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Reply to comment by Igor Esau on "Do stable atmospheric layers exist?"

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[1] We would like to thank *Esau* (2009) for attempting to save the classical notion of stable layers; his argument is very close to one raised up by an anonymous reviewer of *Lovejoy et al.* (2008b). Since a similar argument is often invoked to justify atmospheric applications of linear gravity wave theories, it appears to be widespread in the community. We therefore hope this debate will clarify the issue.

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[2] In our paper we argued that the dynamical meteorological concept of stable layers evolved during the era of classical meteorology characterized by rudimentary measurement techniques so that the atmosphere could only be discerned at low resolution. We are now in a golden age of high resolution data and we argue that the notion has become untenable. Our argument was based on state-of-the-art drop sonde data and it used three standard stability criteria. Starting at low vertical resolution (160 m, comparable to operational radiosondes) we systematically increased the resolution down to our limit of 5 m. As the resolution improved, we simply glimpsed smaller and smaller scale alternations of stable and unstable layers, the lowest couple of orders of magnitude of a fractal hierarchy.

[3] Before discussing the central issue, a clarification is needed. In the third paragraph, *Esau* [2009] claims "The L08 criticism is crucially based on two facts: high short distance correlations between two drop sonde time series; and high variability of the local stability." In actual fact, the short distance correlations are peripheral to our argument. They are only used to demonstrate that the unstable layers are real and not simply artifacts of instrumental noise. Second, it is not the magnitude of the variability at any given scale that is important but rather the fact that the variability exists over a huge range of scales, i.e., what is fundamental is its wide range scaling nature.

[4] The core misunderstanding becomes apparent in the next sentences when *Esau* [2009] states "L08 interpreted variations in the local and instantaneous atmospheric stability as representative of variations in stability of the entire atmospheric layer. Hence, to advocate the Stable Atmospheric Layer Concept one has to show that these variations are largely of local and transient nature". Starting with the first sentence let us carefully analyse this. First, the real world is indeed local and instantaneous: this is the reason why it is possible for the governing fluid equations to involve spatial and temporal derivatives of fluid quantities. These equations are of course based on very strong local and instantaneous assumptions - indeed about the fields at infinitesimal neighbourhoods of space-time points! Second, *Esau* [2009] claims that we take the stability variations "as representative of the entire atmospheric layer". This statement is only true in as much as we believe that the layers analyzed are typical and form a *statistically* representative sample; we nowhere claim that the precise, detailed small spatial scale structures that we observe have significant horizontal extents or temporal durations. In other words, while the profiles may indeed be representative of the *statistics*, the precise fractal structure observed on a given profile at a given instant clearly do *not* "represent" the state of the atmosphere over significant space-time domains.

[5] If *Esau* [2009] wants to develop a stability theory (and as he suggests, gravity wave theory) that would apply to low resolution atmospheric fields (i.e., fields smoothed in space-time), then the onus is on him to do so: existing theories only apply to local and instantaneous fields. The development of such a theory would be necessary to substantiate his idea that the classical theory can somehow be saved by suitable space-time averaging. Indeed, the rest of his argument (based on numerical simulations) is simply an attempt to confirm that sufficient averaging (spatial, temporal or by using EOF's) do indeed suppress the small scale vertical variability and hence as expected, it suppresses the fine scale structures including many of the unstable layers (the layers are of course still there but the data/simulations are unable to discern them). This argument is in fact an indirect repetition (using smoothing in the horizontal and/or temporal domains) of our direct demonstration that vertical smoothing suppresses the structures.

[6] *Esau* [2009] is thus correct that space-time averaging will suppress the small scales and the attendant small scale layers. The problem is that if this is done artificially either through the use of low resolution models or by ex-post facto data filtering/smoothing - or as in his comment – by both, then it will not justify the application of the stable layer concept to the real world. However, if there were a physical smoothing mechanism leading to a real-world scale separation between the small and large scales, then it might indeed be possible to exploit this scale separation to develop stability and gravity wave theories based on large scale average fields i.e. to average out the "local, instantaneous" variability as *Esau* [2009] attempts to do.

[7] We can now understand why the existence of a scaling/fractal structure is so important for our argument:

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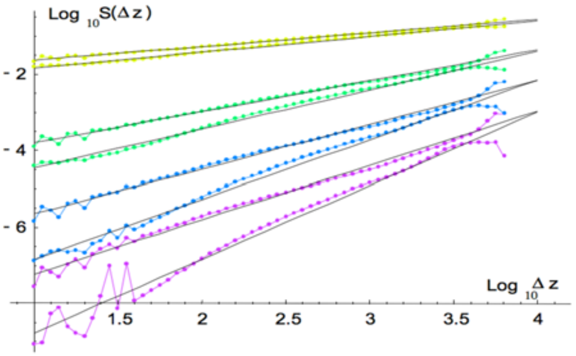

Figure 1. This shows the q th order structure functions for q = 0.4, 1, 1.6, 2.2 (top to bottom). In each pair, the top is for the log equivalent potential temperature and the bottom, the log potential temperature. 216 sondes from the Pacific 2004 experiment were used (see *Lovejoy et al.* [2008b]). The layer thickness Δz is in m, only sondes dropped from above 10km with at least 1000 points were used. Fluctuations were defined using three measurements to determine deviations from linearity. The slopes ($\xi(q)$) are 0.41, 1.02, 1.56, 1.93 and 0.36, 0.81, 1.16, 1.42 for log θ and log θ_E, respectively).

by definition in scaling regimes there are no scale separations. The weak part of the L08 demonstration is that it only considered the scaling of the places (the geometric set of points) where various vertical derivatives (e.g. ∂log $\theta/\partial z$, θ is the potential temperature) changed sign i.e. the fractal set of the unstable layers. To be more convincing we should directly consider the scaling of the fields (potential temperature, equivalent potential temperature; for brevity we do not consider the Richardson number, but see *Lovejoy et al.* [2007]). This is done in Figure 1 (for 216 drop sondes, i.e., about 10 times more than in L08). Figure 1 shows that the statistics of the (local, near instantaneous) fluctuations in log potential temperature ($\Delta\log\theta$) and log equivalent potential temperature ($\Delta\log\theta_E$) over layers thickness Δz are of the power law form:

$$S(\Delta z) = \langle \Delta \log \theta(\Delta z)^q \rangle \approx \Delta z^{\xi(q)}$$

("$\langle . \rangle$" indicates averaging over all the sondes, all the lags). Since over a layer of thickness Δz, the (squared) Brunt Väisälä frequency $N^2(\Delta z) = g\Delta\log\theta/\Delta z$, (with a corresponding relation for the equivalent potential temperature based frequency N_E^2 which takes the humidity into account), Figure 1 implies the multiscaling of the statistics of N^2, N_E^2. From Figure 1, we can see that the scaling holds with remarkable accuracy from several kilometers down to at least 10 m, and there is no compelling reason to believe that physical scale break required for *Esau*'s [2009] argument occurs anywhere above the (submillimetric) dissipation scale. The vertical scaling (and indeed cascade structure) of this and other atmospheric fields has recently been more fully analyzed by *Lovejoy et al.* [2009].

[8] As a final point, a very similar debate arises in the context of gravity wave theories. If one attempts to apply standard linear gravity wave theory to the real world (i.e., local and instantaneous), then one finds that the Reynolds numbers are huge and the assumptions of the linearized equations completely break down (and as mentioned by *Esau* [2009], the waves would have trouble propagating through the observed fractal hierarchy of unstable layers). However as discussed by *Lovejoy et al.* [2008a], the wide range scaling associated with a strongly nonlinear, (high Reynolds number) atmosphere can itself give rise to dispersion relations very close to those derived by assuming quasi-linearity. This is fortunate since it means that we needn't live in a low resolution world in order to have turbulence driven wave-like phenomenologies.

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