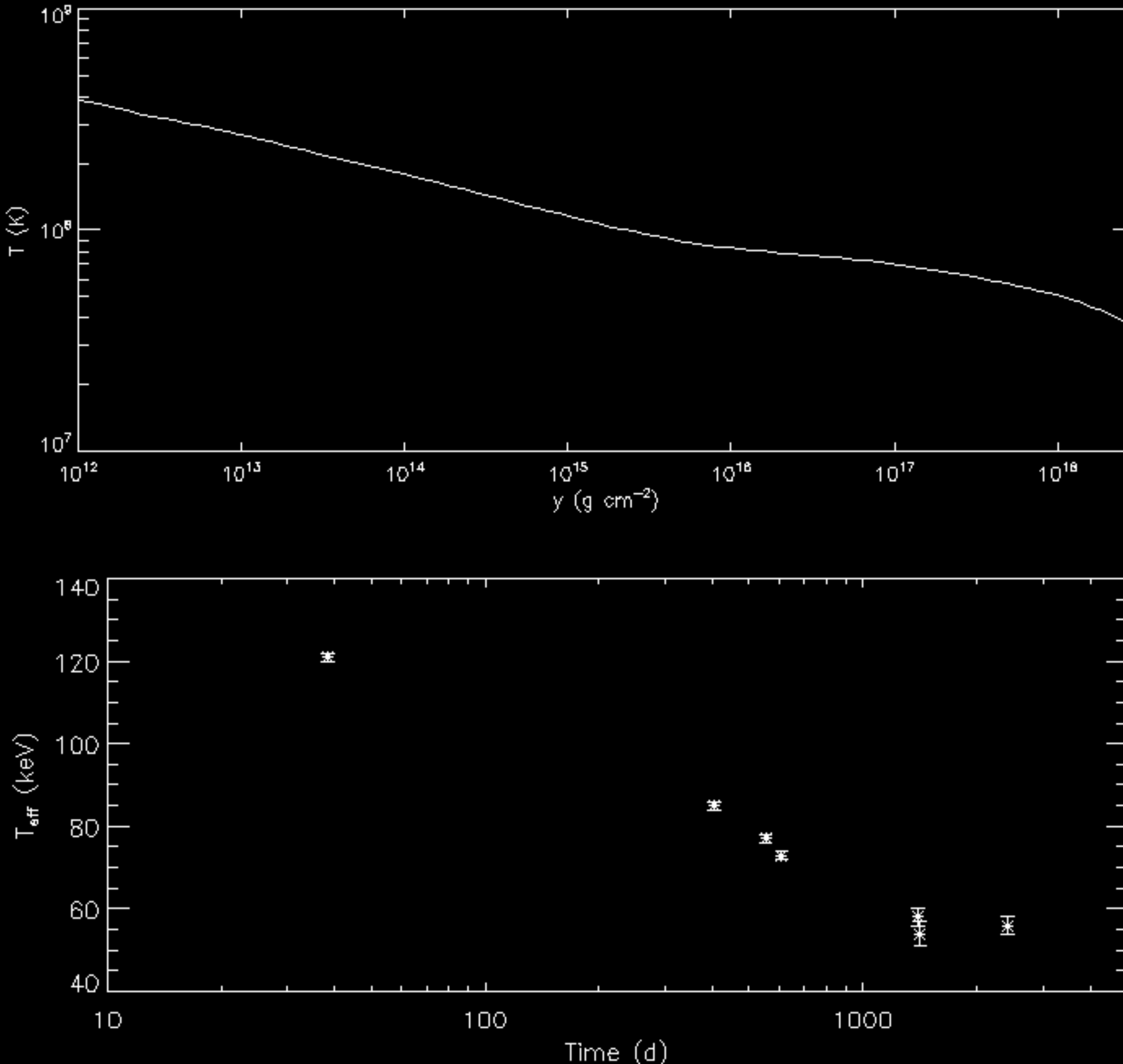
$$K = \frac{\pi^2}{3} \frac{n_e k_B^2 T \tau}{m_\star}$$



Outer crust | Inner crust

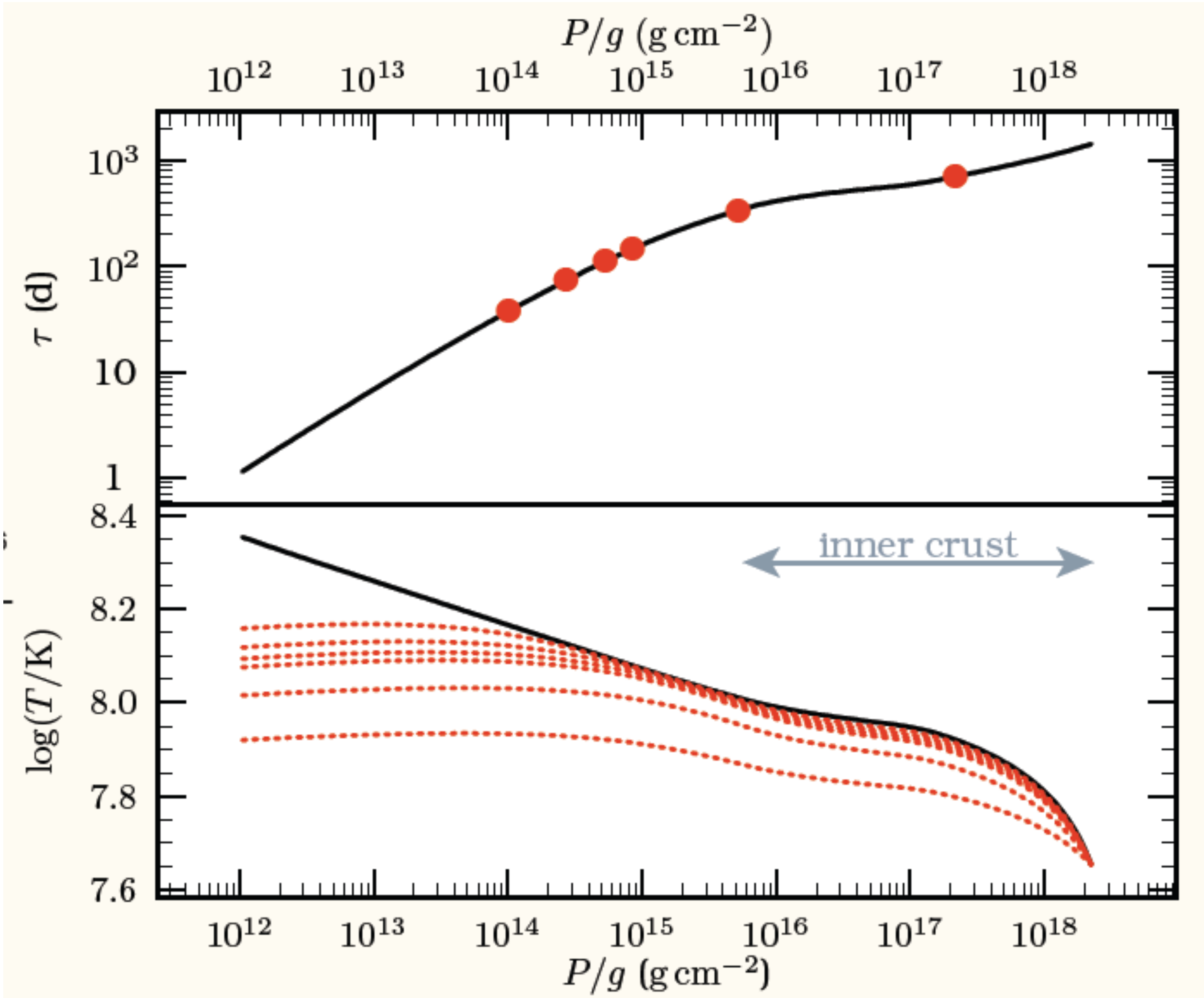
Cooling model for MXB1659-29

Brown & Cumming (2009)

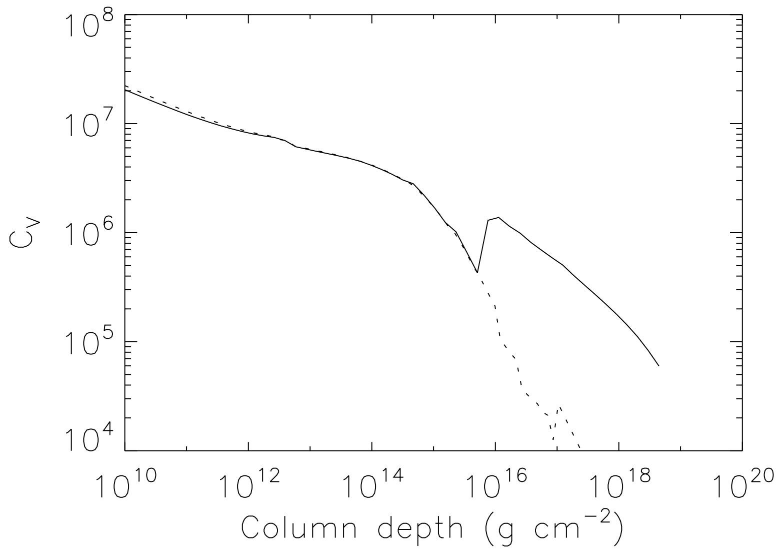


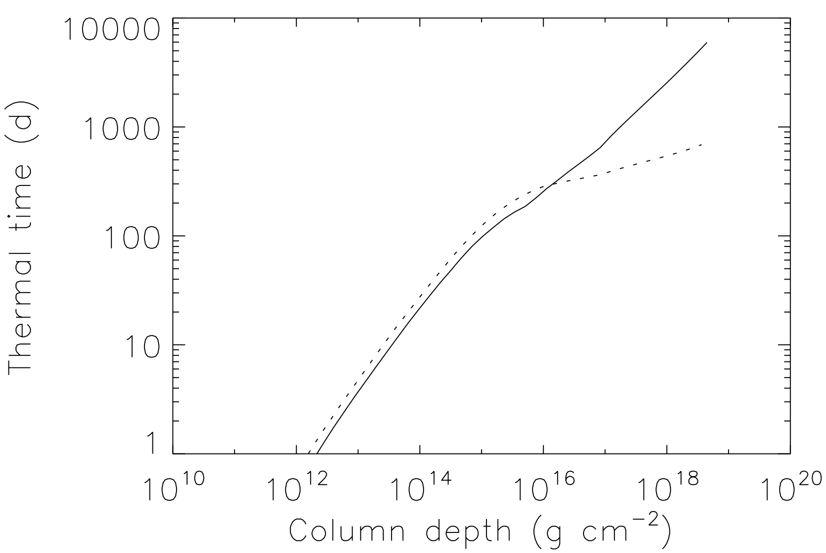
Simple understanding of the lightcurve

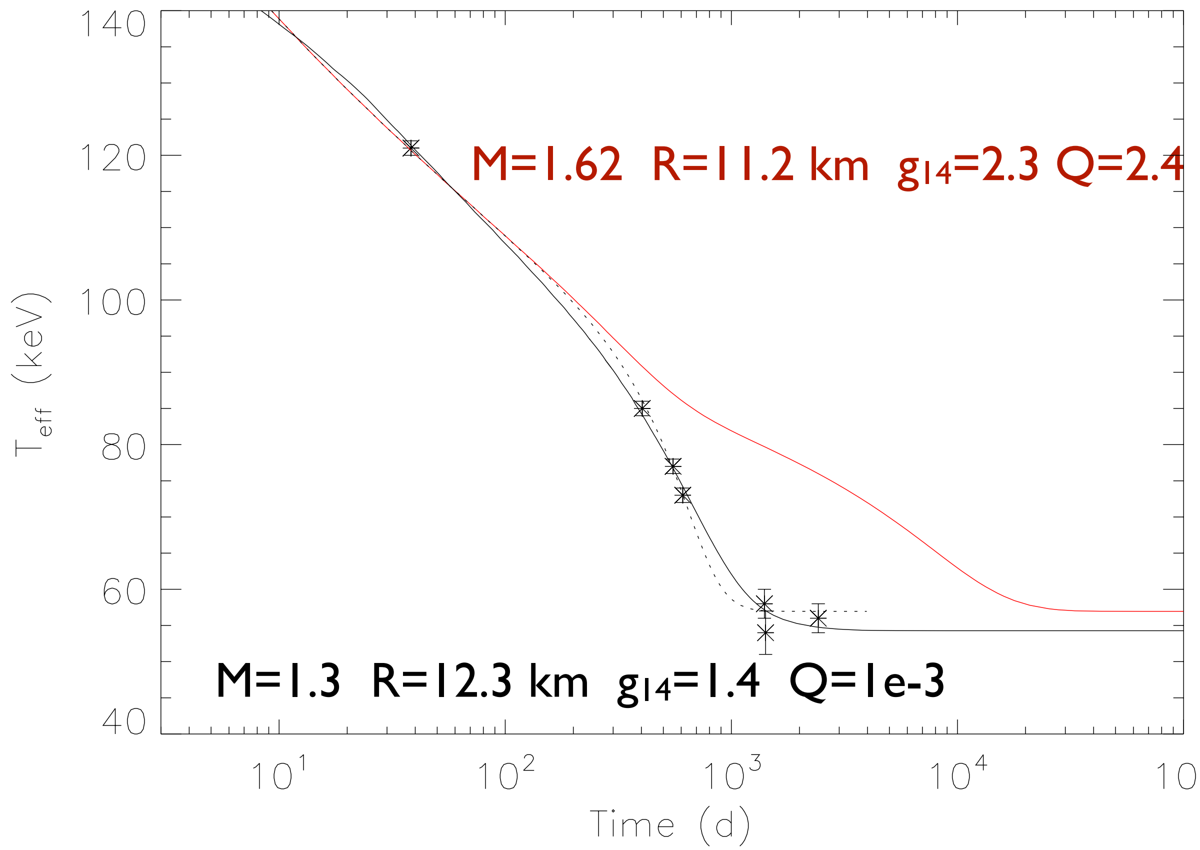
Brown & Cumming (2009)



Normal neutrons would significantly delay the cooling







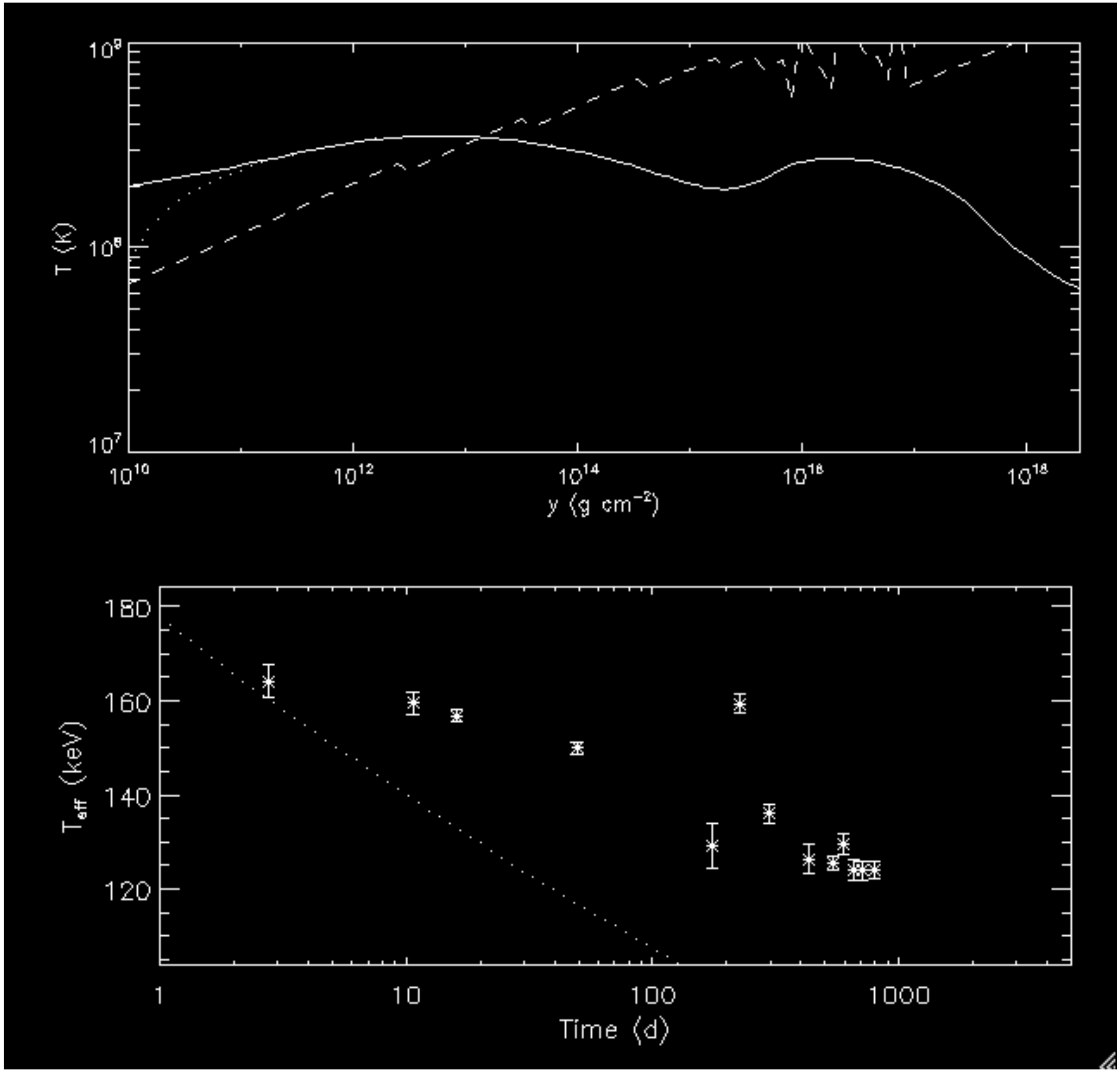
$$\tau^\infty \propto (1+z)/g^2$$

Cooling model for XTEJ1701-462

Much hotter than 1659 and 1731.

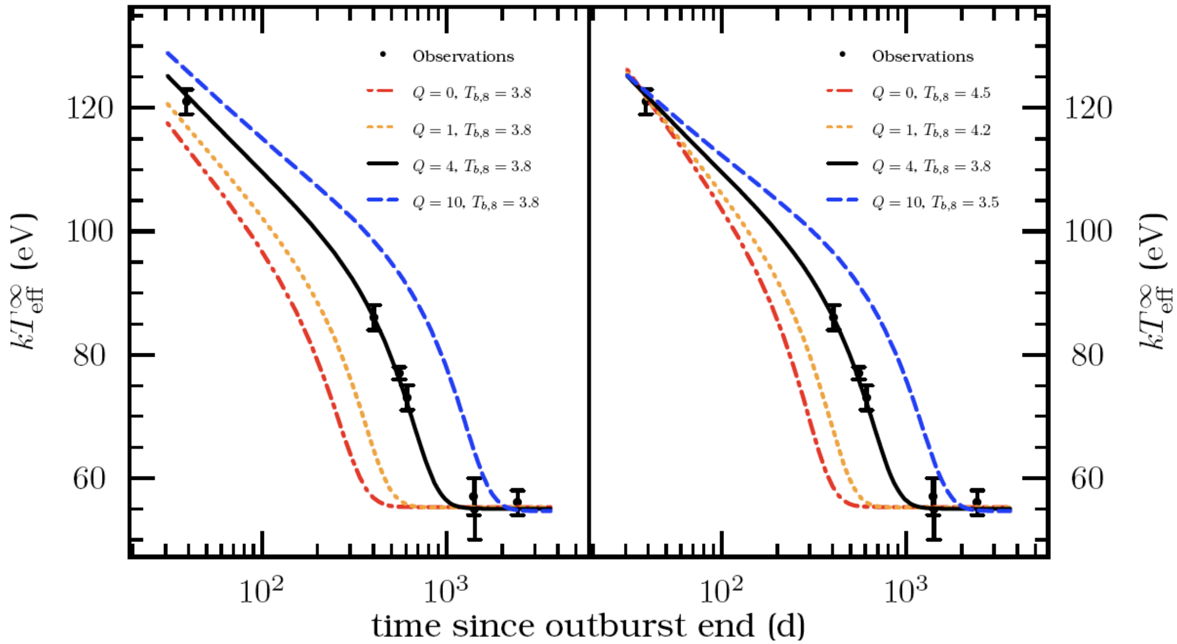
Crust does not reach thermal equilibrium during the outburst.

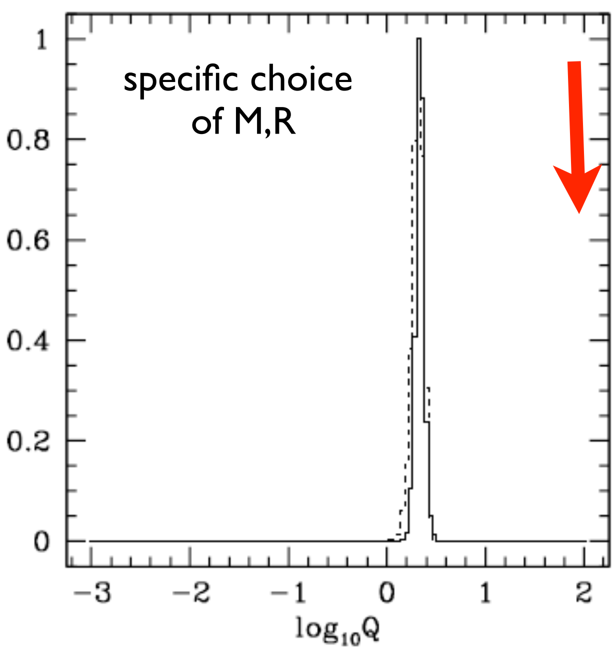
Need a large impurity parameter / low thermal conductivity in the inner crust' (here Q=100)

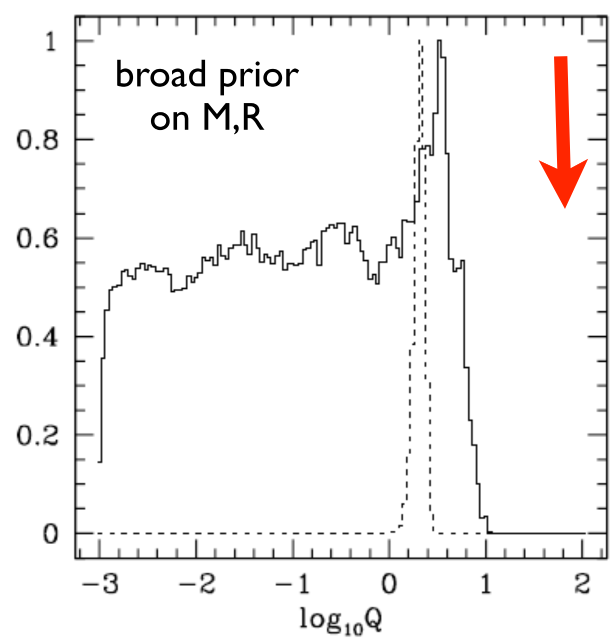


Constraint on the impurity parameter

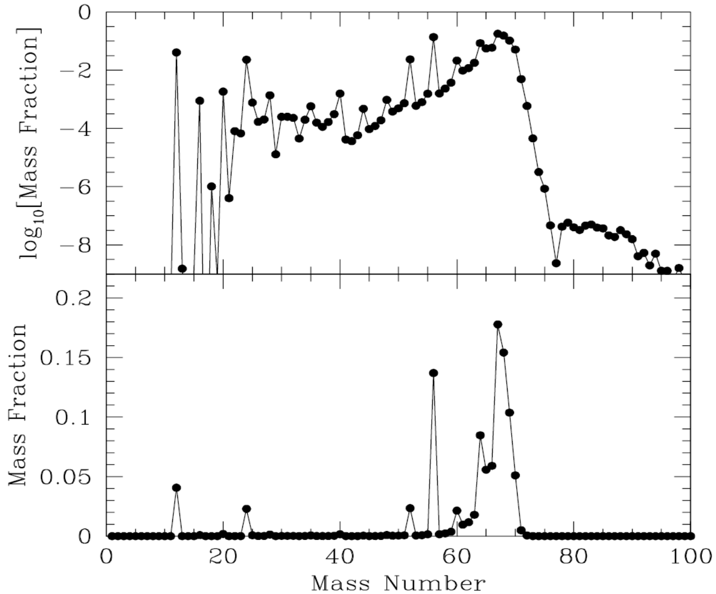
Brown & Cumming (2009)



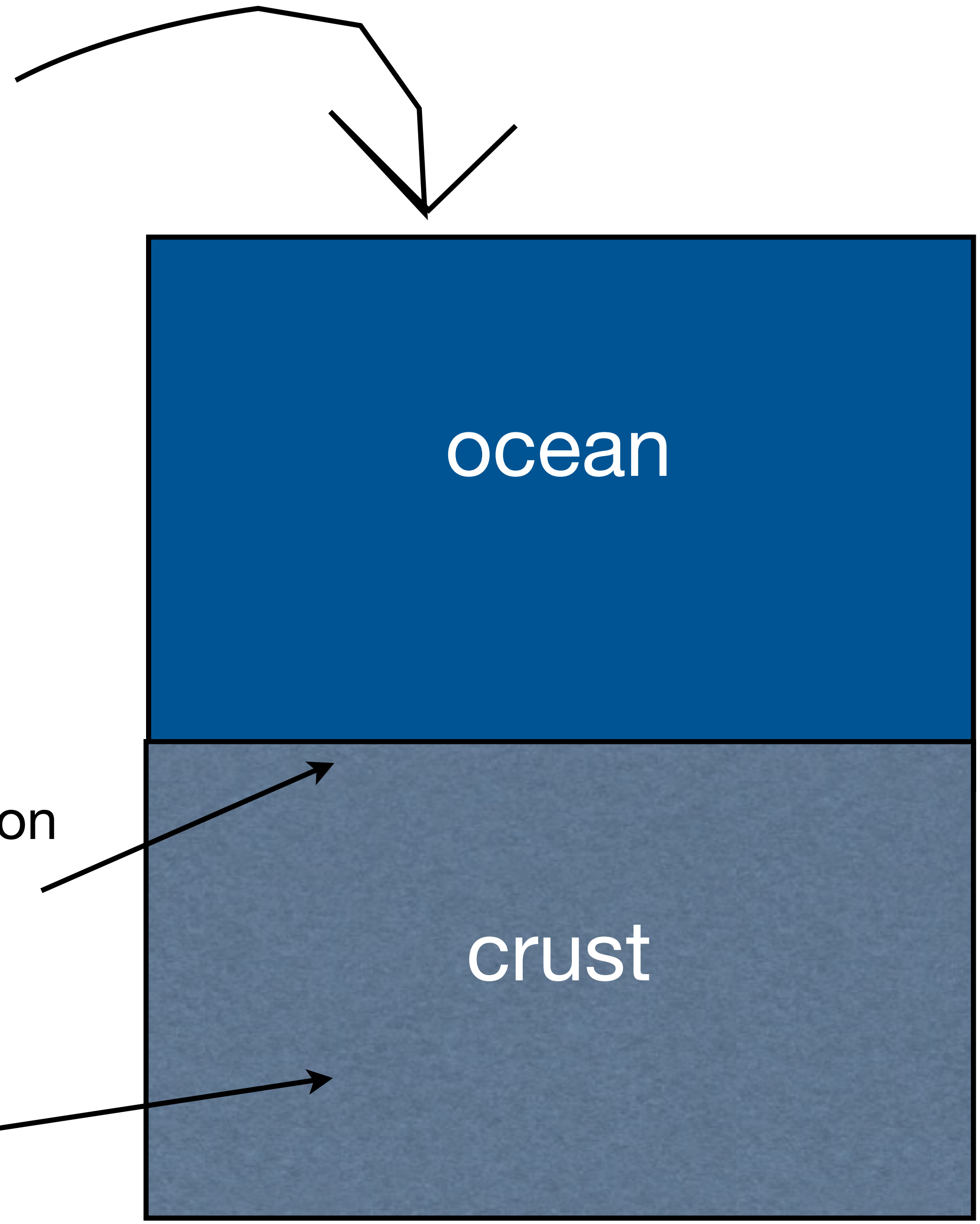




rp process ash



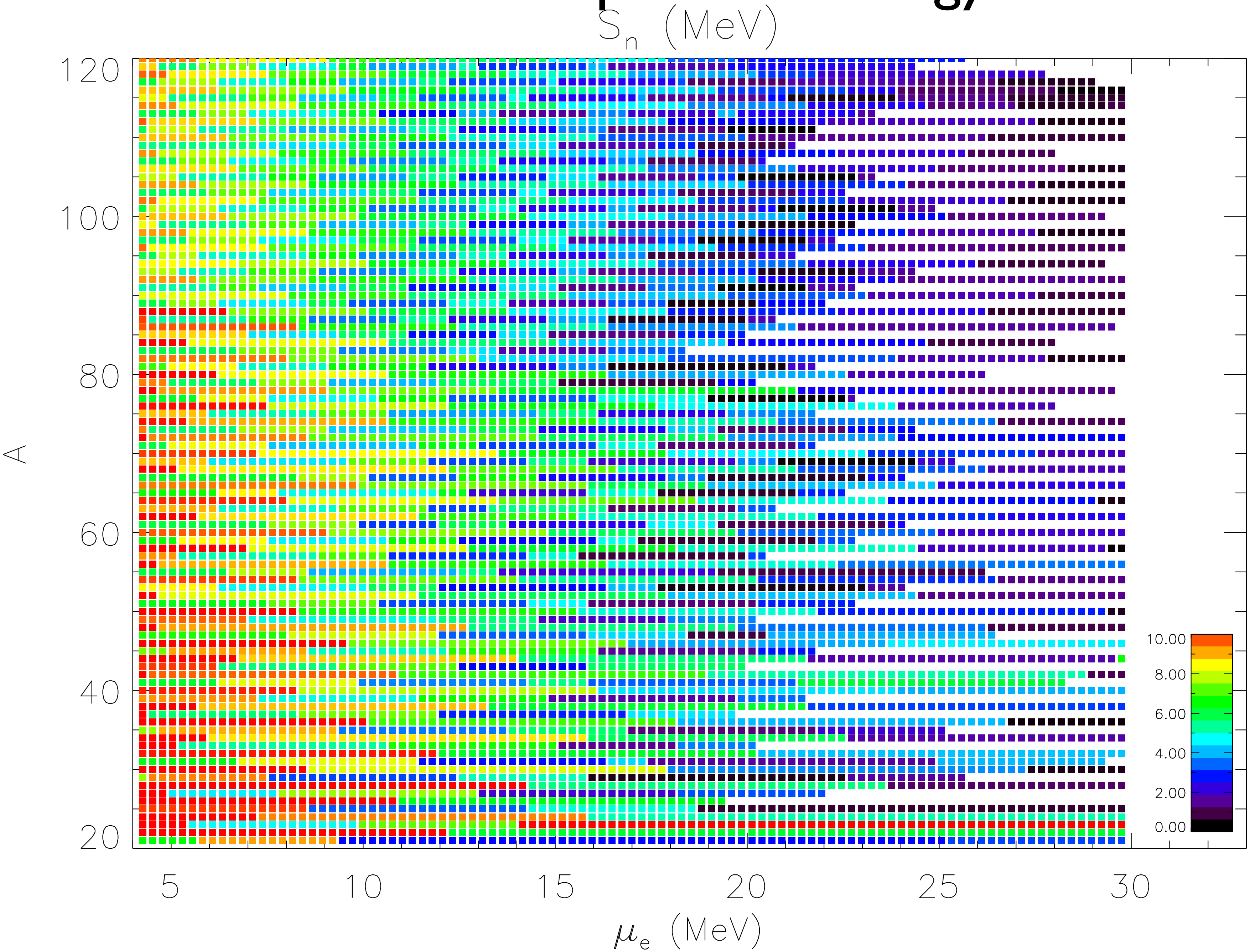
Schatz et al. 1999



chemical separation on freezing

nuclear processing: electron captures, pynonuclear reactions

neutron separation energy



Simple model of evolution of a mixture through the crust

- Monte Carlo approach: follow N nuclei to increasing pressure. At each pressure, follow all reactions that are energetically favorable.

- Reactions considered

TABLE 1

REACTIONS CONSIDERED ($Q > 0$ INDICATES THAT THE REACTION IS FAVORABLE)

Reaction	Q value
n emission	$Q_1 = M_{ex}(A,Z) - M_{ex}(A-1,Z) - (\mu_n + m_n - m_u)$
$2n$ emission	$Q_2 = M_{ex}(A,Z) - M_{ex}(A-2,Z) - 2(\mu_n + m_n - m_u)$
n capture	$Q_3 = M_{ex}(A,Z) + \mu_n + m_n - (M_{ex}(A+1,Z) + m_u)$
p capture	$Q_4 = M_{ex}(A,Z) + \mu_n + m_n - (M_{ex}(A+1,Z+1) + m_u - m_e + \mu_e)$
$2n$ capture	$Q_5 = M_{ex}(A,Z) + 2(\mu_n + m_n) - (M_{ex}(A+2,Z) + 2m_u)$
np-capture	$Q_6 = M_{ex}(A,Z) + 2(\mu_n + m_n) - (M_{ex}(A+2,Z+1) + 2m_u + \mu_e - m_e)$
n transfer	$Q_7 = M_{ex}(A_i,Z_i) + M_{ex}(A_j,Z_j) - M_{ex}(A_i-1,Z_i) - M_{ex}(A_j+1,Z_j)$

\+ pycnonuclear according to Yakovlev et al. but with constant S factor

- use FRDM (Möller et al) and Mackie & Baym mass models to calculate Q values

Compressible liquid drop model for nuclei in the crust

Mackie & Baym (1977)

$$W_N(A,x,k,k_n) = \left[(1-x)\,m_n c^2 + x m_p c^2 + W(k,x)\right] A \cdot$$

energy of bulk matter with x=Z/A and density

$$n_i = 2k^3/3\pi^2$$

$$+ W_{\rm surf}(A,x,k,k_n) + W_{\rm Coul}(A,x,k,k_n)$$

surface energy takes into account the external neutron gas

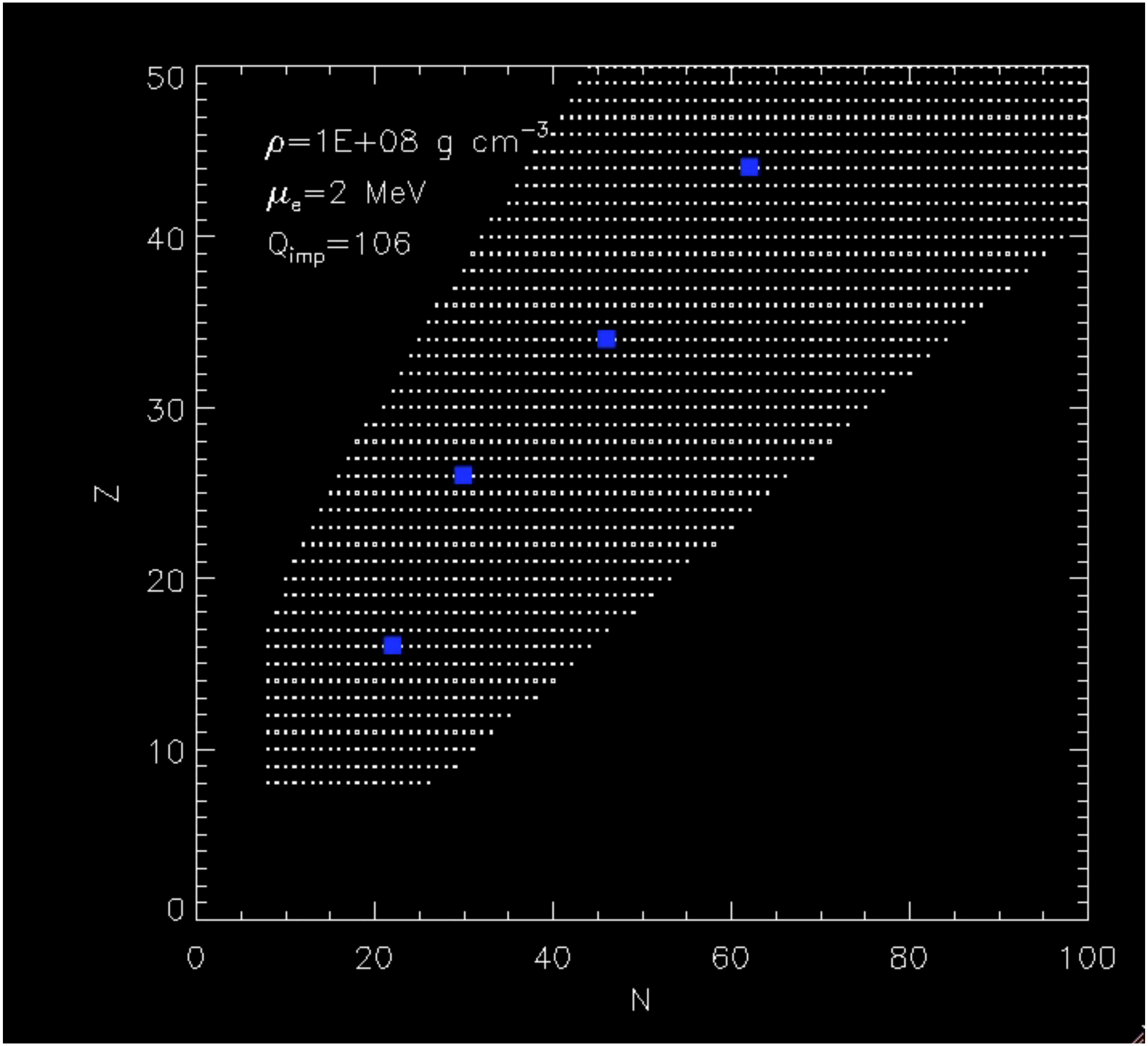
Coulomb energy includes the lattice energy

$$W^0_{\rm surf} = \sigma \frac{(W_0 - W_i)^{1/2}}{w_0^{1/2}} \frac{(n_i - n_0)^2}{n^2_{NM}} \frac{k_0^2}{k^2} A^{2/3}$$

For a given neutron density, Z and A, solve for the size of the nucleus (k) such that there is pressure balance

$$\frac{Ak}{4\pi r_N^3}\frac{\partial W}{\partial k} = P_0 + \frac{(2W_{\rm surf} - W_{\rm Coul})}{A}$$

Nuclear evolution through the crust



The journey of a fluid eler