17

Wind Extremes and Scales: Multifractal Insights and Empirical Evidence

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Summary. An accurate assessment of wind extremes at various space-time scales (e.g. gusts, tempests, etc.) is of prime importance for a safe and efficient wind energy management. This is particularly true for turbine design and operation, as well as estimates of wind potential estimates and wind farm implementation. We discuss the consequences of the multifractal behaviour of the wind field over a wide range of space-time scales, in particular the fact that its probability tail is apparently a power-law and hence much "fatter" than usually assumed. Extremes are therefore much more frequent than predicted from classical thin tailed probabilities. Storm data at various time scales are used to examine the relevance and limits of the classical theory of extreme values, as well as the prevalence of power-law probability tails.

17.1 Atmospheric Dynamics, Cascades and Statistics

Further to his "poem" [1] presenting a turbulent cascade as the fundamental mechanism of atmospheric dynamics, Richardson [2] showed empirical evidence that a unique scaling regime for atmospheric diffusion holds from centimeters to thousands of kilometers. It took some time to realize that a consequence of a cascade over such a wide range of scale is that the probability tails of velocity and temperature fluctuations are expected to be power-laws [3, 4]. Indeed, this is a rather general outcome of cascade models, independently of their details [5]: the mere repetition of nonlinear interactions all along the cascade yields the probability distributions with the slowest possible fall-offs, i.e. power-laws, often called Pareto laws. The critical and practical importance of the power law exponent q_D of the probability tail, defined by the probability to exceed a given large threshold s, could be understood by the fact that all statistical moments of order $q \geq q_D$ are divergent, i.e. the theoretical moments – denoted by angle brackets <.> – are infinite:

$$\text{for large} \quad s : \Pr(|\Delta v| > s) \approx s^{-q_D} \Leftrightarrow \text{any} q > q_D : \langle |\Delta v|^q \rangle = \infty \tag{17.1}$$

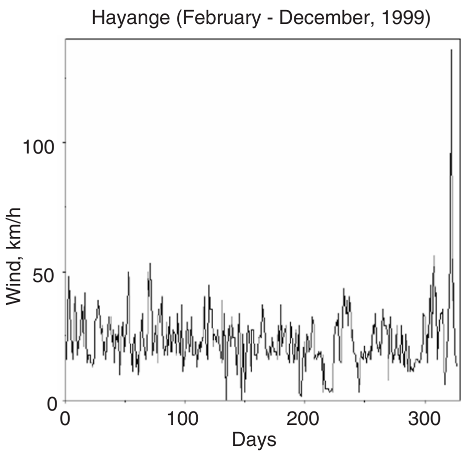

Fig. 17.1a. Daily wind from February to December 1999 at Hayange station

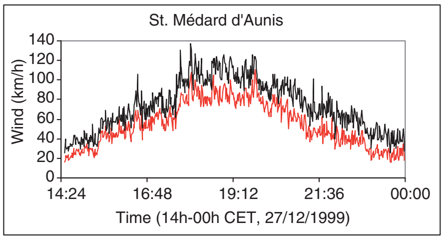

Fig. 17.1b. Mean (red) and maximal (black) wind (both at 15 min resolution) at Saint Médard d'Aunis (16:00-20:00 (*top*); 20:00-24:00 (*bottom*))

and their empirical estimates diverge with the sample size. The probability of having an extreme 10 times larger decreases only by the factor 10^{q_D}, κg$1/q_D$ is often called the form parameter.

However, by the end of 1950's Richardson's cascade was split [6, 7] into a quasi-2D macro turbulence cascade, a quasi-3D micro turbulence cascade, separated by a meso-scale (energy) gap necessary to avoid contamination of the former by the latter. This gap got some initial empirical support [8], but was more and more questioned [e.g. [9]]. Starting in the 1980's through to the present, this atmospheric model has became untenable thanks to various empirical analyses [10–15] which showed that a new type of strongly anisotropic cascade operates from planetary to dissipation scales. This cascade is neither quasi-2D, nor quasi-3D, but has rather an "elliptical" dimension $D_{\text{el}} = 23/9 \approx 2.555$ (in space-time, $29/9 = 3.222$).

Indeed, a scaling anisotropy could be induced by the (vertical) gravity and the resulting buoyancy forces that generate two distinct scaling exponents for horizontal and vertical shears (H_{h}, respectively, H_{v}) of the velocity field and the elliptical dimension value is defined by $D_{\text{el}} = 2+\ H_{\text{h}}/H_{\text{v}}$, the theoretical values $H_{\text{h}} = 1/3$, $H_{\text{v}} = 3/5$ being obtained by a reasoning "à la Kolmogorov" [16] and, respectively, "à la Bolgiano–Obukhov" [17, 18]. The theoretical possibility of having power-law pdf tail got also some empirical support with $q_D \approx 7$ for the velocity field [12, 13].

17.2 Extremes

If wind correlations had only short ranges, then the statistical law of the extremes over a given a period would be determined by the classical extreme value theory [19, 20]: the power-law probability tail of the wind series would

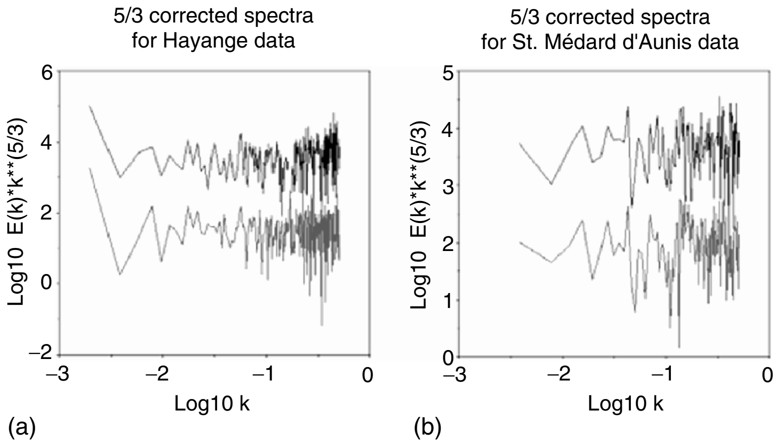

Fig. 17.2. Compensated spectra of the two horizontal components of the data of Fig. 17.1a and of Fig. 17.1b, respectively. The horizontal plateau correspond to the Kolmogorov scaling

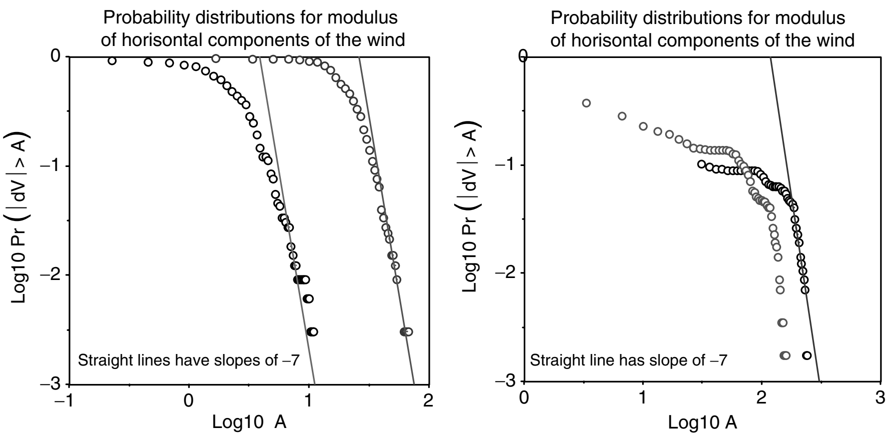

Fig. 17.3. Probability tails of the wind increments of the two horizontal components of the data of Fig. 17.1a and of Fig. 17.1b, respectively

imply that the wind maxima distribution is a Frechet law [21], often called Generalized Extreme Value distribution of type 2 (GEV2), instead of the more classical Gumbel law (GEV1). GEV1 and GEV2 are quite distinct, since GEV1 has the same power law exponent $q_D$as the original series, whereas GEV2 has a very thin tail corresponding to a double (negative) exponential. This sharp contrast results from the fact that a Frechet variable corresponds to the exponential of a Gumbel variable, therefore its distribution is often called "Log-Gumbel." We will illustrate this with the help of Météo-France wind data, in particular those collected during the two "inland hurricanes"

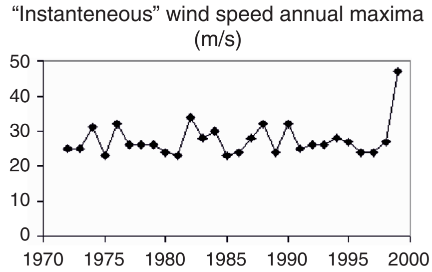

Fig. 17.4. Yearly wind maxima at Montsouris station during the period of 1970–1999

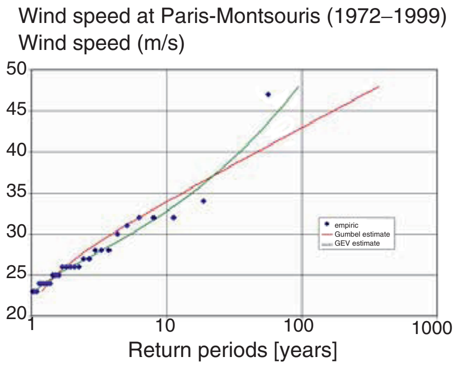

Fig. 17.5. GEV1 (*red straight asymptote*) and GEV2 (*green convex curve*) distributions fitted to the data of Fig. 4

that swept across France by the end of the last century (25-26/12/1999 and 27-28/12/1999). They correspond to sharp spikes in daily wind time series at various meteorological stations (Fig. 17.1a). However, intermittent fluctuations are also present at scales of 1 min (Fig 17.1b) and all these fluctuations respect the Kolmogorov scaling (Fig. 17.2 a-b). Furthermore, their probability distributions (Fig. 17.3 a-b) display a rather clear power-law fall-off with an exponent $q_D \approx 7$, in agreement with previous studies.

Looking to larger time scales, let us consider the yearly wind maxima in Paris area during the period of 1970–1999 (Fig. 17.4), as well as the fitted GEV1 and GEV2 distributions (Fig. 17.5) in the so-called Gumbel paper (i.e. wind speed vs. double logarithm of the empirical probability distribution) in which the asymptote of GEV1 is a straight line, whereas GEV2 curve remains convex. It is rather obvious, that both fits are rather equivalent up to the year 1998, whereas the latter can be only captured by the convexity of GEV2.

17.3 Discussion and conclusion

The hypothesis of short range correlations, which is necessary to derive the classical extreme value theory, is not satisfied by cascade processes. Indeed, the latter introduce power law dependencies. It is therefore indispensable to look for a generalization of extreme value theory in a multifractal framework. This task might be not as difficult as it looks at first glance, since for instance cascade processes may have (statistical) 'mixing' properties [22, 23], which have been used to extend the extreme value theory from uncorrelated time series to those having short range correlations. This may partially explain why GEV2 fits rather well the empirical extremes, although one may expect that its asymptotic power-law exponent might differs from that of the probability tail of the original time series. Larger data bases and numerical cascade simulations are currently being analysed to clarify these issues. However, it may be already timely to use multifractal wind generators in numerical simulations for the design and management of wind turbines.

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