Electricity and Magnetism PHYS-340: 2014

(8:35-9:25 Monday, Wednesday, Friday, Rutherford 114)

http://www.physics.mcgill.ca/~gang/PHYS340/PHYS340.home.htm

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Math background: **Prerequisites**: Math 222A,B (Calculus III= multivariate calculus), 223A,B (Linear algebra), **Corequisites**: **314A** (Advanced Calculus = vector calculus), 315A (Ordinary differential equations)

Primary Course Book: "Introduction to Electrodynamics" by D. J. Griffiths, Prentice-Hall, (2013, fourth edition).

Similar books:
-"Electromagnetism", G. L. Pollack, D. R. Stump, Addison and Wesley, 2002.
-"Electromagnetic fields" by R. K. Wangsness, 1979, John Wiley and Sons,
-"Classical Electromagnetism" by R. H. Good, 1999, Harcourt Brace College publishers.
-"Electromagnetic fields and waves" by P. L. Lorrain, D. P. Corson, F. Lorrain, 1988 (3rd edition) W. H. Freeman and co., New York.
Reference: "Classical Electrodynamics", J.D. Jackson, 1998 Wiley.

Outline:

1. Vector Analysis:
   Algebra, differential and integral calculus, curvilinear coordinates, Dirac δ function, potentials.
2. Electrostatics:
   Definitions, basic notions, laws, divergence and curl of the electric potential, work and energy.
3. Special techniques:
   Laplace's equation, images, seperation of variables, multipole expansion.
4. Electrostatic fields in matter:
   Polarization, electric displacement, dielectrics.
5. Magnetostatics:
   Lorenz force law, Biot-Savart law, divergence and curl of B, vector potentials.
6. Magnetostatic fields in matter:
   Magnetization, field of a magnetic object, the auxiliary field H, magnetic permeability, ferromagnetism.
7. Electrodynamics:
   Electromotive force, Faraday's law, Maxwell's equations.

PHYS 340 Assessment:

Homework **(10%)**:	Class-tests/midterm **(2x20%)**:	Final **(50%)**:
There will be 6 assignment sheets each with about 10 problems from the course textbook. All students will be required to hand in the homework which will be marked by the TA. Deadlines will be typically 2 weeks later; the suggested total worth of all the submitted problems is 10%. The rationale for the low percentage is that you'll need to do the problems simply in order to understand the material and do the exams; the 10% is simply a small *extra* incentive.	There will be two class tests. These will assess where you are with respect to where you should be to allow me to assess the class progress. The tests will be in class (50 minutes each) and will consist of three problems, you will be able to use one two sided crib sheet. I suggest 20% overall grade weight each (however, see below).	The final will consist of 5 problems (3 hours) 4 of which will be on material covered since the last class test; you will be able to use one two sided crib sheet. The final will be worth 50%. Escape clause: if one of your class tests is below the final, it will not be counted, the final will be worth 70%, the other class test 20%. (If both class tests are below the final, only the best of the two will be counted). This means that if one of the class tests is poor, you have a chance of redemption.

Outcomes

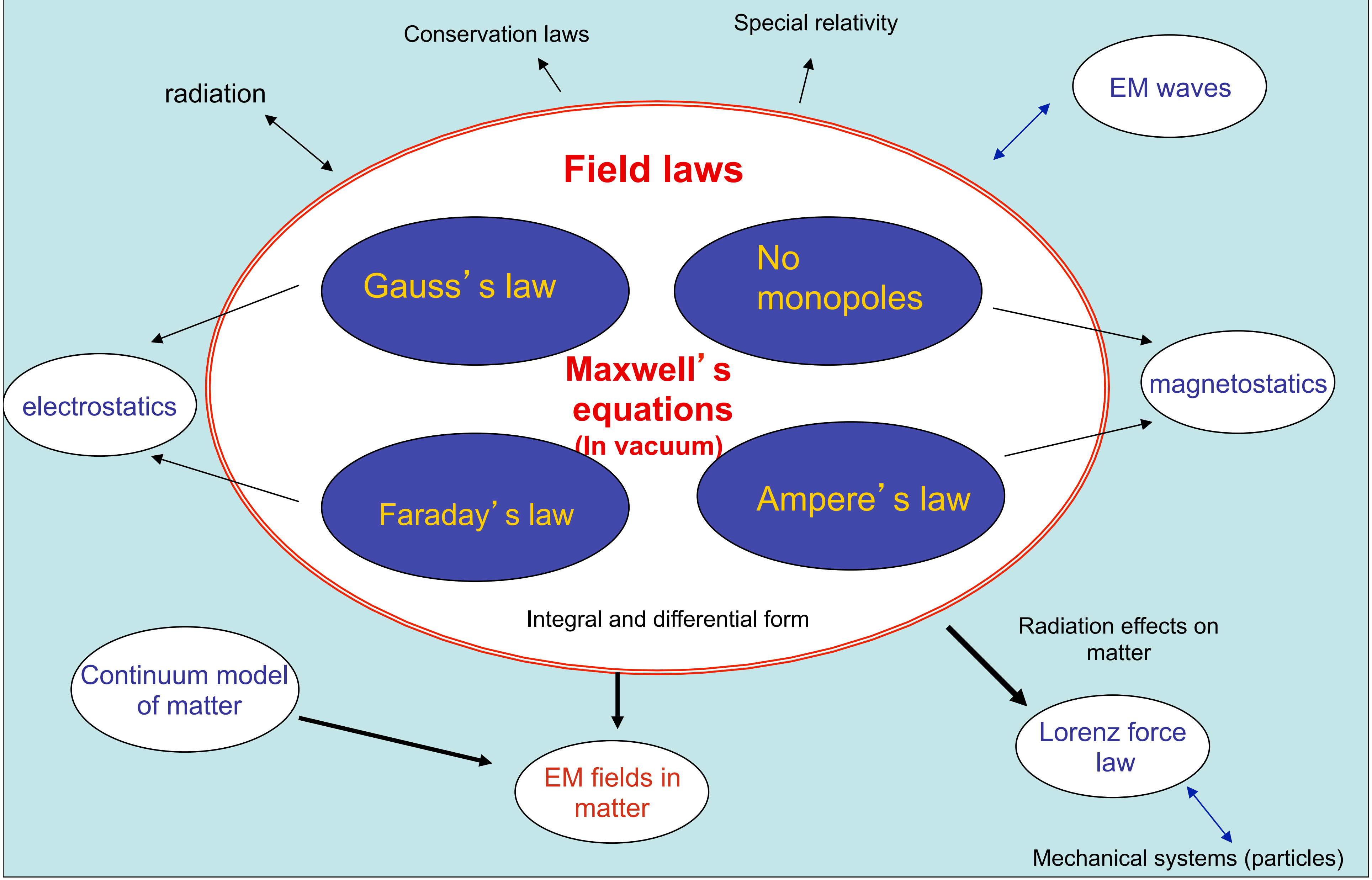
Concept	Outcomes	Rough time on topic (roughly the percentage of problems on a given topic)
Electrostatics	Solve problems involving static electric fields from charge distributions. Able to use scalar potentials, solve problems involving conductors, multipoles, image charges, the Laplace equation.	40%
Magnetostatics	Solve problems involving static current distributions. Able to use vector potentials, solve problems involving magnetic dipoles, Lorenz force law.	25%
EM Fields in matter	Be able to solve (static) problems involving polarization, magnetization, electric displacement and *H* fields	20%
Electrodynamics	Solve problems involving time varying E and B fields: electromotive force, electromagnetic induction.	15%

PHYS 340: Course Calender 2014

Homework #1 due Sept. 17
Homework #2 due Oct. 1
Class test #1: Oct. 10
Homework #3 due Oct. 20
Homework #4 due Nov. 5
Class test #2: Nov. 12th
Homework #5 due Nov. 21
Homework #6 due Dec. 4 (last class)

Note that these deadlines may be subject to some change, check the course web site.

EM concept map



Brief Chronology of the early development of Electricity Magnetism and optics (1)

≈300BC: The Greeks discover that amber displays electrical properties:

16thC: William Gilbert extends this to glass, sealing wax, sulphur, precious stones. He also showed that magnetism and electricity were different; the former could orient (e.g. iron filings) while the latter could not.

1621: Willebrod Snell discovered the correct law for the diffraction of light.

1637: Descartes proposed that light is particulate and derived Snell's law from that assumption.

1665: Francesco Grimaldi discovered that the edges of shadows were coloured and the shadows a little too big, phenomena he ascribed to waves in the "light fluid". He also suggested that different frequencies corresponded to different colours.

1672-76: Olaus Rohmer proposed that light travels with a finite velocity which he estimated from transit times of Jupiter's moons.

1678-1690: Christian Huygens proposed that light was longitudinal vibrations in the "luminiferous ether".

1680-1704: Newton proposed that light was corpuscular, and showed that white light was a mixture of colours.

1745: The Leiden jar and electric shock are discovered.

Chronology (2)

1750: John Mitchell, showed that $F \approx 1/r^2$ for magnetic repulsion.

1752: B. Franklin shows lightning is an electric phenomenon. He also proposed that an electrical fluid pervaded all space and material bodies. An excess of electrical fluid renders the body positively charged.... many problems (since excess charge found to "stick to surfaces").

1767: Priestly showed that no force is exerted on a charge within a hollow charged sphere, hence concludes (following Newton in gravity) that $F \approx 1/r^2$.

1785-1789: Coulomb showed that $F \approx 1/r^2$ for both E+M.

1799: Voltaic cell discovered (Volta), giving continuous current (unlike Leiden jar).

1801: Young resuscitated Huygen's wave theory of light and showed the existence of diffraction patterns.

1817: Fresnel derived the known laws of optics by assuming that light was a transverse wave.

1820: Oersted shows the magnetic effects of such currents. In particular, an electric current would rotate about a magnetic pole... first example of non-central force. This is the principle of the electric motor.

1820: Ampere deduces that magnetism = result of circular electric currents.

1831: Faraday discovers electromagnetic induction linking mechanical motion and magnetism to the production of current. This is the principle of the dynamo.

1834: Wheatstone showed that current electricity travels at speeds one and a half times (!) the speed of light.

1837: Electric condensers and dielectrics (Faraday).

Chronology (3)

1845: Analogous behaviour of magnetic materials (Faraday).

1846: Faraday suggests that light = "vibrations" of EM force lines (not quite right, but close).

1850: Fizeau showed that the speed of current ranges from 1/3 to 2/3 c depending on composition of wire.

1850-1862: Foucault accurately measured the speed of light using rotating mirrors.

1857: Kirchoff showed that static and current electricity were related by a constant of the order of the speed of light.

1862: Maxwell adds the last effect: that a changing electric field generates a magnetic field, thus discovering the last of "Maxwell's equations". He then proposed that light is an EM wave. He imagined that E+M fields were manifestations of disturbances in rotating tubes of ether with tiny particles acting as ball-bearings.

1883: Fitzgerald pointed out the possibility of generating EM waves from oscillating current.

1886: Hertz proved this experimentally by building a “detector” (antenna).

Maxwell’s equations

VIII. *A Dynamical Theory of the Electromagnetic Field.* By J. CLERK MAXWELL, *F.R.S.*

Received October 27,—Read December 8, 1864.

PART I.—INTRODUCTORY.

(1) THE most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance according to mathematical laws.

For Electromagnetic Momentum	F	G	H
,, Magnetic Intensity	α	β	γ
,, Electromotive Force	P	Q	R
,, Current due to true conduction	p	q	r
,, Electric Displacement	f	g	h
,, Total Current (including variation of displacement)	p'	q'	r'
,, Quantity of free Electricity	e		
,, Electric Potential	Ψ		

ween these twenty quantities we have found twenty equations, viz.

Three equations of Magnetic Force	(B)
,, Electric Currents	(C)
,, Electromotive Force	(D)
,, Electric Elasticity	(E)
,, Electric Resistance	(F)
,, Total Currents	(A)
One equation of Free Electricity	(G)
,, Continuity	(H)

20 equations

These equations are therefore sufficient to determine all the quantities which occur in them, provided we know the conditions of the problem. In many questions, however, only a few of the equations are required.

The equations

The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity, which we may call p', q', r', so that

$$\left.\begin{aligned} p' &= p + \frac{df}{dt}, \\ q' &= q + \frac{dg}{dt}, \\ r' &= r + \frac{dh}{dt}, \end{aligned}\right\} \quad \text{(A)}$$

Equations of Magnetic Force.

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz}, \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx}, \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy}. \end{aligned}\right\} \quad \text{(B)}$$

Similarly,

$$\left.\begin{aligned}\frac{d\gamma}{dy}-\frac{d\beta}{dz}&=4\pi p'.\\ \frac{d\alpha}{dz}-\frac{d\gamma}{dx}&=4\pi q',\\ \frac{d\beta}{dx}-\frac{d\alpha}{dy}&=4\pi r'.\end{aligned}\right\} \qquad \text{(C)}$$

We may call these the Equations of Currents.

Equations of Electromotive Force.

$$\left.\begin{aligned}P&=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx},\\ Q&=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy},\\ R&=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}.\end{aligned}\right\} \qquad \text{(D)}$$

Equations of Electric Elasticity,

$$\left.\begin{array}{l}P=kf,\\ Q=kg,\\ R=kh.\end{array}\right\} \quad \text{(E)}$$

Equations of Electric Resistance,

$$\left.\begin{array}{l}P=-\varrho p,\\ Q=-\varrho q,\\ R=-\varrho r.\end{array}\right\} \quad \text{(F)}$$

Equation of Free Electricity,

$$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0. \quad \text{(G)}$$

Equation of Continuity,

$$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0. \quad \text{(H)}$$

Modern form of Maxwell's equations

(Heaviside 1884)

$$\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_0}$$

Gauss's law

$$\nabla \cdot \underline{B} = 0$$

No magnetic monopoles

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}$$

Ampere's law

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday's law

Empirical status of some the axioms of E+M:

Note: The *E* field is unobservable so that all empirical axioms tests assume F=qE

a) Inverse square law <-> mass of photon

i) Assume $1/r^{2+\varepsilon}$ and put limits on ε

ii) Assume a Yukawa potential $V = r^{-1}e^{-\mu r}$ (with $\underline{E} = -\nabla V$) and quote a value for μ.

According to QM, $\Delta p \Delta x \approx h$ and $\Delta x \approx 1/\mu$, $\Delta p = m_\gamma c$ so $m_\gamma \approx h\mu/c$.

Experiments use the idea that E = 0 inside a conductor only if $\varepsilon = m_\gamma = 0$.

D.C. Cavendish 1772: $|\varepsilon| < 0.02$

Maxwell 1879: $|\varepsilon| < 5x10^{-5}$

$4x10^6$ Hz Plimpton and Lawton 1936 $|\varepsilon| < 2x10^{-9}$

Williams et al 1971 $|\varepsilon| < 2.7 \pm 3.1x10^{-16}$ This implies $m_\gamma < 1.6x10^{-50}$Kg

Geophysical estimates

If the photon had mass, then the magnetic field of the earth would have a constant field in addition to a dipole field. Satellites show that this is $<4\text{x}10^{-3}$ times smaller than the dipole, hence:

$$m_\gamma \approx < 4\text{x}10^{-51}\text{Kg, i.e. } \mu^{-1} > 10^{8}\text{m}$$

Astrophysical:

If photons were massive, they would be directly affected by the galactic vector potential and would lead to observable effects on the galactic plasma. The absence of such effects places a bound of $\approx 10^{-63}$ Kg on photon masses.

Range of scales of direct tests: $1/r^2$ tests from 10^7m to 10^{-18} m (100 GeV electrons)

b) Linear Superposition

i) Macroscopic level: Many phenomena including microwave transmission of thousands of calls show valid to better than 0.1%

ii) Atomic level: Huge fields: 10^{11} – 10^{17} V/m for e- in atoms, 10^{21} V/m in nucleus… classical linearity holds to better than one part in 10^{6} even here.

iii) QM nonlinearity: arises because of "vacuum polarization" in which E-M fields can be screened by virtual e+, e- pairs (light scattering light, also virtual pair exchanges)… but this is predicted

c) Force law:

Since E is unobservable, $\underline{F} = q\underline{E}$ is needed to test axioms a,b. However if relativistic law of motion is added, then $\underline{F} = q\underline{E}$ implies particle trajectories which are extraordinarily accurate.