Layered Convection and Convective Layers in Jupiter and other gas giant planets

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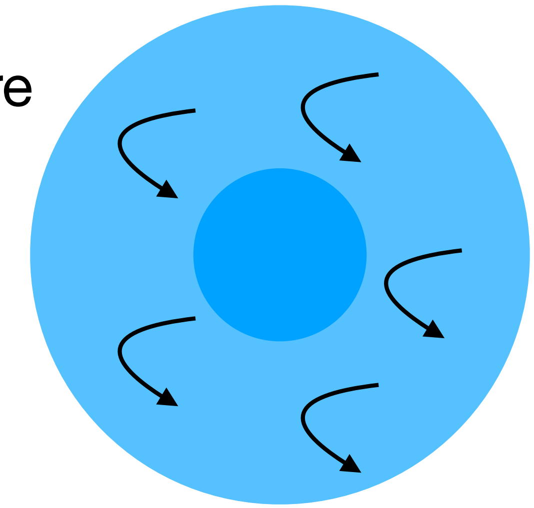
Motivation

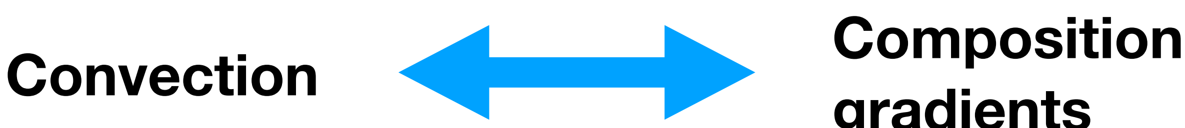
- Composition gradients are a natural outcome of planet formation because heavy elements condense and are delivered separately (pebbles, planetesimals)

- Core accretion => core-envelope structure

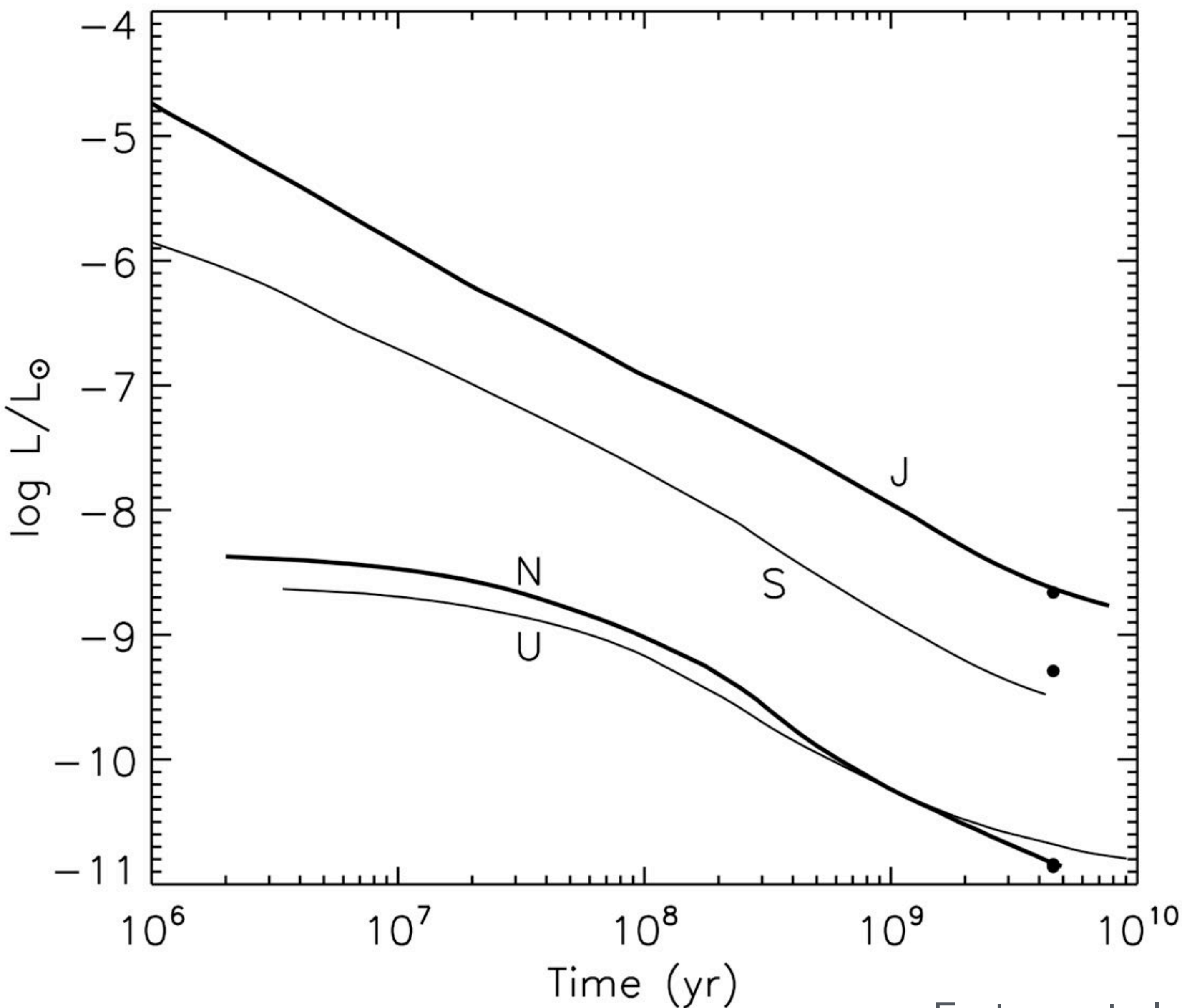


- Elements can separate during the evolution, e.g. helium rain

- Energy transport is usually/often by convection. How do they interact?

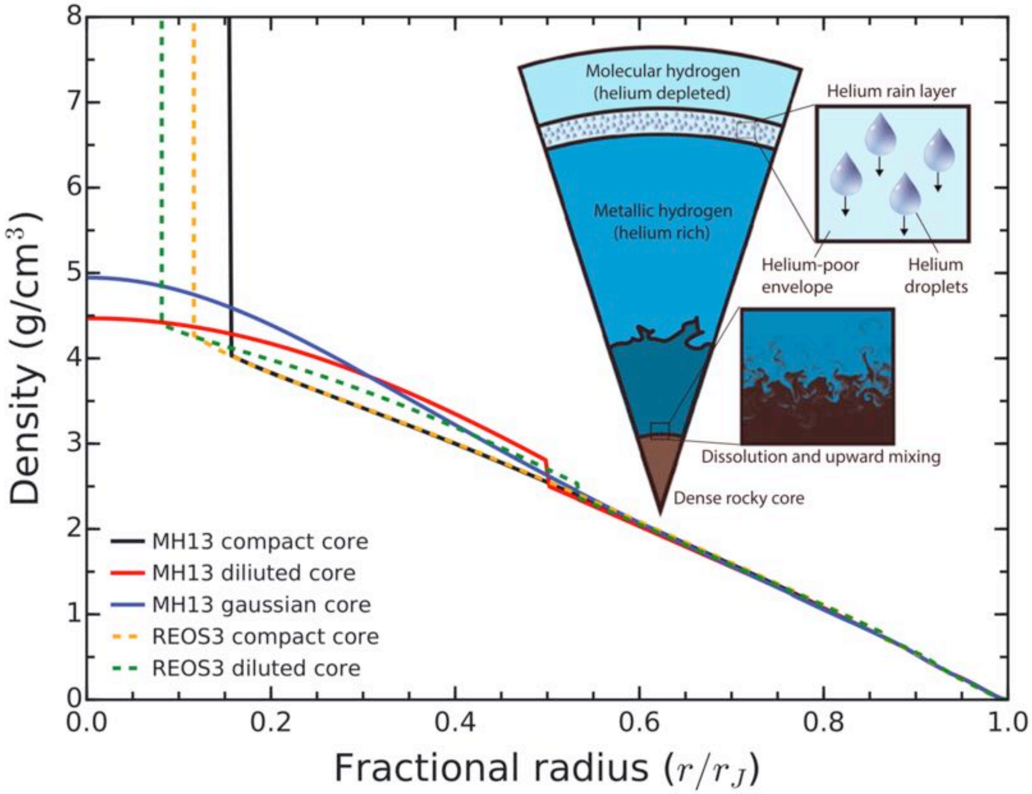


Composition effects likely explain some puzzles in the cooling of the Solar System giant planets



Fortney et al. (2011)

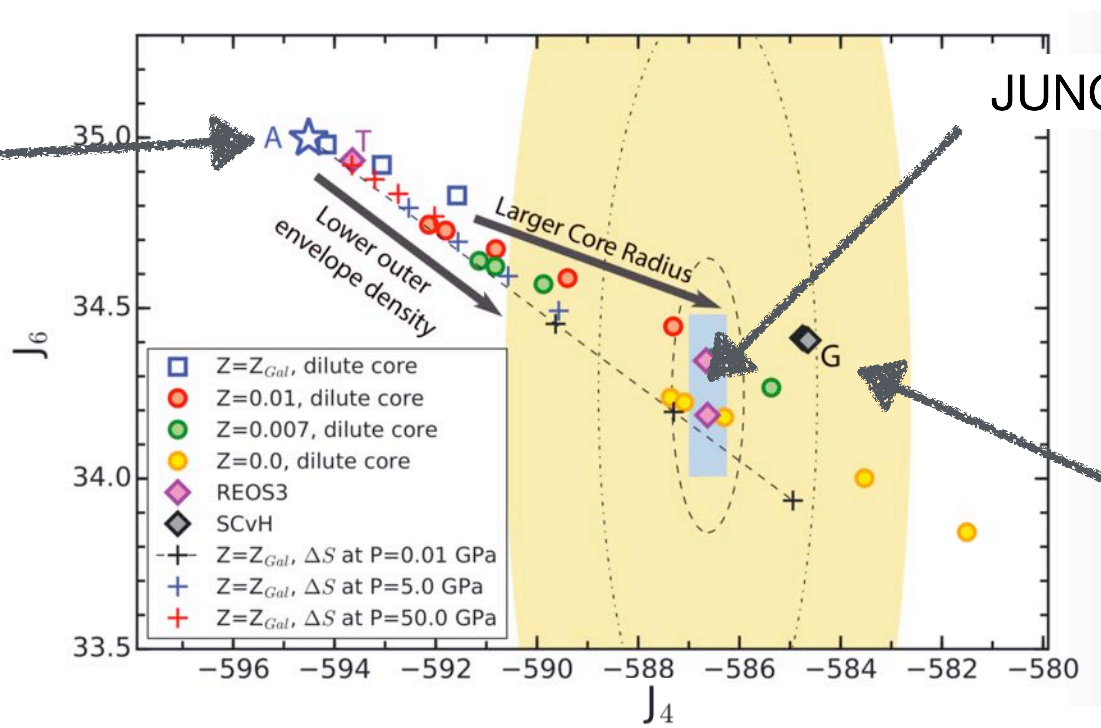
Juno gravitational moments suggest a diluted core in Jupiter



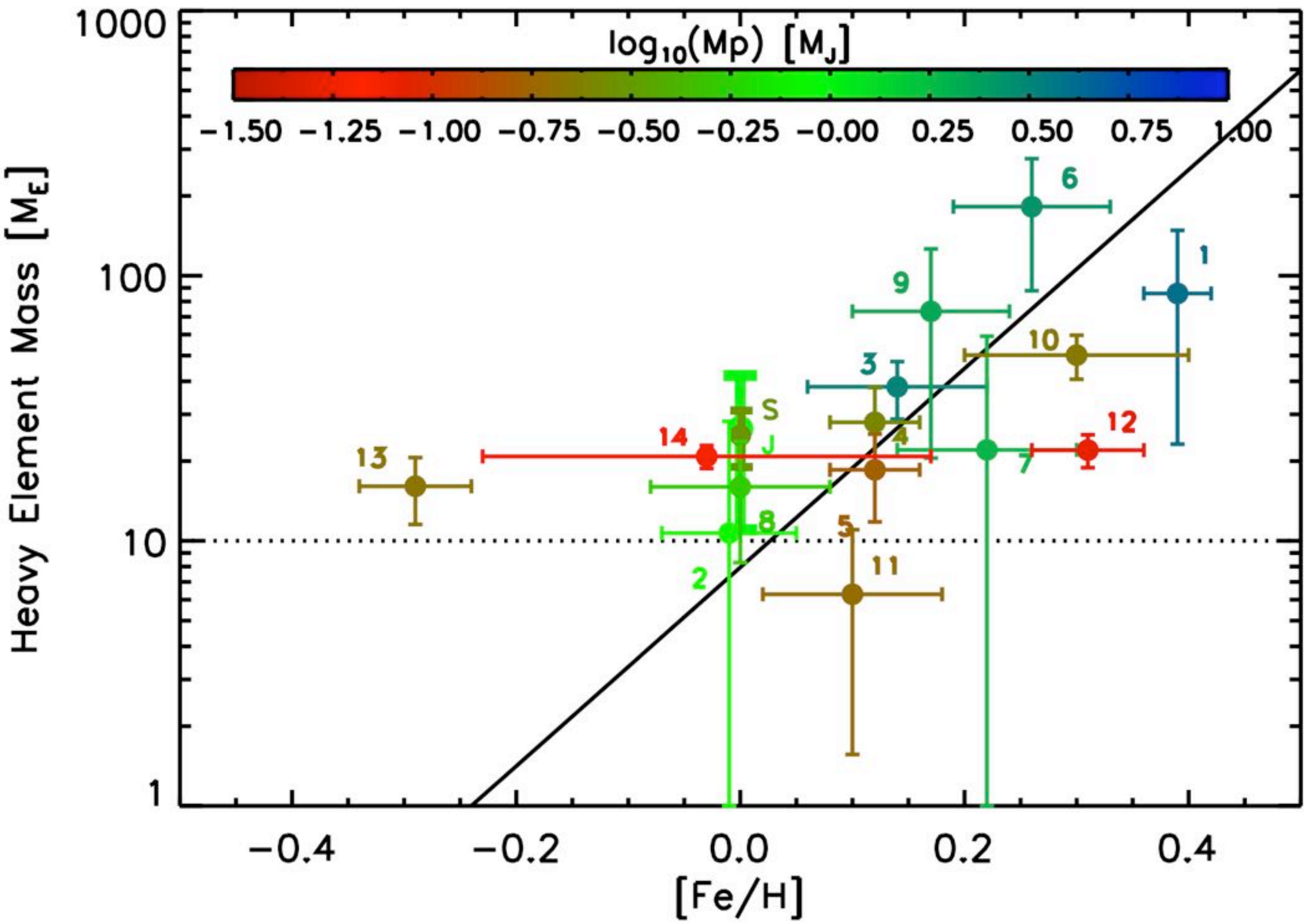
Wahl et al. (2017)

Standard Jupiter model

JUNO



SCVH



Miller & Fortney (2011)

This talk

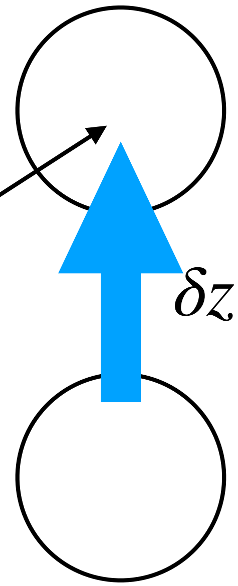
- Interaction of convection with composition gradients and how it might apply to Jupiter
- Convective layers in 1D
- 2D Boussinesq simulations of a salinity gradient cooled from above

Interaction between convection and composition gradients

Thermal convection

Fluid element

$$\frac{\delta\rho}{\rho} = \left(\frac{d\ln P}{dz}\delta z\right)^{1/\gamma}$$



Background

$$\frac{\delta\rho}{\rho} = \frac{d\ln\rho}{dz}\delta z$$

The adiabatic gradient is marginally stable

$$\nabla > \nabla_{\rm ad} \quad \text{=> convection}$$

$$\nabla = \frac{d\ln T}{d\ln P}$$

$$\nabla_{\rm ad} = \left.\frac{\partial\ln T}{\partial\ln P}\right|_S$$

Interaction between convection and composition gradients

Thermal convection

Large heat flux $F \sim \rho v_c c_P T(\nabla - \nabla_{\rm ad})$

$$v_c^2 \sim g\ell(\nabla - \nabla_{\rm ad})$$

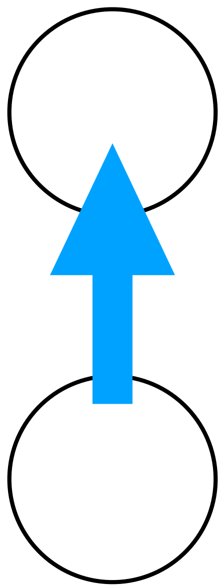
mixing length

Typically need $\nabla - \nabla_{\rm ad} \lll 1$

Convection <=> Adiabatic interior

Interaction between convection and composition gradients

With composition gradients:



Light fluid

$$\nabla_\mu = \frac{d \ln \mu}{d \ln P}$$

Heavy fluid

Ledoux criterion

$$\nabla > \nabla_{\rm ad} + \frac{\chi_\mu}{\chi_T} \nabla_\mu$$

=> convection

There is then a composition flux $F_X \sim \rho v_c \ell \nabla_\mu$

Convection <=> Uniform composition

Huppert & Turner (1981)

What happens when $\nabla_{\mathrm{ad}} < \nabla < \nabla_{\mathrm{ad}} + \nabla_{\mu}$?

Hot salty

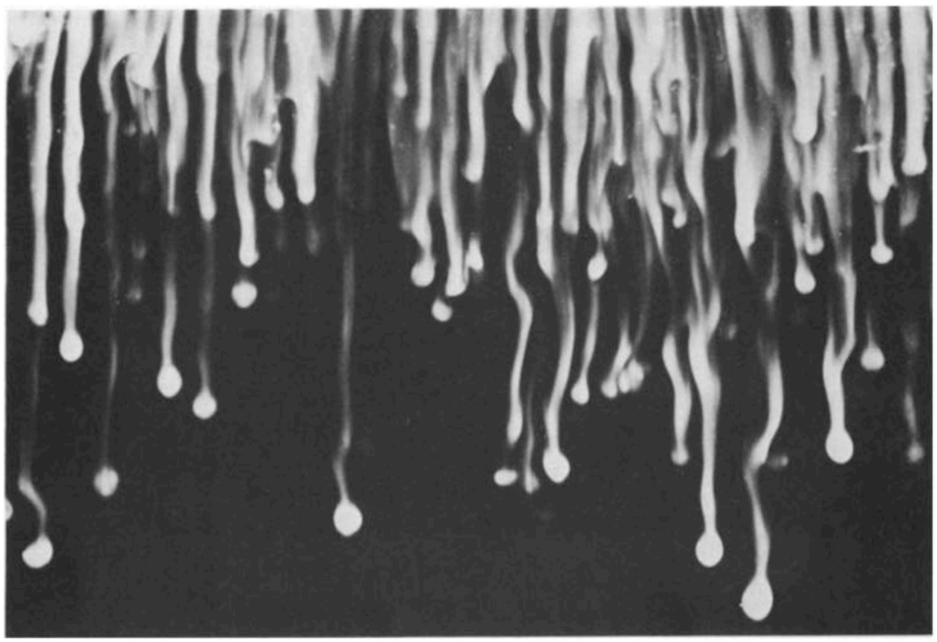


FIGURE 1. A field of salt fingers formed by setting up a stable temperature gradient and pouring a little salt solution on top. The downward-moving fingers were made visible by adding fluorescein to the salt and lighting through a slit from below.

Cold fresh

"Fingering regime"

Thermally stable
compositionally unstable

Cold fresh

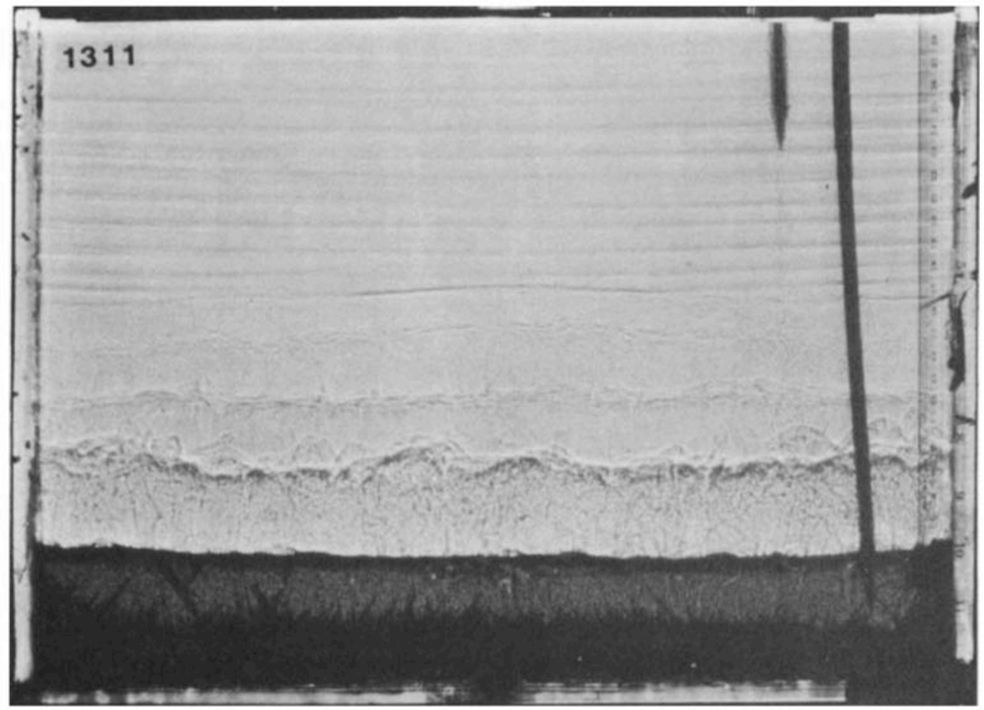


FIGURE 2. A series of convecting layers and 'diffusive' interfaces, formed by heating a gradient of K_2CO_3 solution from below. In this experiment the heating was provided by a hot layer of

Hot salty

Thermohaline convection

Thermally unstable
compositionally stable

Non-linear outcome in both cases: layering and staircases

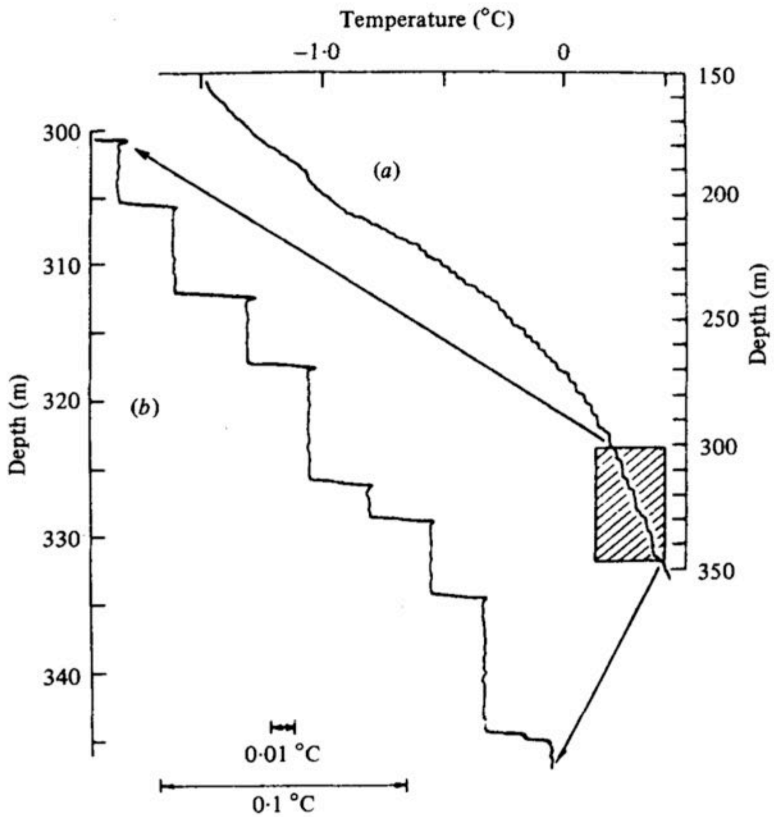
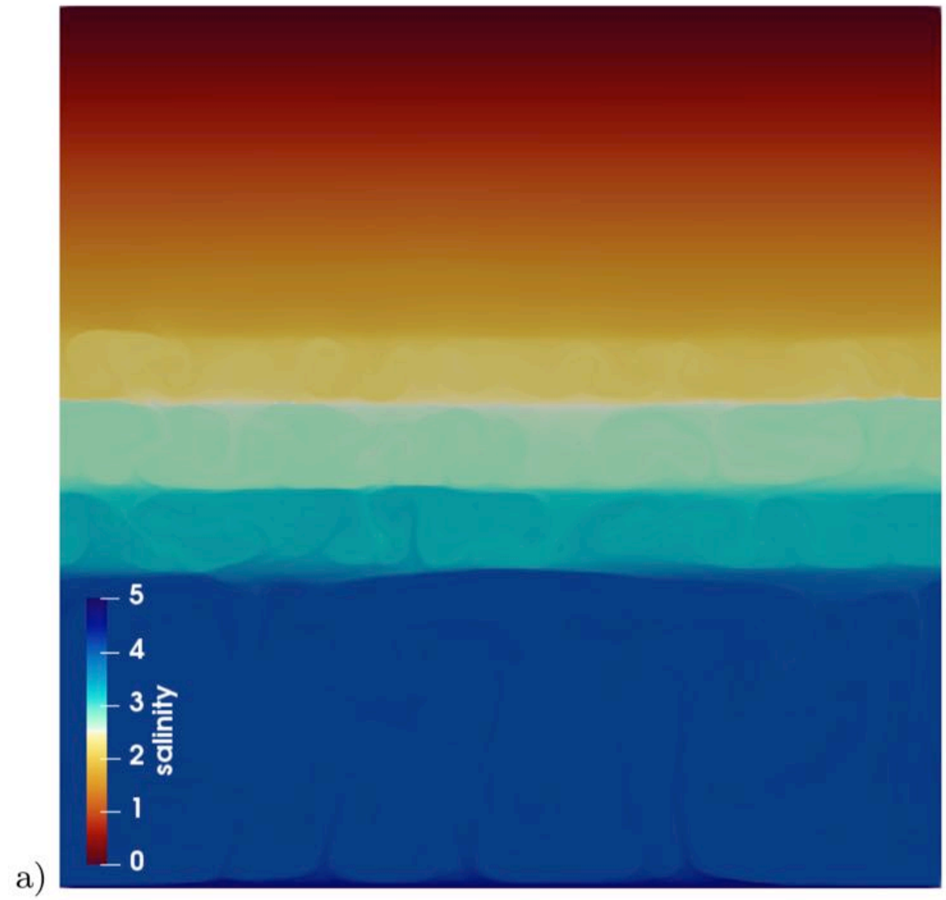


FIGURE 8. A temperature profile obtained under an Arctic ice island by Neal, Neshyba & Denner (1969) showing steps formed by the double-diffusive mechanism. (a) Typical temperature profile section. (b) Section of profile recorded at high gain.

Huppert & Turner (1981)



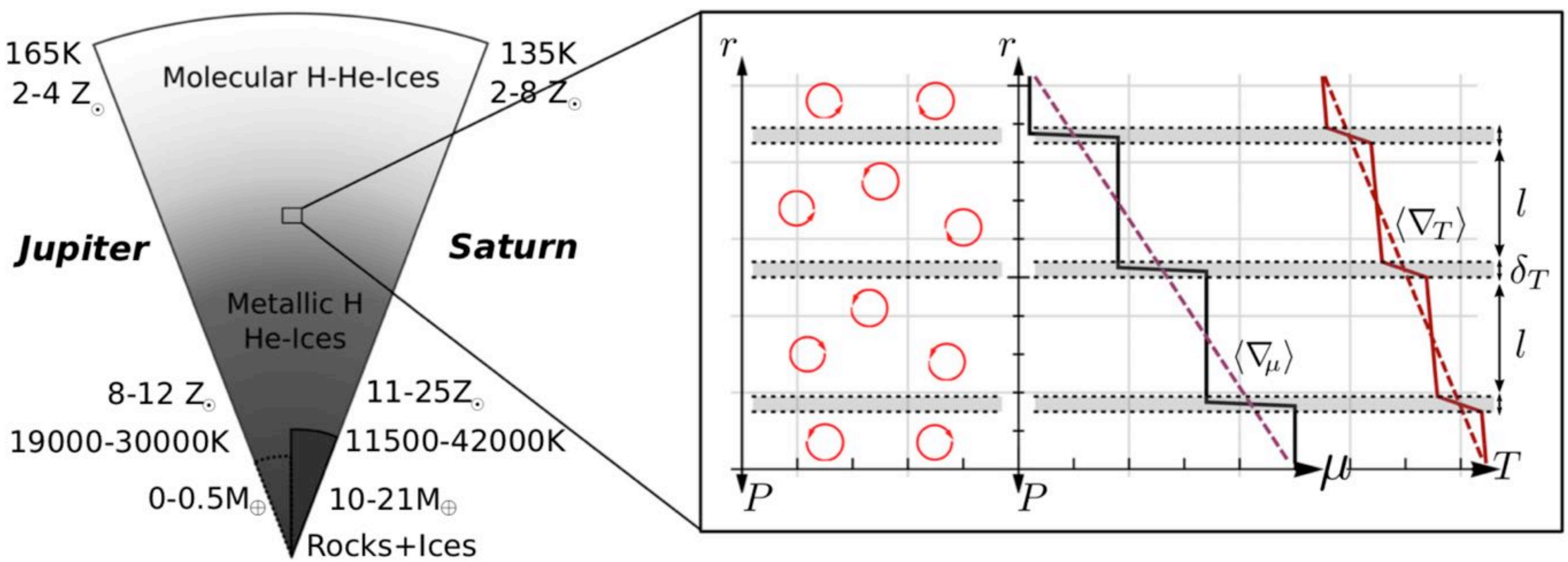
Zaussinger & Kupka (2018)

Layered convection in giant planets

Chabrier & Baraffe (2007): (see also Stevenson 1979,1985)

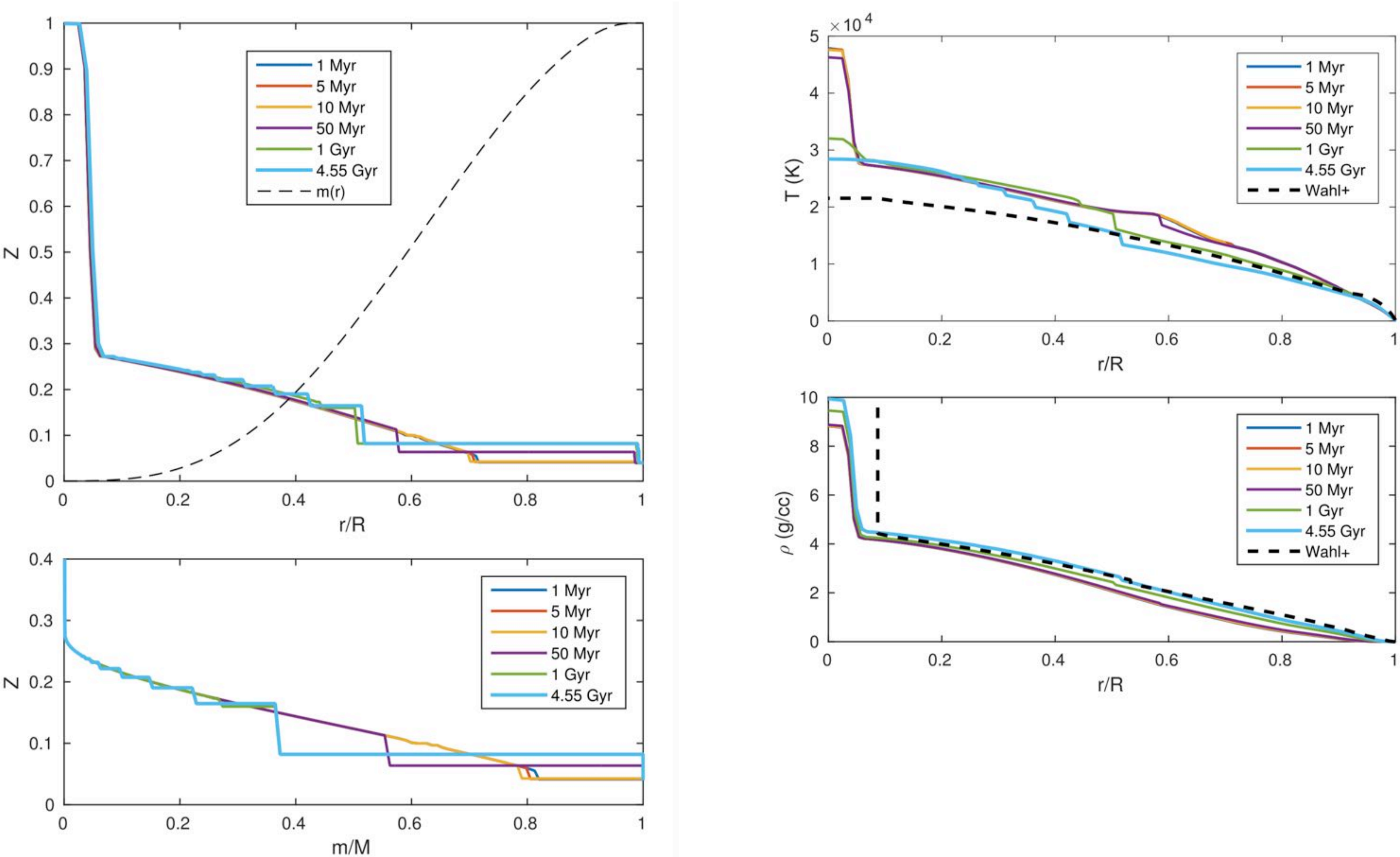
- compositional gradients could inhibit convection in giant planet interiors
- inefficient heat transport could explain inflated hot jupiters?

- estimated layer thickness as $\delta_T \approx (\kappa_T l/v)^{1/2}$ => ~10^6 layers!



Leconte & Chabrier (2012)

Evolution to Jupiter today with heavy elements



Vazan, Helled, Guillot (2018)

Simple model of staircase generation

Solve

$$\frac{\partial T_i}{\partial t} = -\frac{F_{i+1/2} - F_{i-1/2}}{\Delta x}$$
$$\frac{\partial X_i}{\partial t} = -\frac{F_{X,i+1/2} - F_{X,i-1/2}}{\Delta x}$$

on a Cartesian grid with fluxes

$$F_{i+1/2} = a\,K\,\nabla_{i+1/2} + a\left(\nabla_{i+1/2} - \nabla_{\rm ad}\right)$$

$$F_{X,i+1/2} = b\,\nabla_X$$

convective terms only turned on when

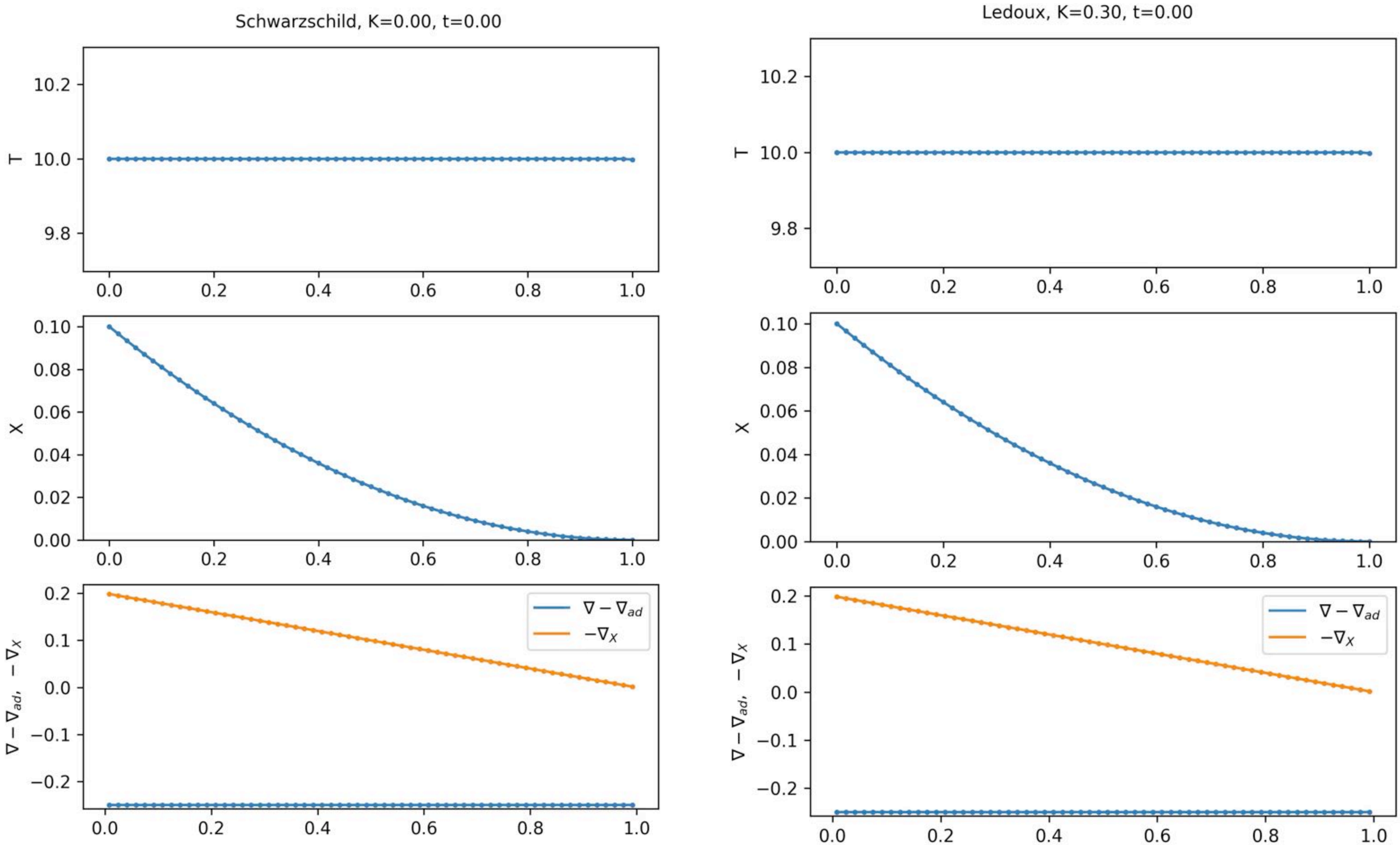
$$\nabla - \nabla_{\rm ad} - \nabla_X > 0$$

Gradients (written to be positive if decreasing outwards)

$$\nabla_{i+1/2} = \frac{T_i - T_{i+1}}{\Delta x}$$
$$\nabla_{X,i+1/2} = \frac{X_i - X_{i+1}}{\Delta x}$$

Show some movies:
- stairs form only for K>0
- dependence on resolution

Layer formation is a competition between convection and thermal conduction



(see also discussion in Vazan et al. 2015)

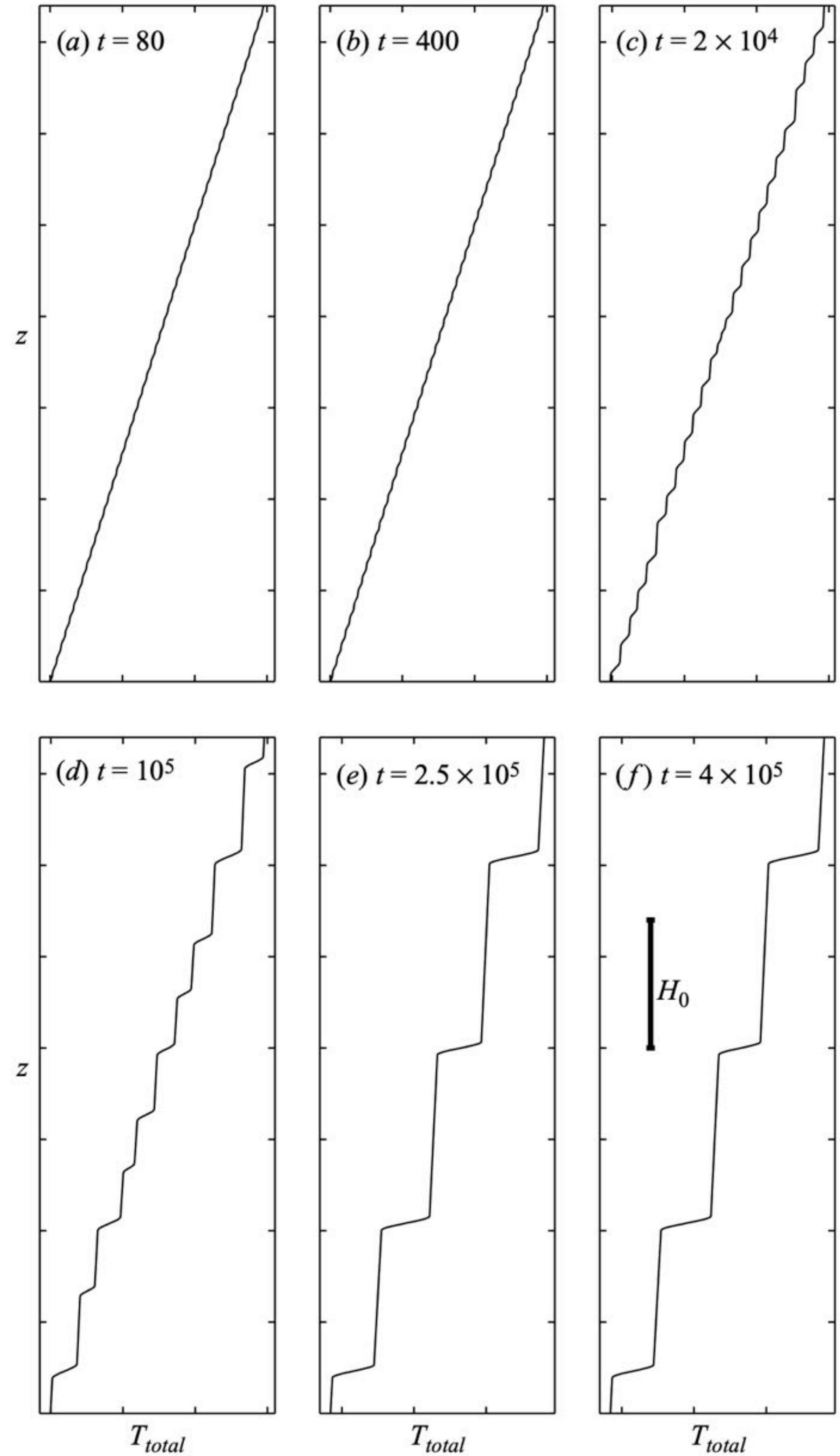
Radko (2005) solved a similar set of equations to study the growth of staircases on a linear background

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} F_T - \mu \frac{\partial^4}{\partial z^4} T$$

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial z} F_S - \mu \frac{\partial^4}{\partial z^4} S.$$

Higher order terms stabilize short wavelength unstable modes

Diffusive layer thickness controlled by the parameter μ



Observations of layered convection in the oceans cannot be extrapolated to planets/stars

(see Moll et al. 2016 for discussion)

Important parameters

$$Pr = \frac{\nu}{\kappa_T} \qquad \tau = \frac{D}{\kappa_T}$$

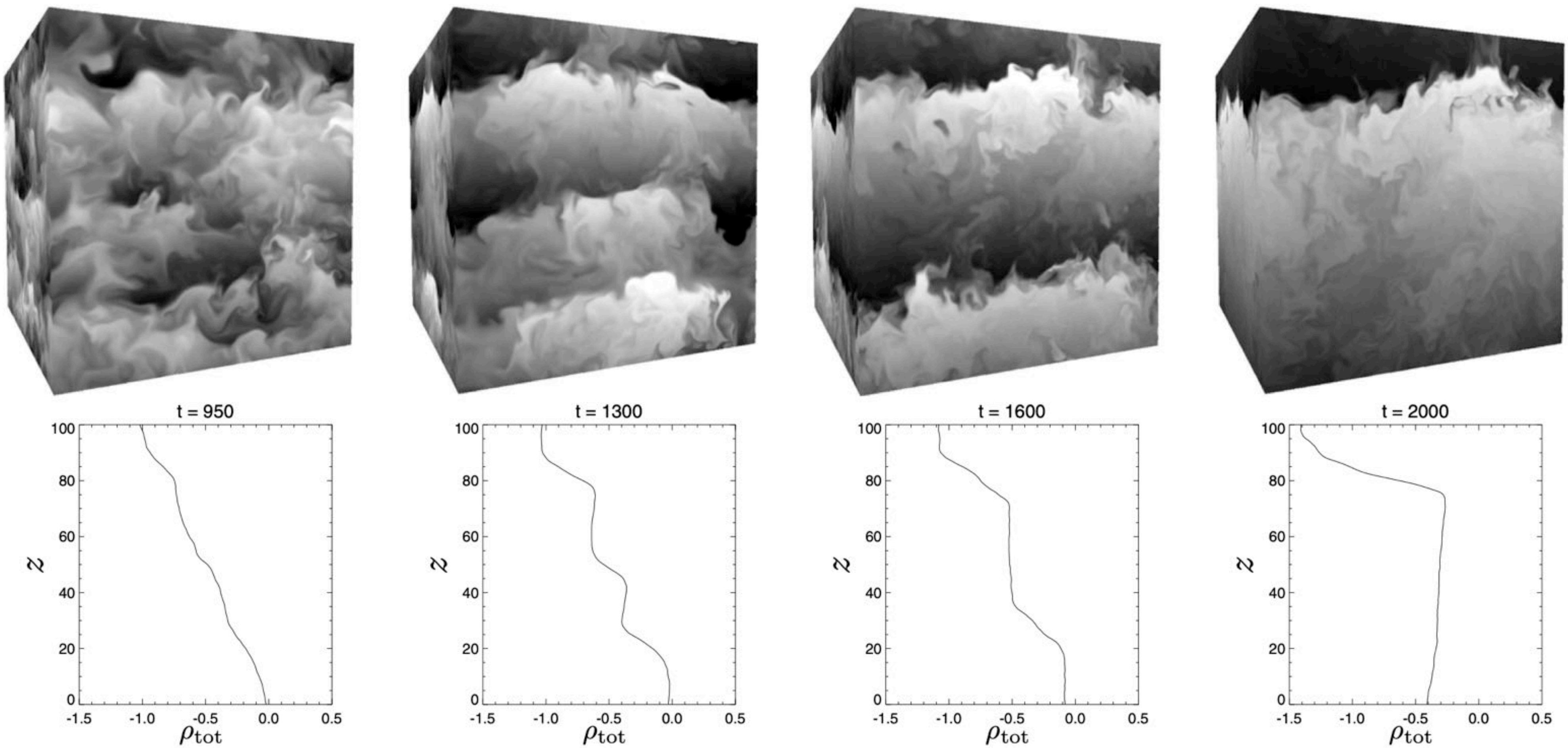
Ocean	**Planet**
$\kappa_T \approx 1.4 \times 10^{-3}\ \mathrm{cm^2/s}$	$\kappa_T \approx 10^{-2} - 10^{-1}\ \mathrm{cm^2/s}$
$\nu \approx 10^{-2}\ \mathrm{cm^2/s}$	$\nu \approx 10^{-3} - 10^{-2}\ \mathrm{cm^2/s}$
$D \approx 10^{-4}\ \mathrm{cm^2/s}$	$D \approx 10^{-4} - 10^{-3}\ \mathrm{cm^2/s}$
$Pr \approx 7$	$Pr \approx 10^{-2} - 1$
$\tau \approx 0.01$	$\tau \approx 0.01$

Need numerical simulations at low Pr

(Stellar conditions are much worse Pr << 1, tau<<1)

Series of numerical studies of semiconvection by Pascale Garaud and collaborators

Rosenblum et al. (2011), Mirouh et al. (2012), Wood et al. (2013), Moll et al. (2016)



Wood et al. (2013)

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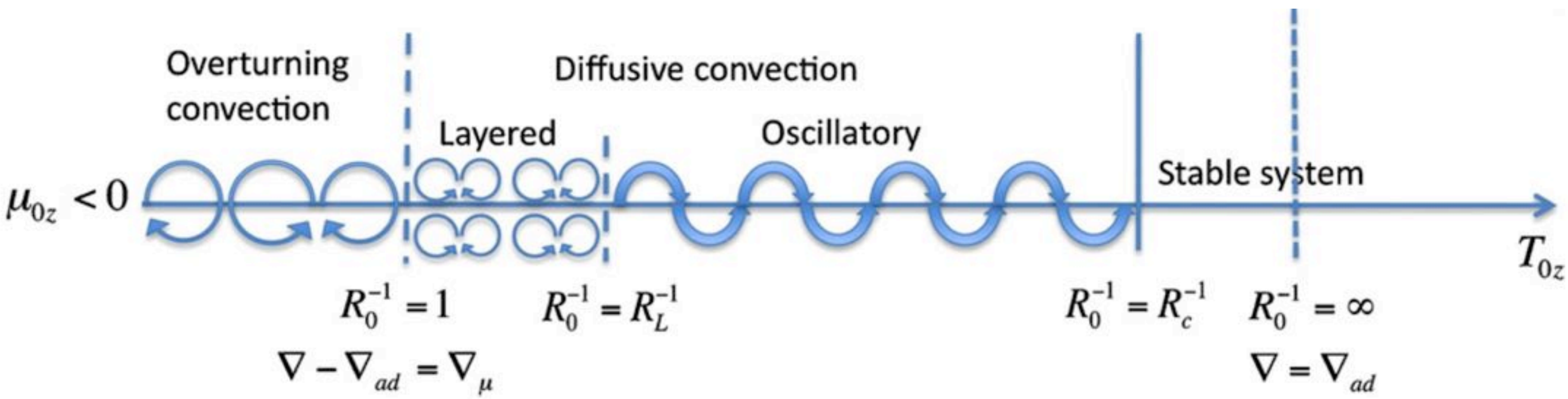
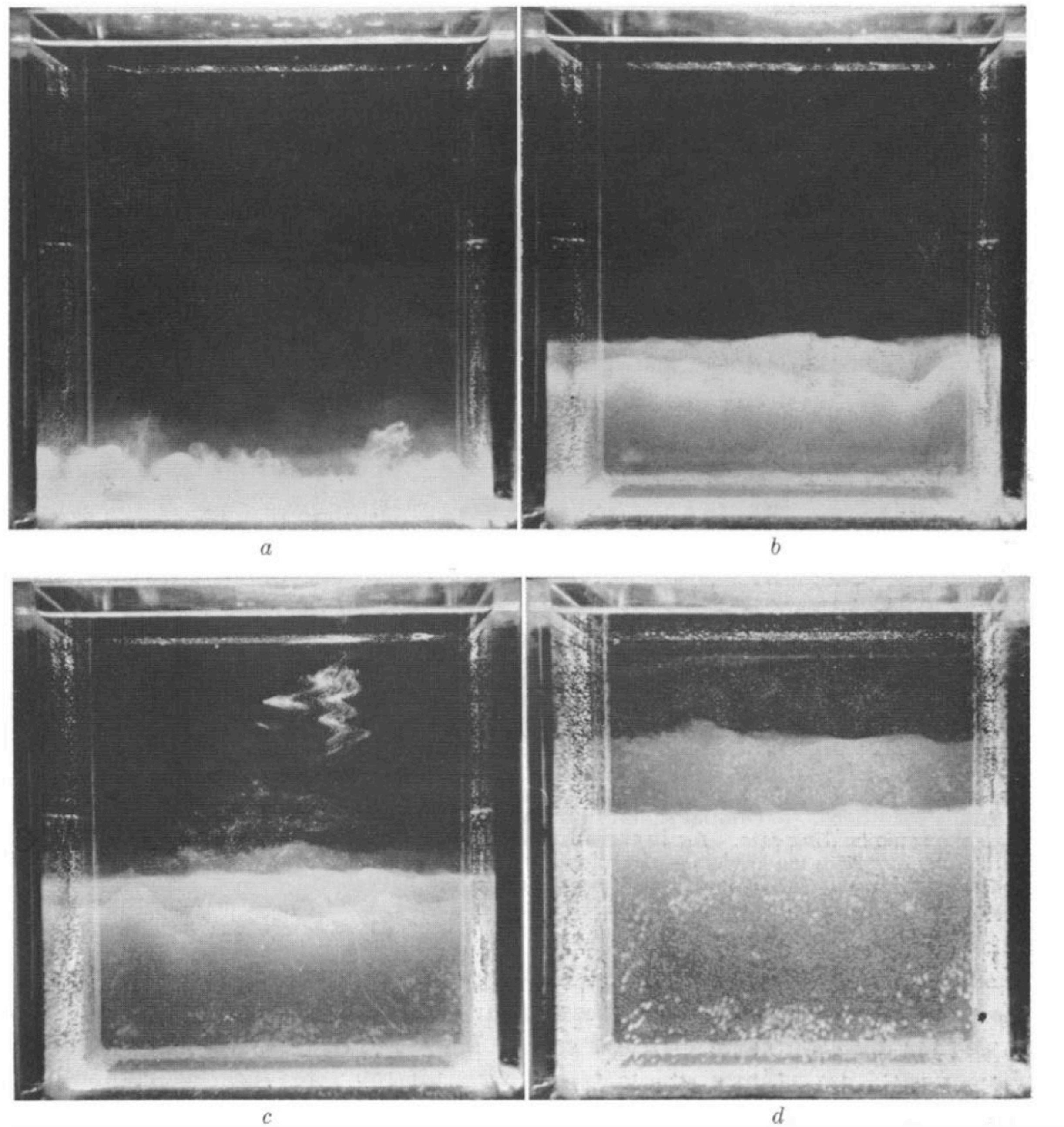


Mirouh et al. (2012)

Turner and Stommel (1964) : A new case of convection in the presence of combined vertical salinity and temperature gradients



<u>Convection and mixing in a linear salt gradient cooled from above</u>

Rafael Fuentes
(PhD student at McGill)

Bousinessq approximation

$$\nabla \cdot \boldsymbol{v}' = 0\,,$$

$$\frac{\partial T'}{\partial t} = -(\boldsymbol{v}' \cdot \nabla)\, T' + \kappa_T \nabla^2 T'\,,$$

$$\frac{\partial X'}{\partial t} = -(\boldsymbol{v}' \cdot \nabla)\, X' + \kappa_X \nabla^2 X'\,,$$

$$\frac{\partial \boldsymbol{v}'}{\partial t} = -(\boldsymbol{v}' \cdot \nabla)\, \boldsymbol{v}' - \frac{\nabla P'}{\rho_b} + \left(\frac{\rho'}{\rho_b}\right)\boldsymbol{g} + \nu \nabla^2 \boldsymbol{v}'$$

Equation of state

$$\rho' = \rho_b(\beta X' - \alpha T')$$

$$\alpha = 2.3 \times 10^{-4}\ \mathrm{K}^{-1}$$

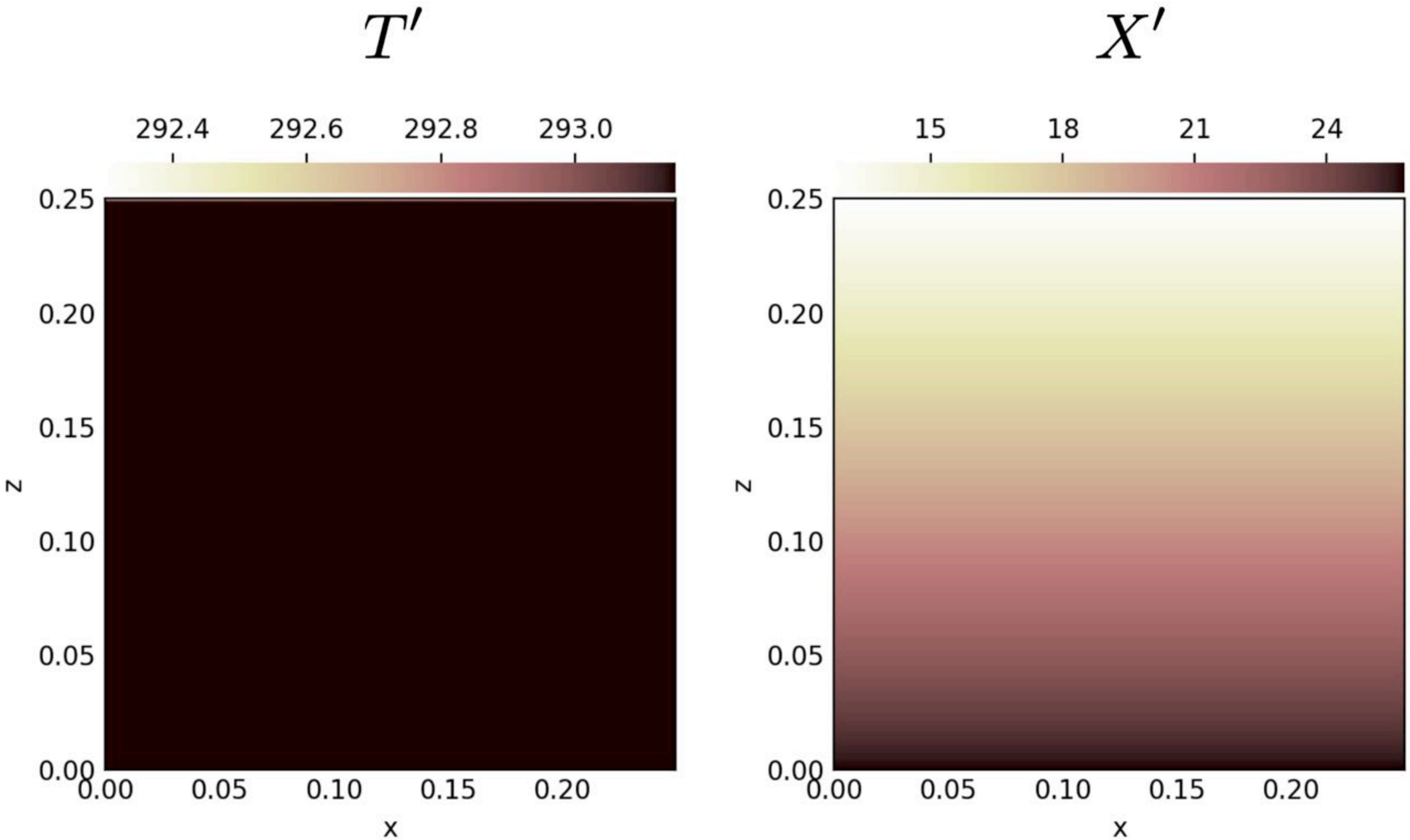
$$\beta = 7.6 \times 10^{-4}$$

Dedalus code http://dedalus-project.org

Open source, python based spectral code for solving PDEs

Initial conditions

1. Uniform temperature
2. Linear gradient of solute

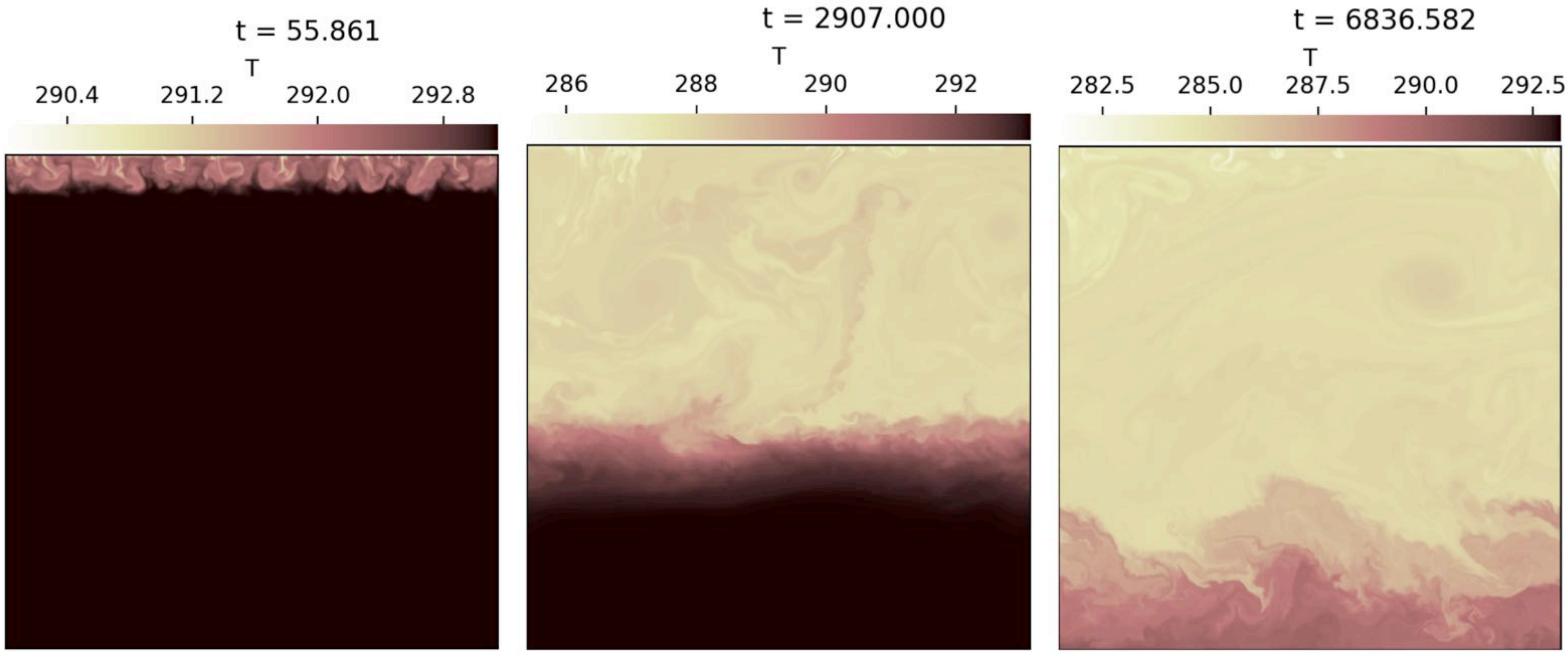


Apply a constant flux at the top of the box, a multiple of the critical flux

$$F_{\mathrm{crit}} = k\frac{\beta}{\alpha}\left|\frac{dX_0'}{dz}\right|$$

Marginally stable temperature gradient

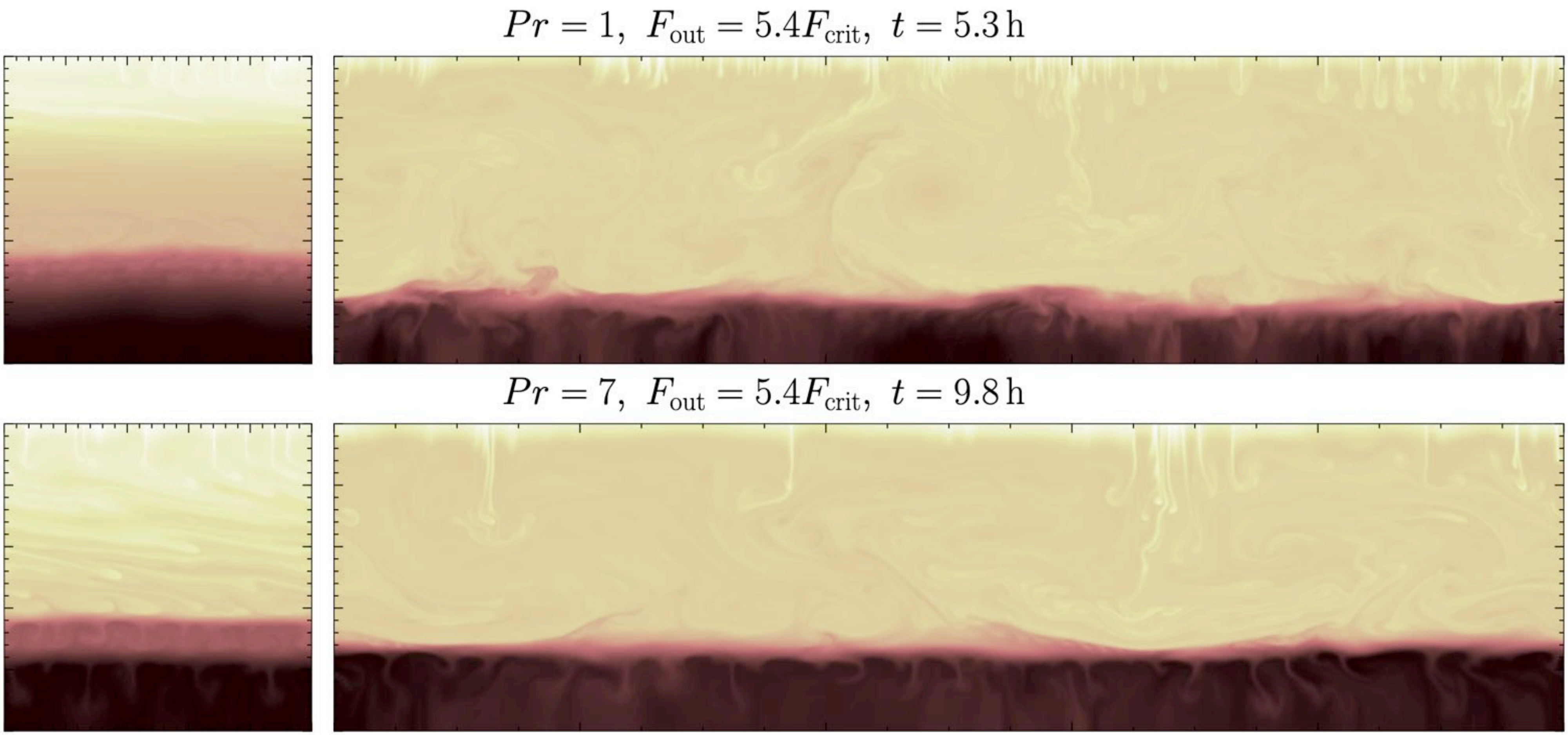
$$Pr = 0.1, \quad \tau = 0.1$$

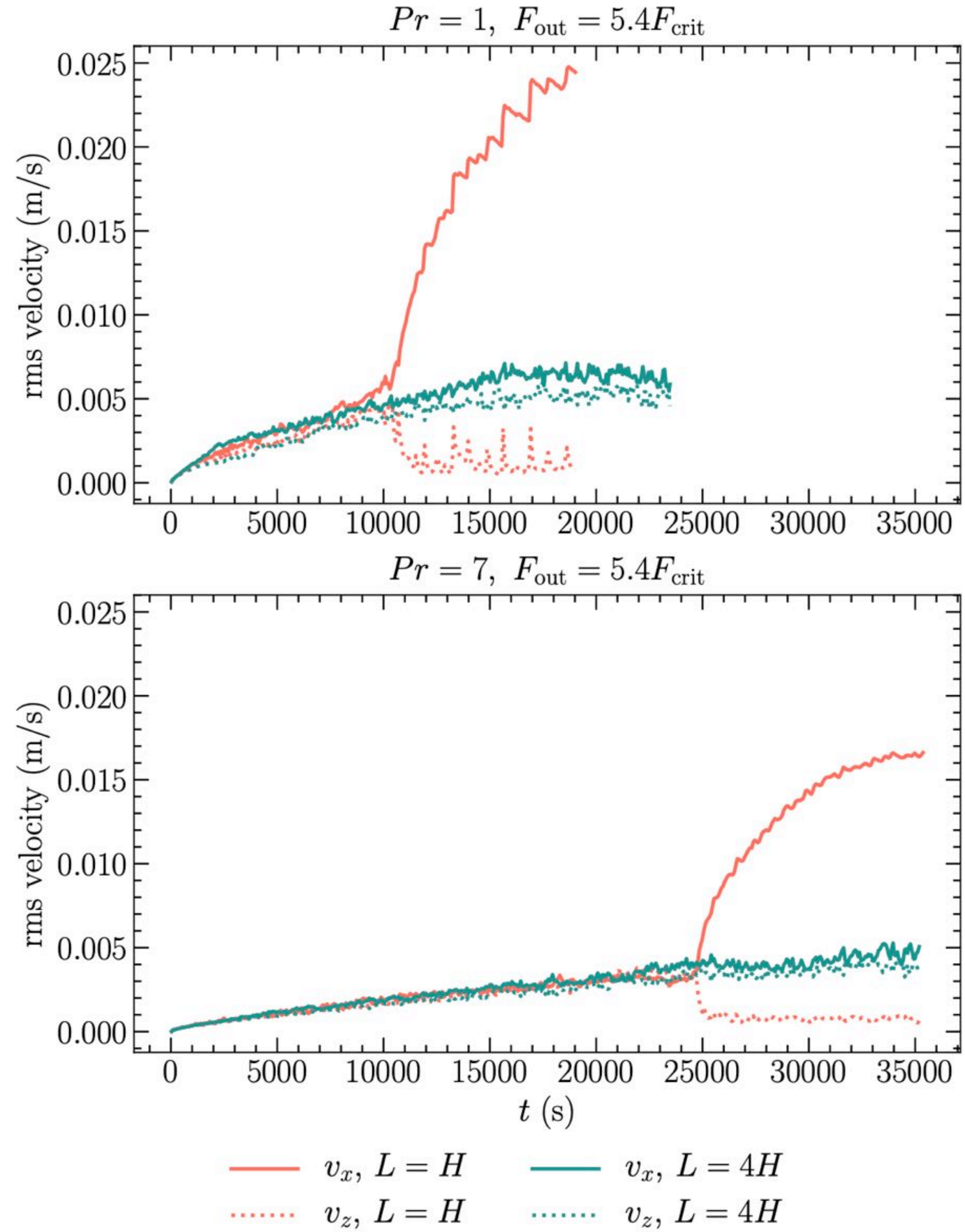


Show some movies:

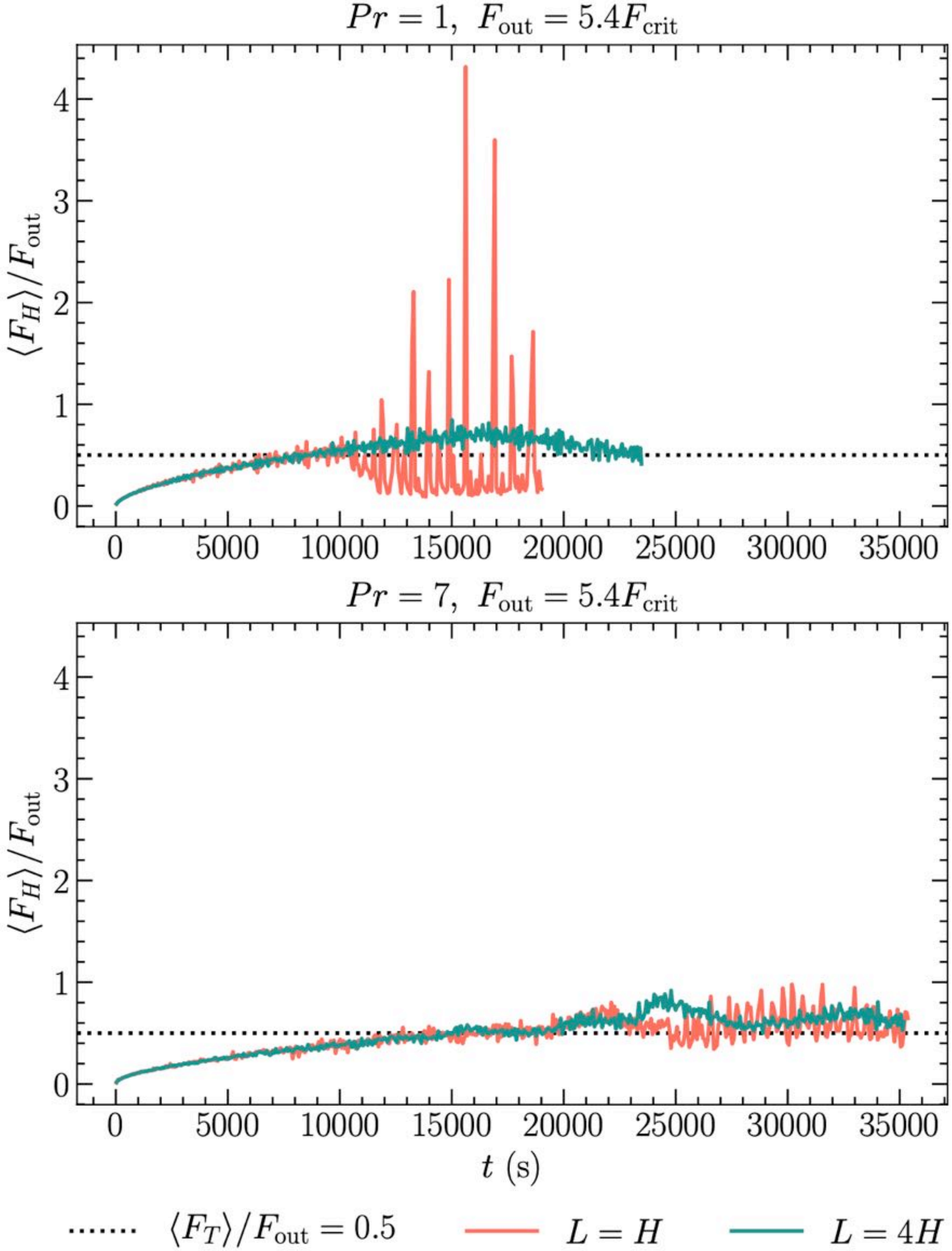
- Shear develops in square boxes
- Goes away with larger aspect ratio
- Differences between Pr = 7 and 1

Larger aspect ratio boxes avoid the problem of shear

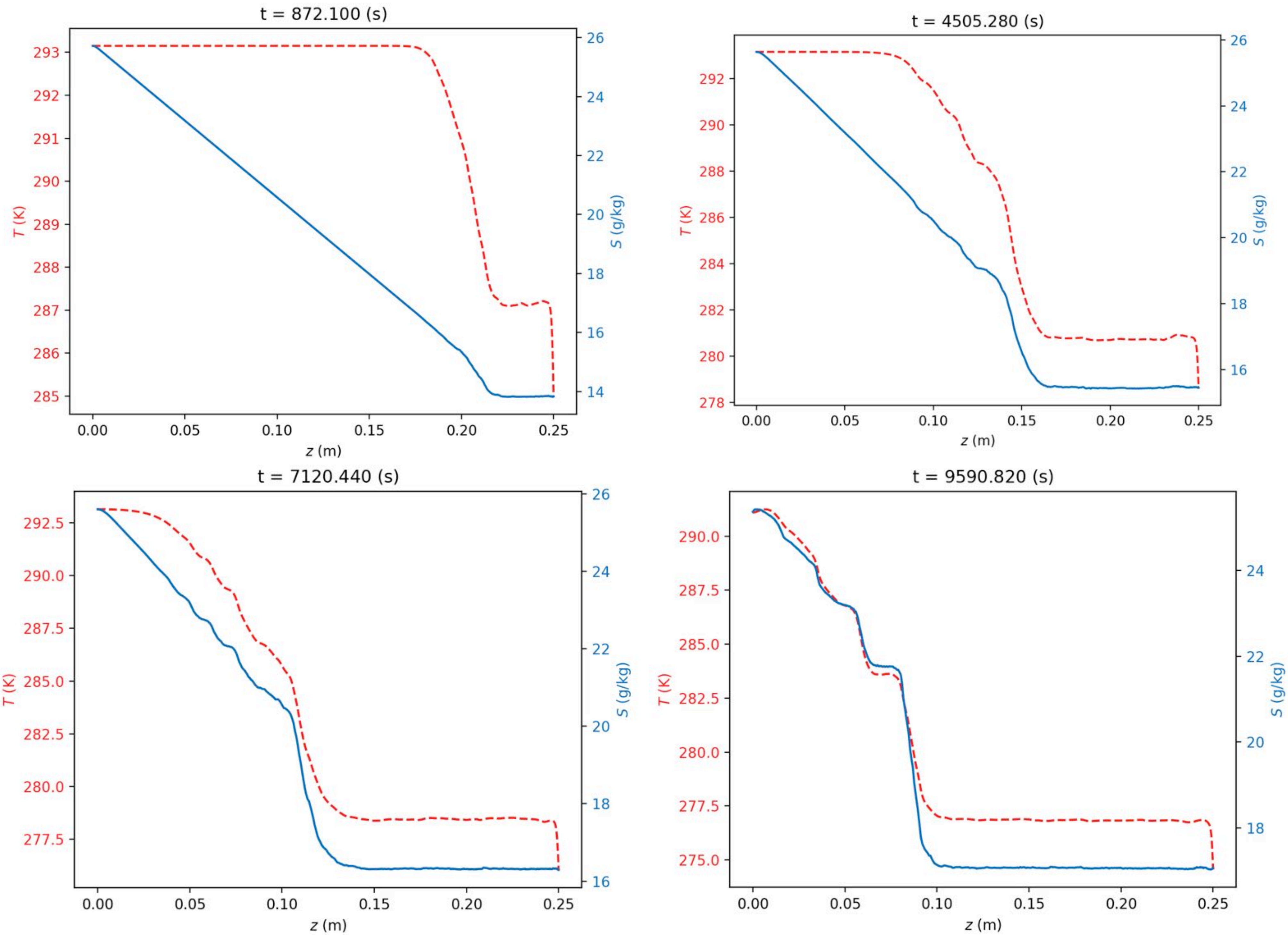




$Pr = 1,\ F_{\rm out} = 5.4F_{\rm crit}$
$Pr = 7,\ F_{\rm out} = 5.4F_{\rm crit}$
rms velocity (m/s)
t (s)
0.000
0.005
0.010
0.015
0.020
0.025
0
5000
10000
15000
20000
25000
30000
35000
$v_x,\ L = H$
$v_z,\ L = H$
$v_x,\ L = 4H$
$v_z,\ L = 4H$

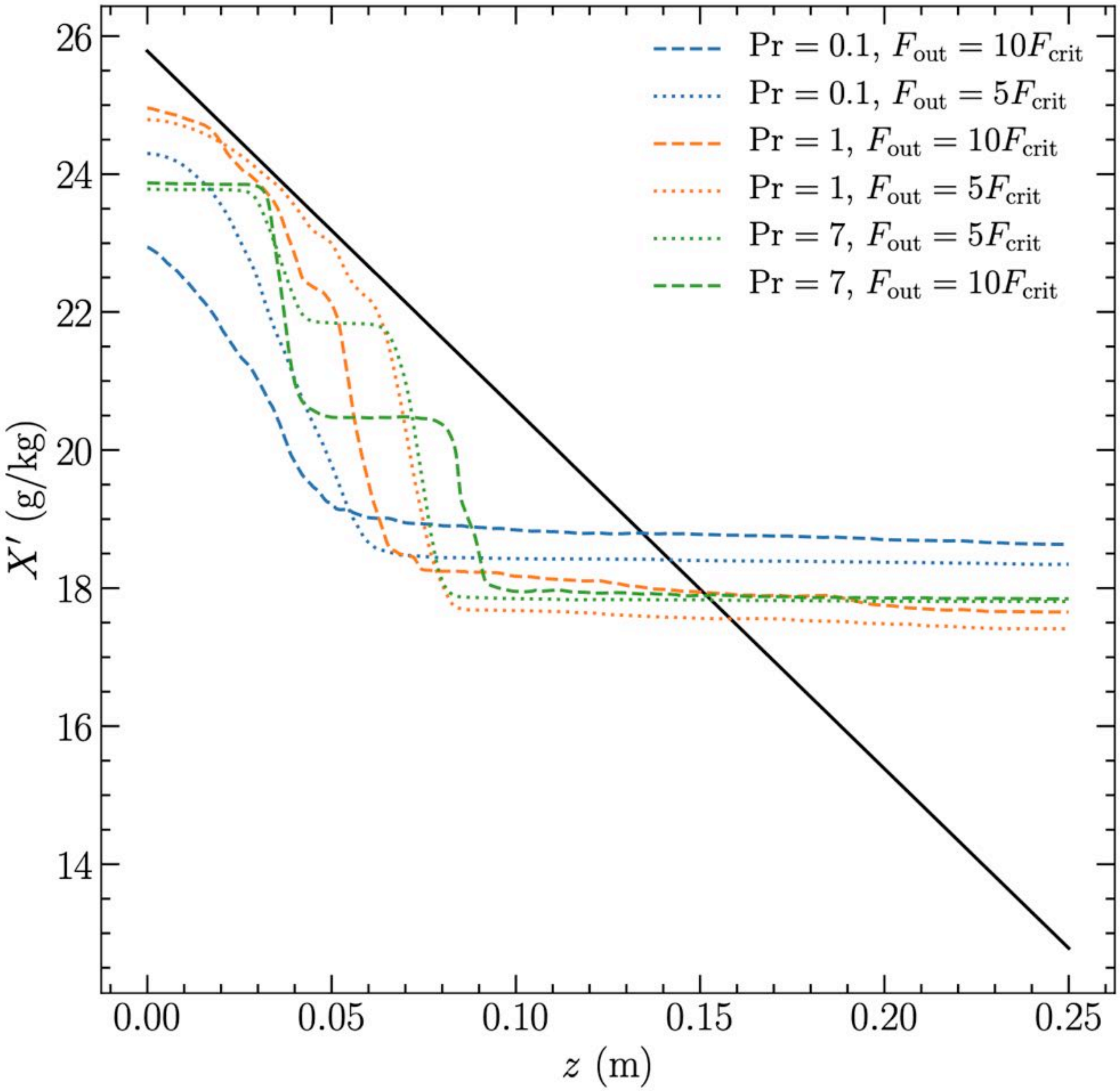


$Pr = 1,\ F_{\rm out} = 5.4F_{\rm crit}$
$\langle F_H \rangle / F_{\rm out}$
0
1
2
3
4
0
5000
10000
15000
20000
25000
30000
35000
$Pr = 7,\ F_{\rm out} = 5.4F_{\rm crit}$
$\langle F_H \rangle / F_{\rm out}$
t (s)
$\langle F_T \rangle / F_{\rm out} = 0.5$
$L = H$
$L = 4H$



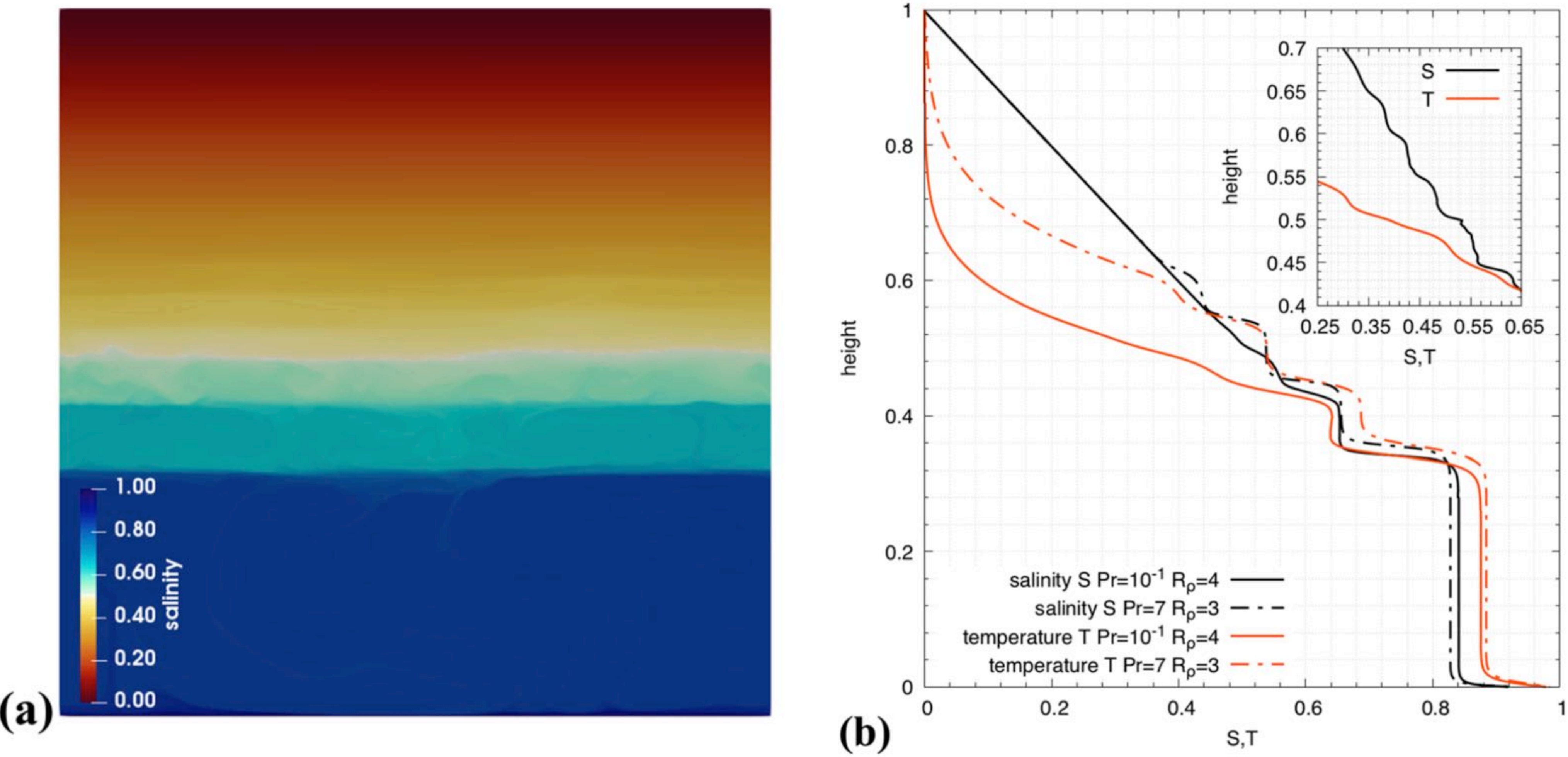
Fuentes et al. (2019)

Steps are much less prominent or absent at Pr < 1



Florian Zaussinger · Friedrich Kupka

Layer formation in double-diffusive convection over resting and moving heated plates



Pr = 0.1

Zaussinger et al. (2019)

How quickly does the outer convection zone move inwards?

Turner (1968) argument: $$\rho_b c_p \Delta T' h = F_{\rm out} t$$

Energy conservation determines the temperature drop

$$\Delta X' = \frac{1}{2}\left|\frac{dX_0'}{dz}\right| h$$

Solute is mixed throughout the convection zone

When is the temperature difference enough to mix the underlying layer upwards?

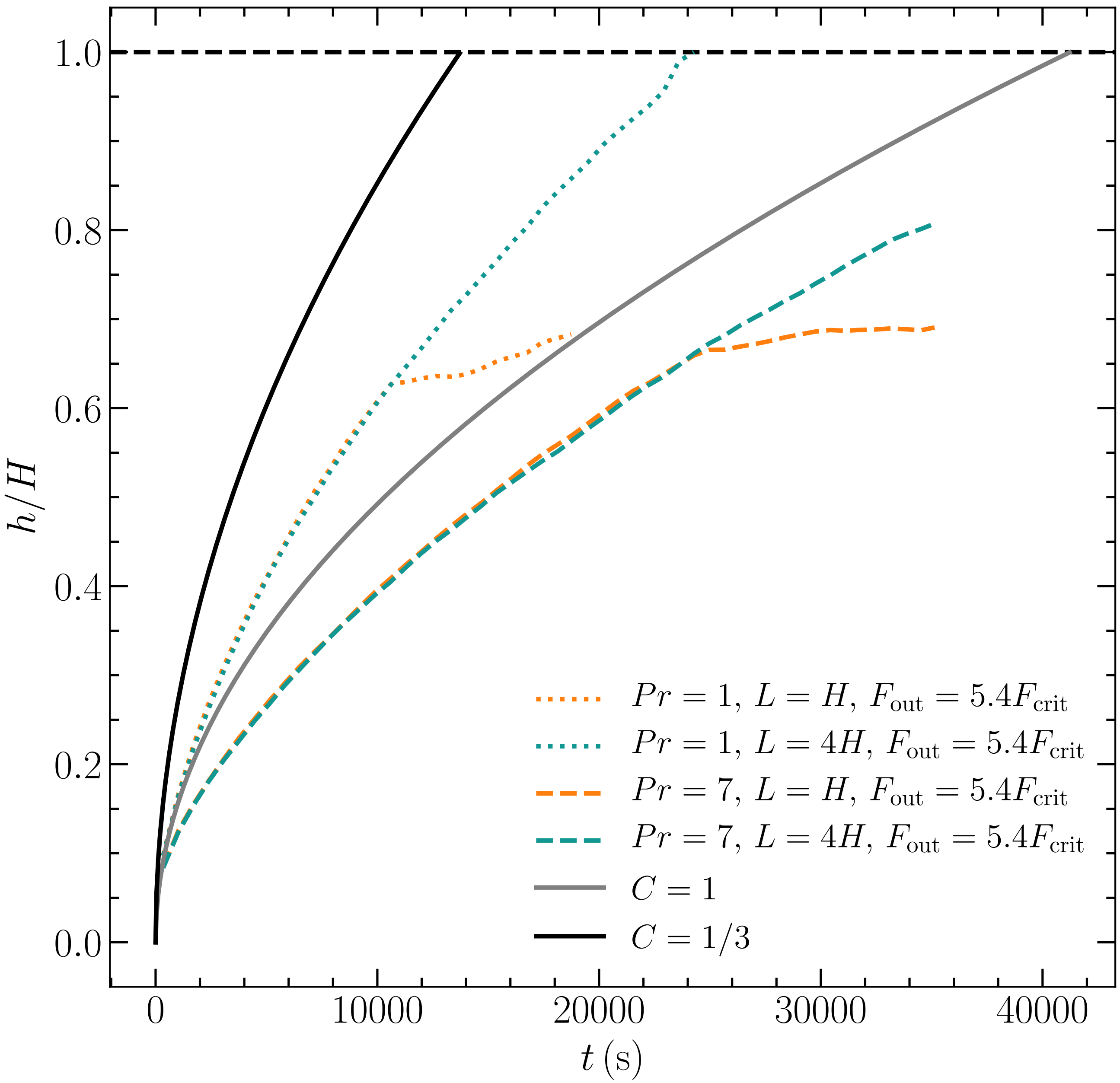
$$\alpha \Delta T' = C\beta\Delta X'$$

Two limits: $C = 1$ The temperature difference eventually overcomes the composition difference (Ledoux)

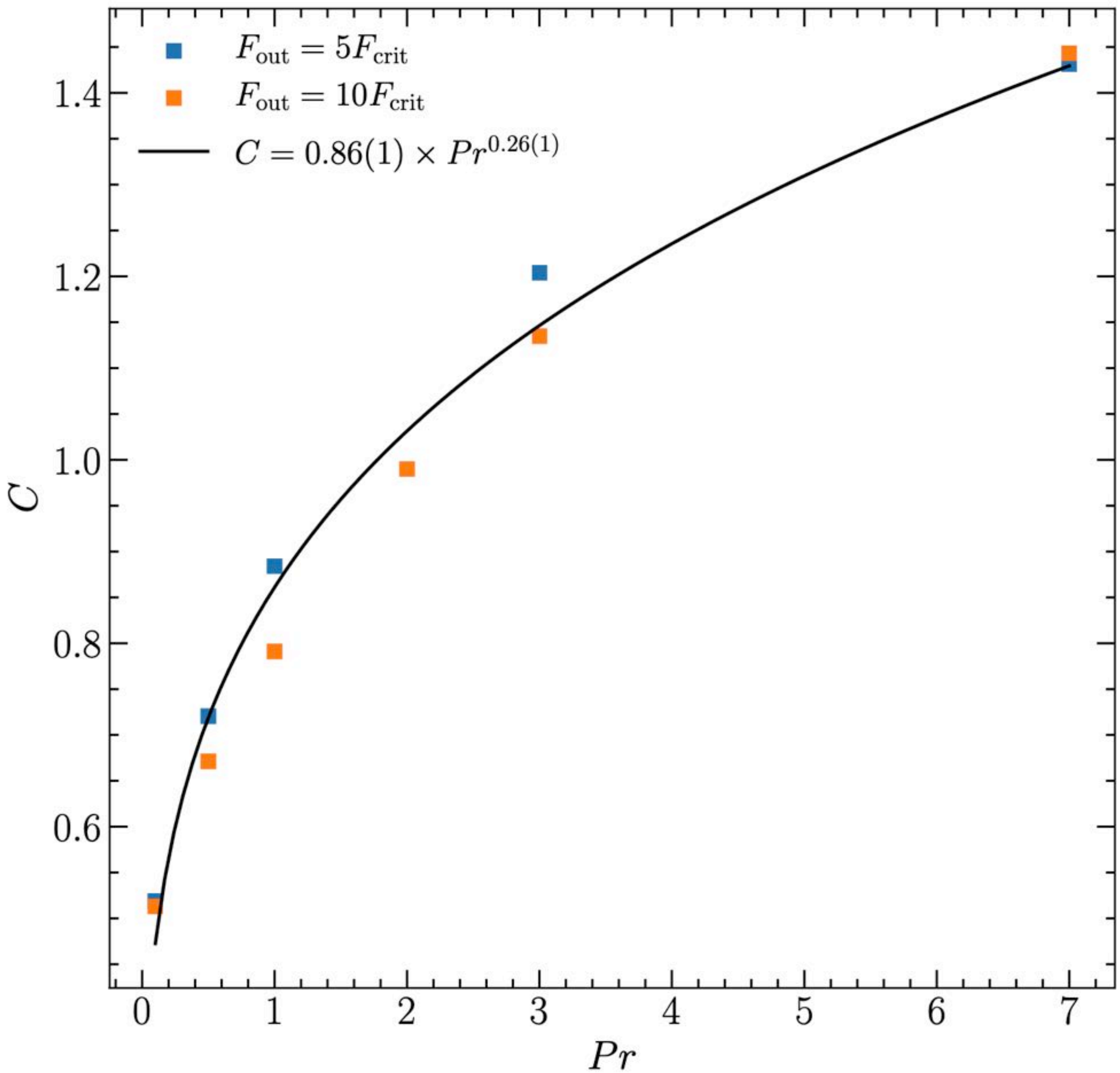
$C = 1/3$ The convection can entrain heavy fluid from below: thermal energy used to lift the material

=> $$h(t) = \sqrt{\frac{2}{C}}\left(\frac{F_{\rm out}}{F_{\rm crit}}\right)^{1/2}(\kappa_T t)^{1/2}$$

The rate at which the convection zone moves inwards is only weakly-dependent on Pr



The rate at which the convection zone moves inwards is only weakly-dependent on Pr



Conclusions

- Composition gradients should be ubiquitous in planets, and need to be considered in formation, evolution, and structure models

- Convective layers appear in 1D when a composition gradient is heated/cooled
- 1D evolution requires some kind of model for the boundary layer between convection zones in order to resolve them
- This gives another parameter … analogous to stellar overshoot… how to determine/calibrate it? What is the correct prescription in 1D?

- We are lucky that we can simulate the parameter regime of planets (Pr ~ 0.01-1, tau~0.01)
- Aspect ratios >1 avoid generation of shear
- Low Pr => more turbulent flow => more efficient mixing across interfaces
- Role of gravity waves
- Next steps: include stratification (anelastic), rotation