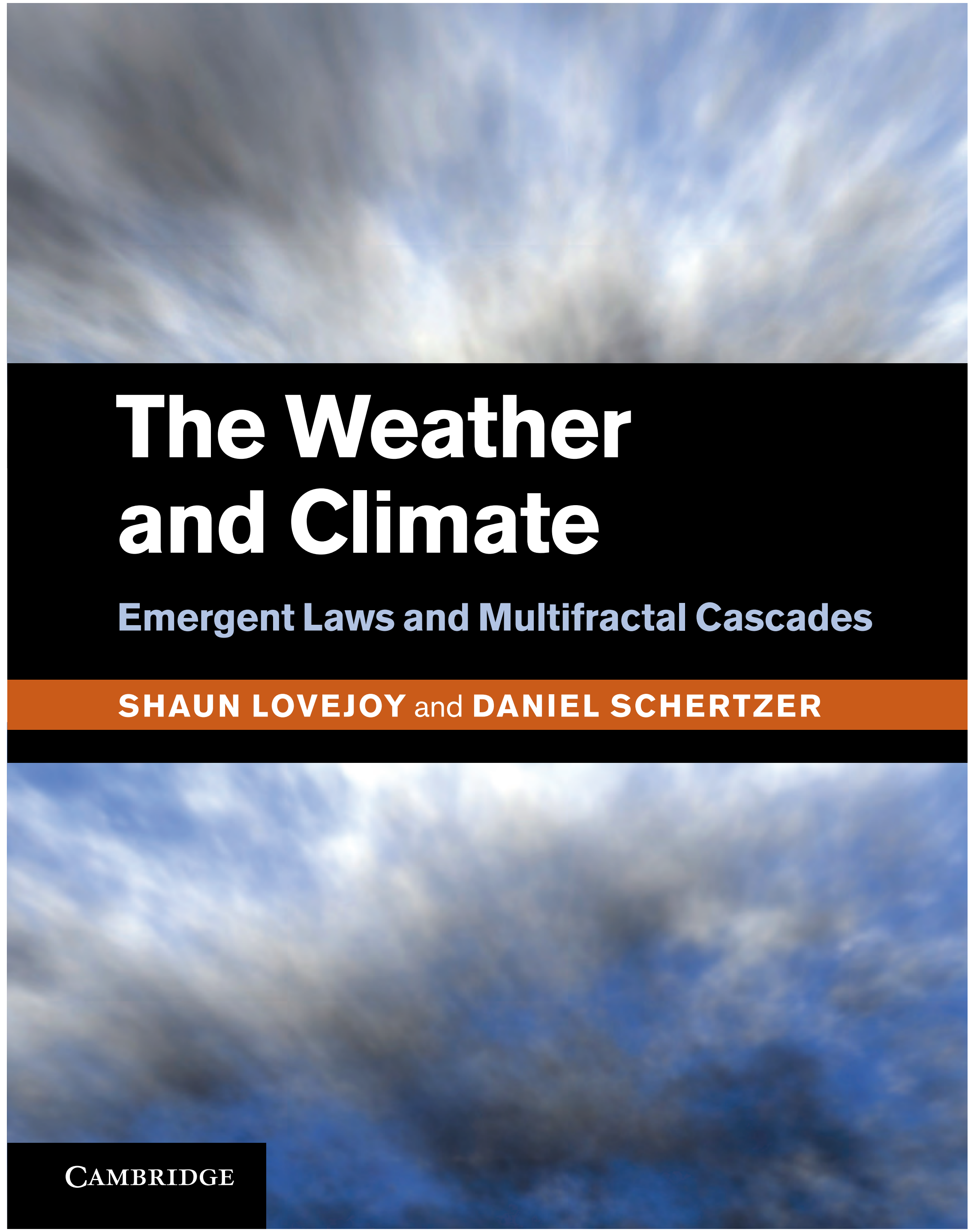
The weather and climate as problems in physics: scale invariance and multifractals

S. Lovejoy McGill, Montreal

McGill
April 13, 2012

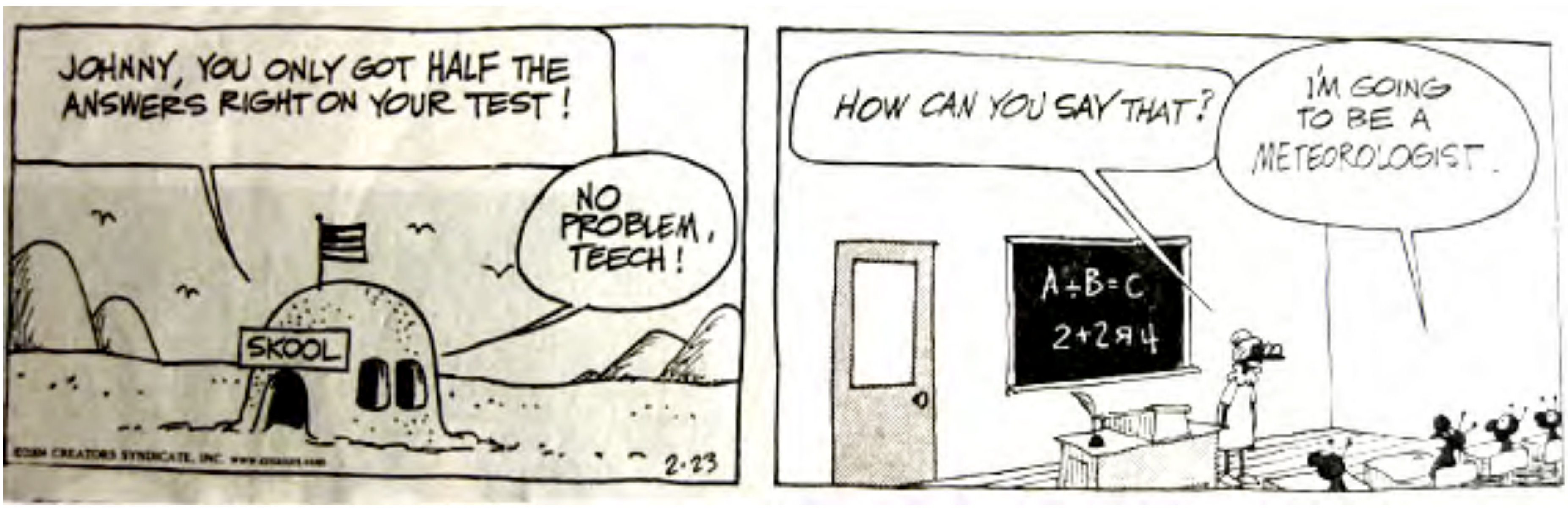
Required reading for this course

(in press, Oct. 2012)



The Weather

Meteorologists

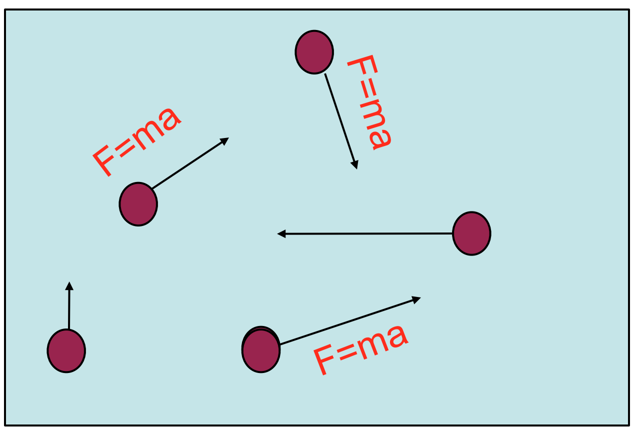


The Emergence of physical laws

Quantum mechanics	→ Large scales (usually)	Classical Mechanics
Statistical mechanics	→ Large numbers of particles	Continuum mechanics, **Fluid mechanics** thermodynamics
General Relativity	→ Low energy mass density	Special Relativity
Special Relativity	→ Velocities << speed of light	Classical (Galilean) Relativity
Low level (fundamental)		high level (simpler if applicable)

Example: The emergence of Thermodynamics from Newton's laws

Newton's laws:



Low level, (difficult to handle for many particles)

Large number of particles

Thermodynamics:

First law: conservation of energy

Second law: increase in entropy

ex.: Boyle's law:
(pressure) x (volume) = constant

High level
(Valid when many particles are present)

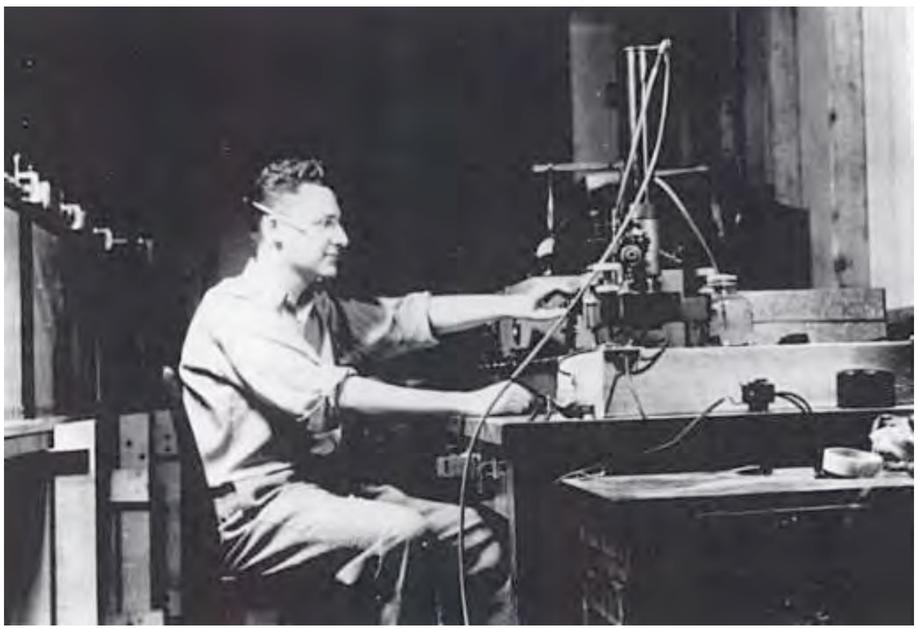
Pioneers of turbulence

Richardson
1881 - 1953

Kolmogorov
1903 – 1987

Corrsin

1920 – 1986



Obukhov

1918 – 1989

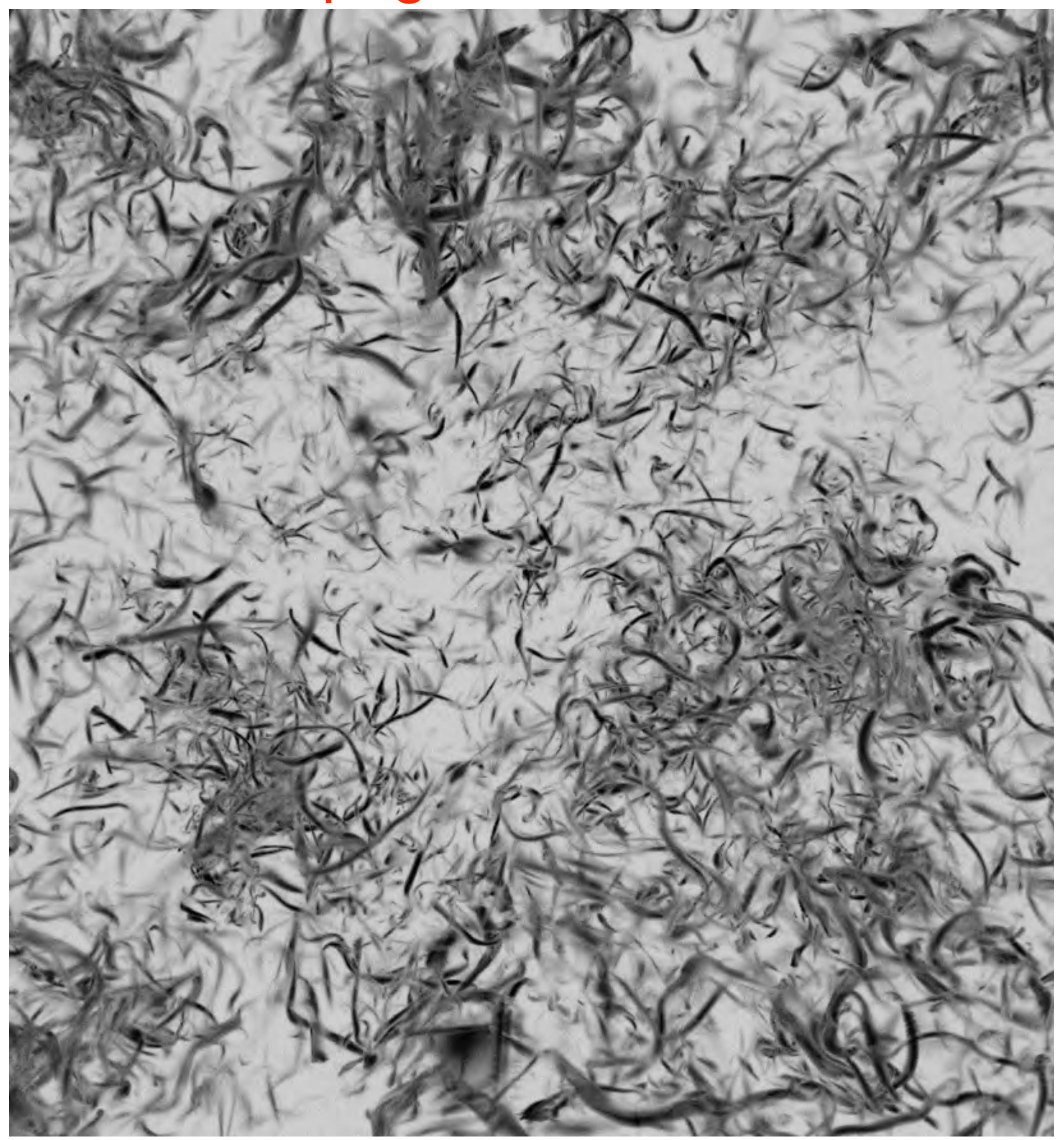
Ralph Bolgiano, Jr.

1922 — 2002

The emergence of turbulence dynamics (Classical)

Vortices in strongly turbulent fluid
(M. Wiczek, numerical simulation, 2010)

"Spaghetti"



Fluid mechanics

Low level
(fundamental)

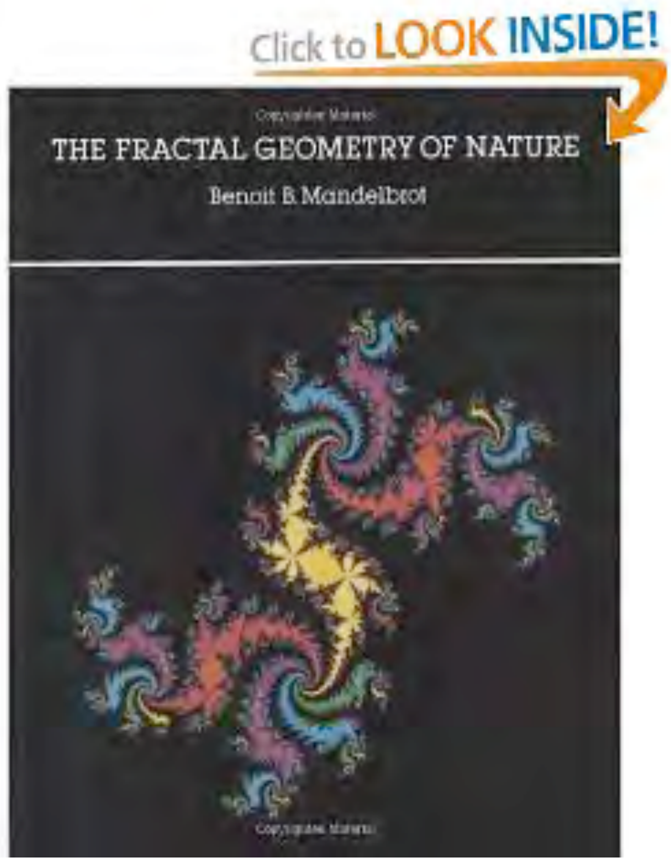
Strong stirring
(nonlinearity)

Laws of turbulence
Classical:
Richardson, Kolmogorov,
Corrsin, Obukhov, Bolgiano

High level

Emergent laws reduce seeming complexity to simplicity at another level

Mandelbrot
1924-2010

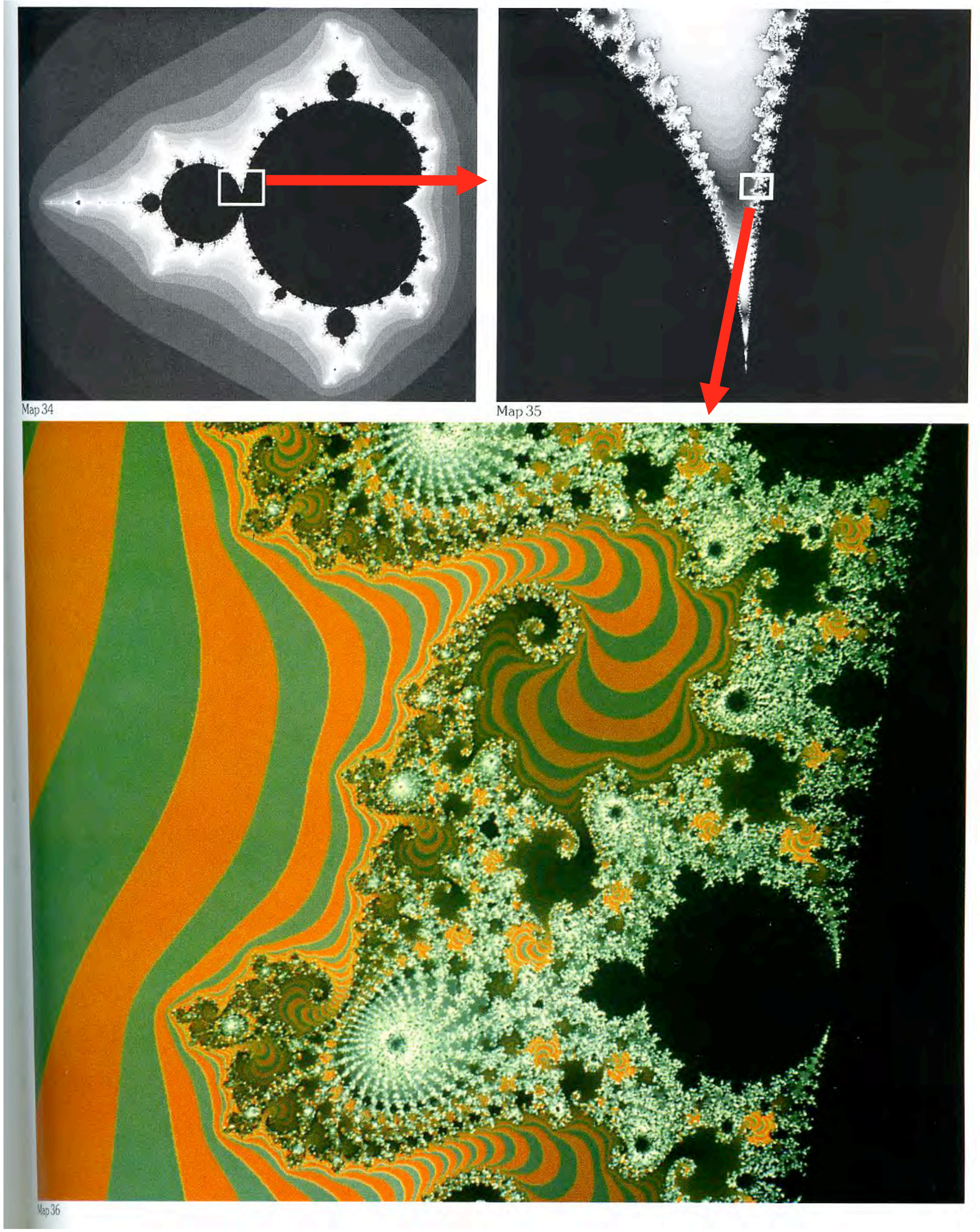


Complex?

Blowing up gives the same type of shapes

The Mandelbrot set

("self-similar", scale invariant, fractal)



Or simple?

Generating the Mandelbrot set

-Take a number.

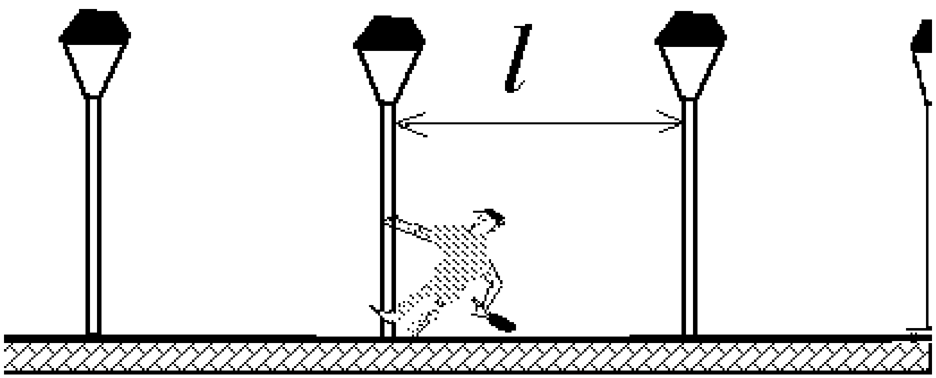
-Multiply it by itself.

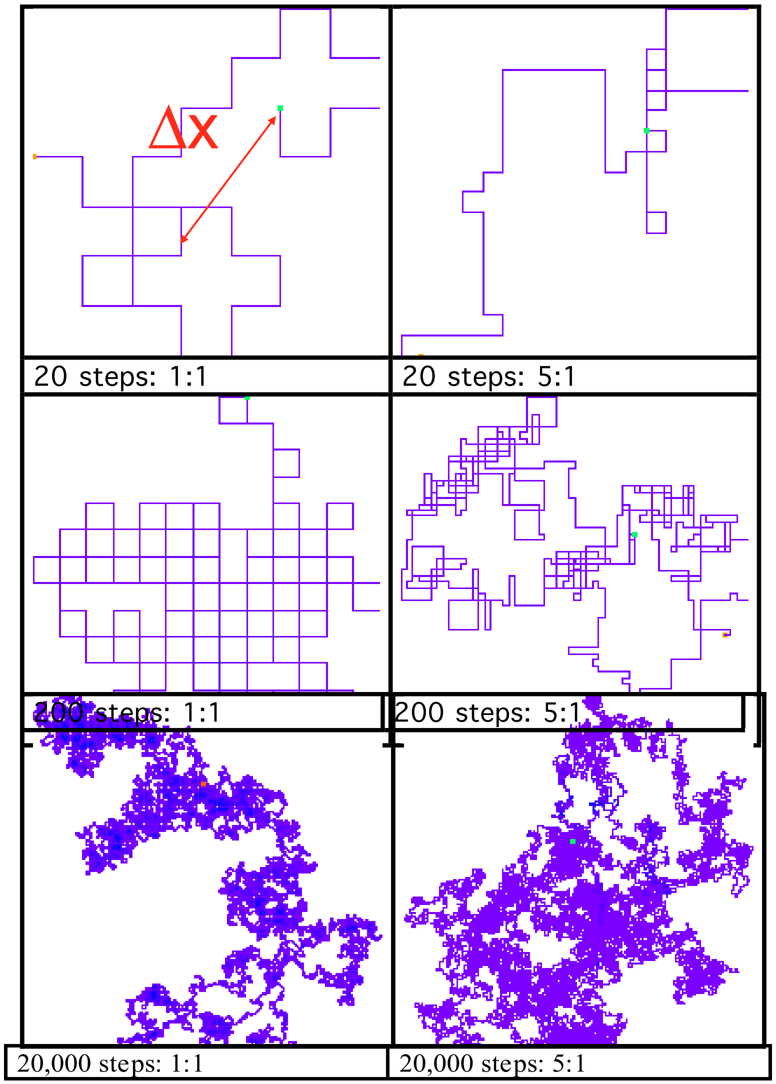
-Add a constant.

-Repeat.

(I forgot to mention: take a COMPLEX number)

Complex?





Drunkard's walk

Or simple?

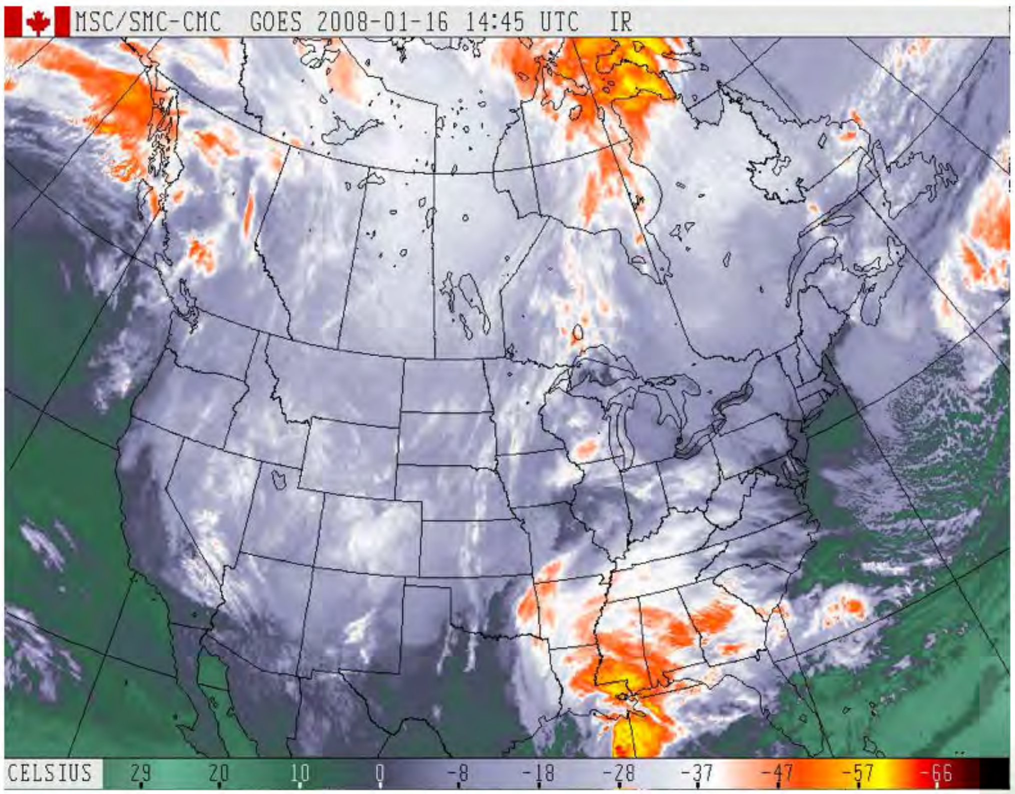
(distance) x (distance) = number of bars visited

From initial bar

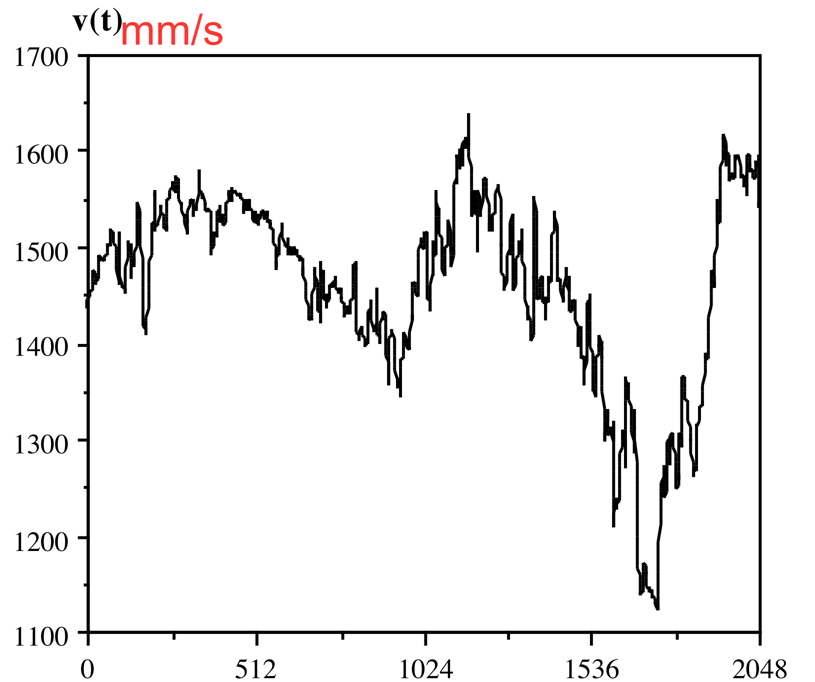
Average number of bars visited
(or displacements made)

(Brownian motion)

Complex?… or simple?



Infra Red satellite effective temperatures, January 16, 2008



1 second of wind data (roof of Rutherford building, McGill)

The Atmosphere

Brute force…

Atmosphere: Laws of Fluid mechanics

(low level)

Gravitational potential

wind

Earth angular velocity

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}.\,\mathrm{grad})\mathbf{u} - 2\boldsymbol{\Omega} \times \mathbf{u} - \alpha\,\mathrm{grad}\,p - \mathrm{grad}\,\Phi + \mathbf{F}$$

Specific volume=1/ρ

Friction

pressure

$$c_v \frac{\partial T}{\partial t} = -c_v(\mathbf{u}.\,\mathrm{grad})T - \frac{p}{\rho}\,\mathrm{div}\,\mathbf{u} + Q$$

Specific heat

temperature

Heating rate

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u}.\,\mathrm{grad})\rho - \rho\,\mathrm{div}\,\mathbf{u}$$

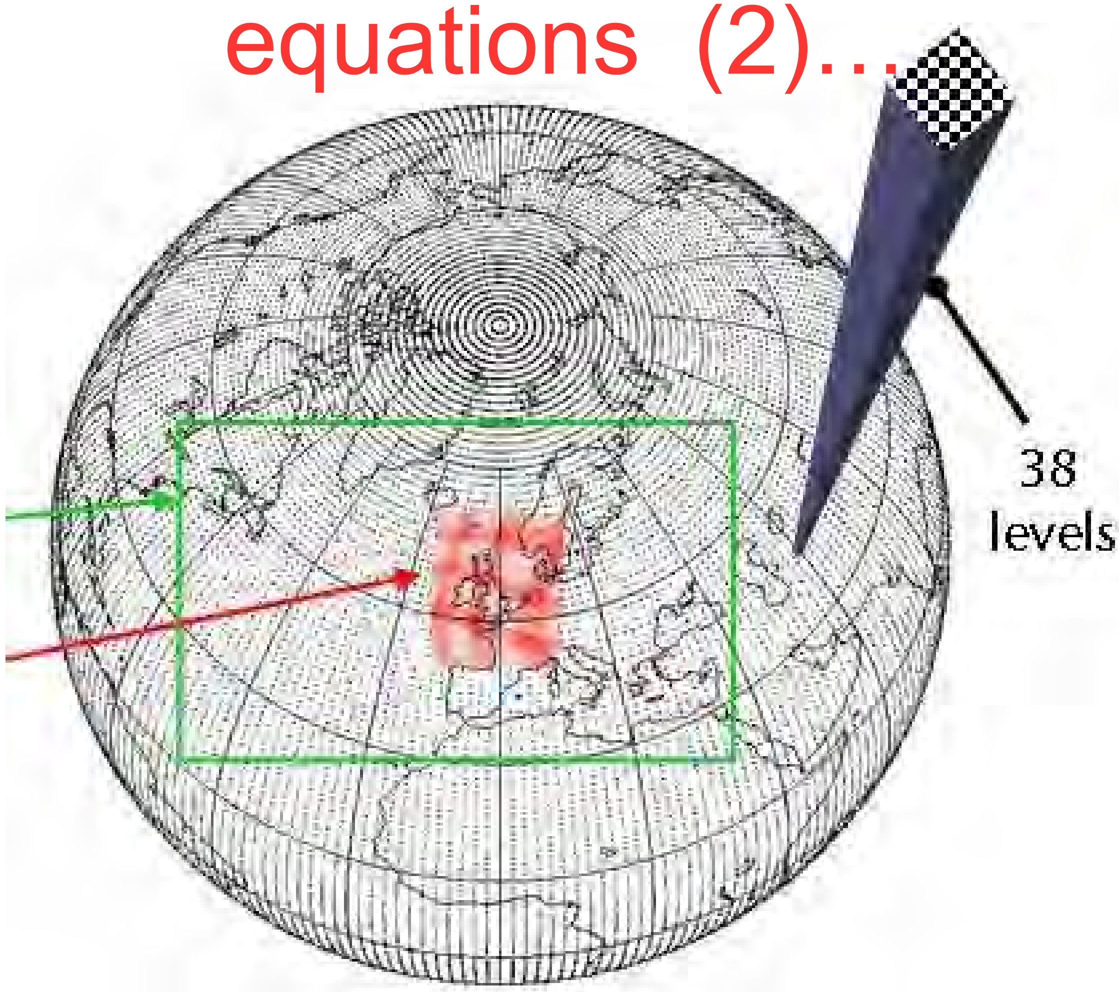
$$p = \rho R T.$$

Gas constant

density

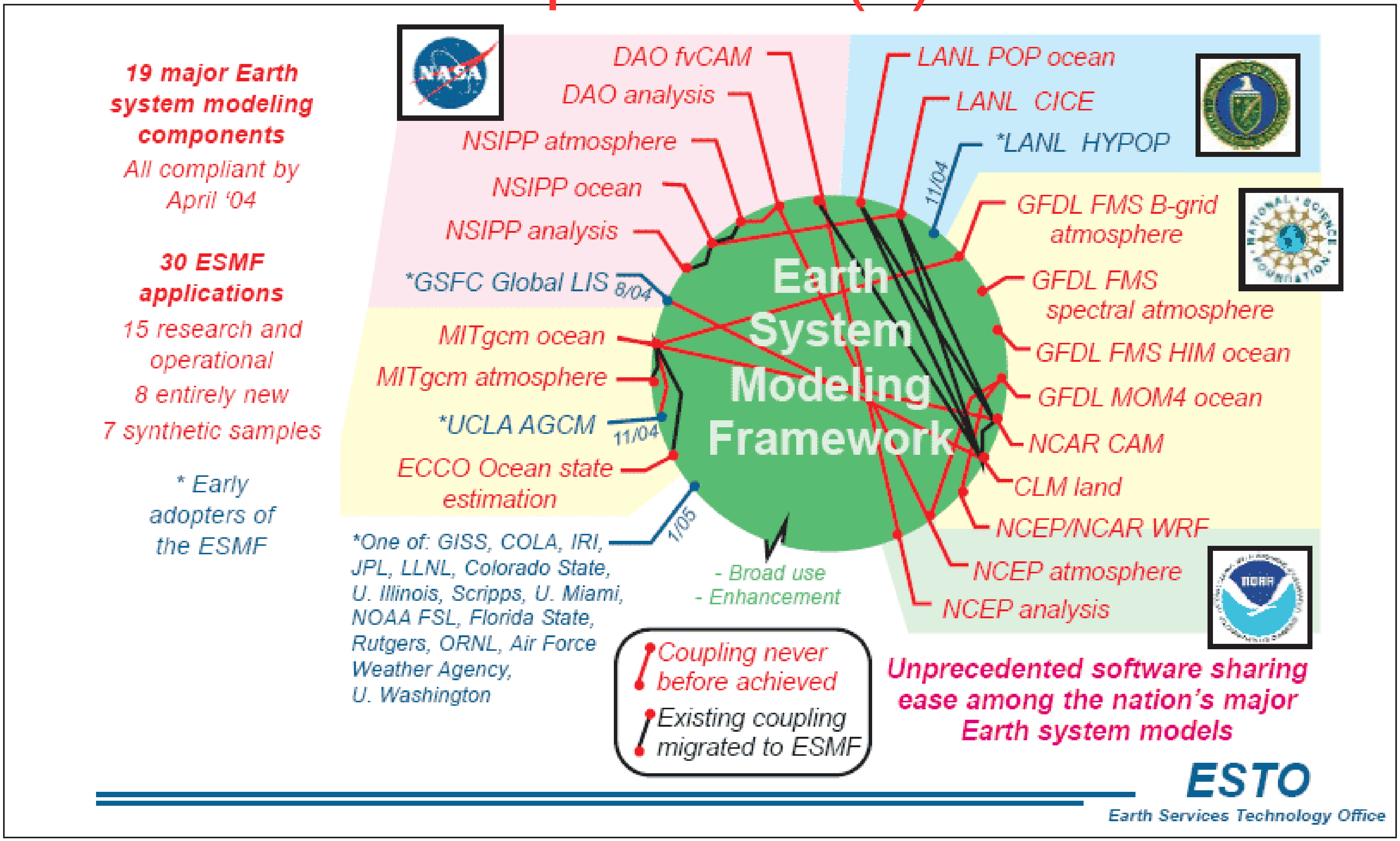
Governing atmospheric laws

Brute force numerical solution of the equations (2)...



Discretization of the equations

Brute force numerical solution of the equations (3)…



Earth system modelling

Or simplicity?

Atmosphere: Emergent laws

(high level)

Power law

$$\boxed{\text{Fluctuations} \approx (\text{turbulent flux}) \times (\text{scale})^{H}}$$

Differences,
tendencies,
wavelet
coefficients

Cascading
Turbulent flux

Size:
Anisotropic
Space-time
Scale function

Fluctuation
/conservation
exponent

These laws
are scale
invariant

Fluctuation = change in time and/or space

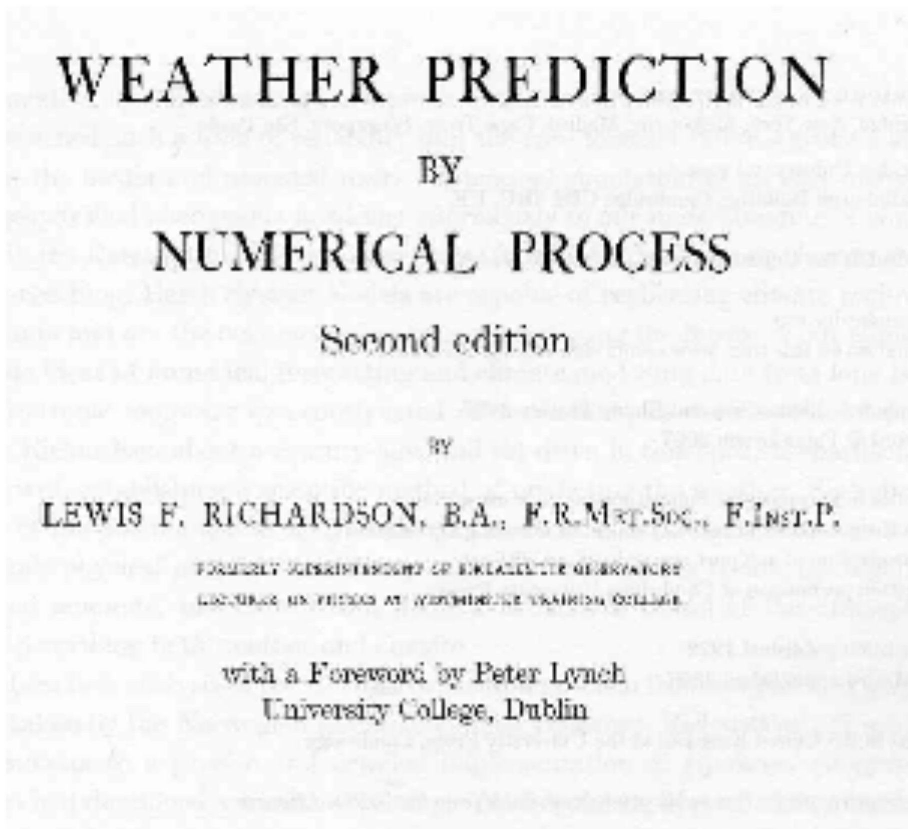
Scale = size

Turbulent flux = strength of stirring

Which Richardson? The father of Numerical Weather Prediction…

L. F. Richardson, 1931

The father of numerical weather prediction



WEATHER PREDICTION

BY

NUMERICAL PROCESS

Second edition

BY

LEWIS F. RICHARDSON, B.A., F.R.MET.SOC., F.INST.P.

FORMERLY SUPERINTENDENT OF ESKDALEMUIR OBSERVATORY

LECTURER ON PHYSICS AT WESTMINSTER TRAINING COLLEGE

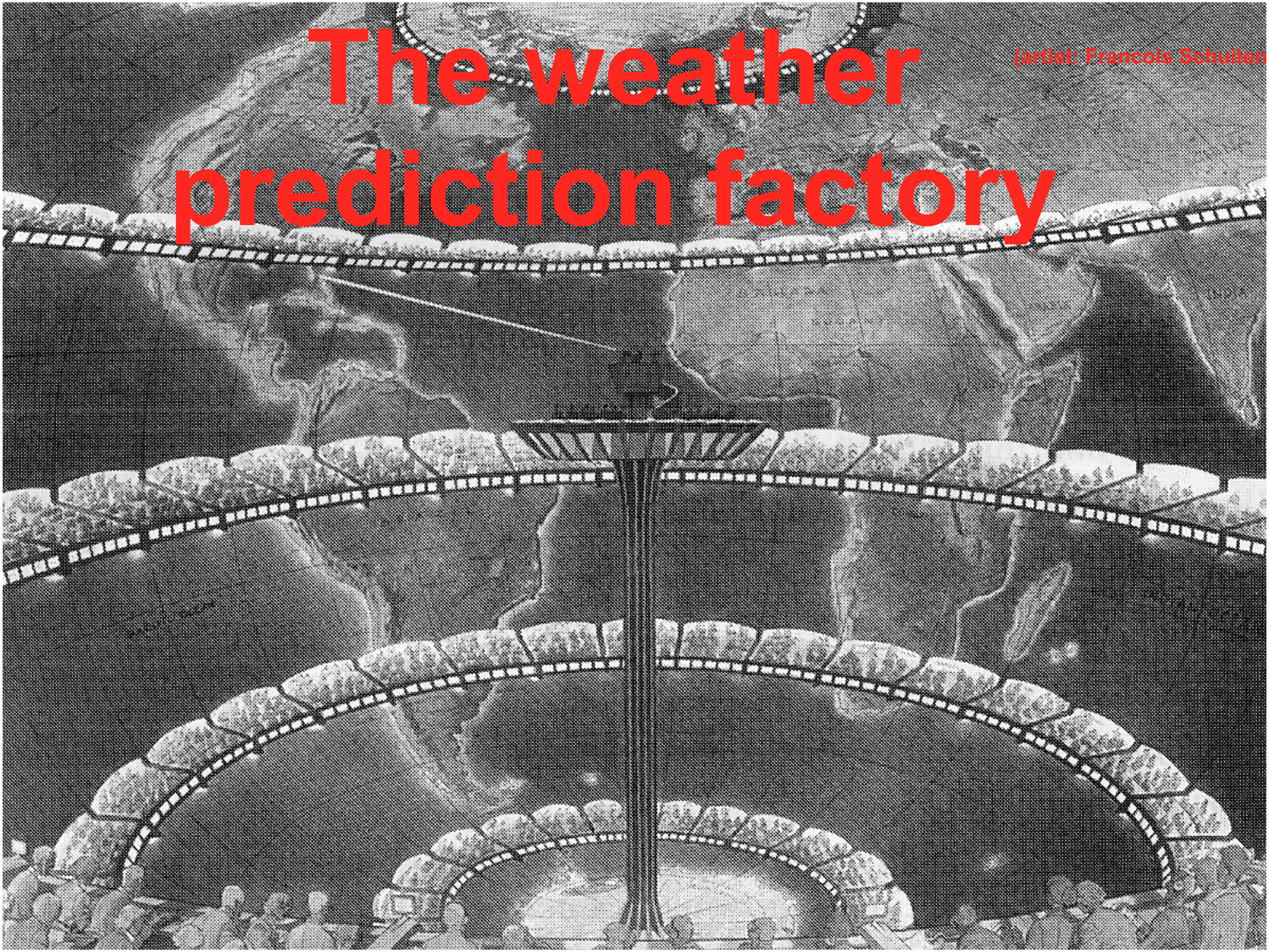
with a Foreword by Peter Lynch

University College, Dublin

1922

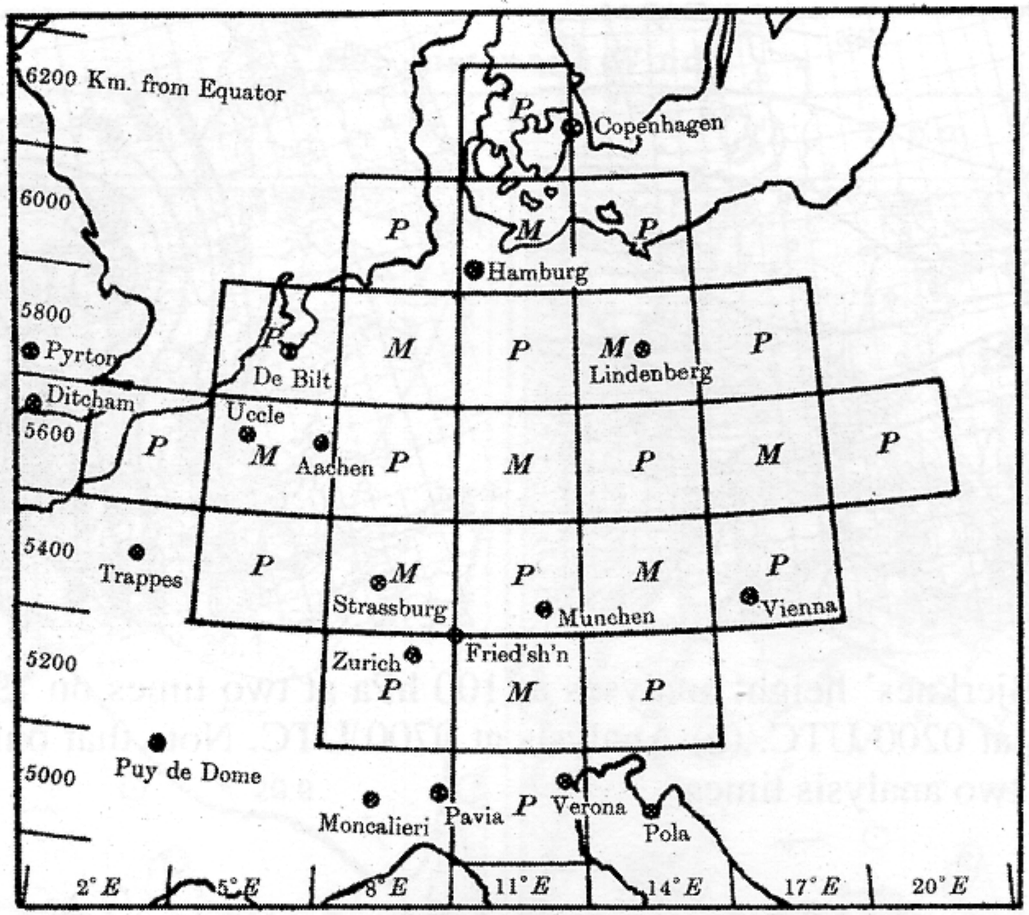
The weather prediction factory

(artist: Francois Schuiten



Richardson's numerical grid for integrating

Each column was divided into 5 vertical cells and defined 7 quantities: pressure, temperature, density, water content, 3 velocity components



"It took me the best part of six weeks to draw up the computing forms and to work out the new distribution in two vertical columns for the first time. My office was a heap of hay in cold rest billet. With practice the work of an average computer might go perhaps ten times faster. If the time-step were 3 hours, then 32 individuals could just compute two-points so as to keep up with the weather."

-Richardson 1922

... or the grandfather of cascades?

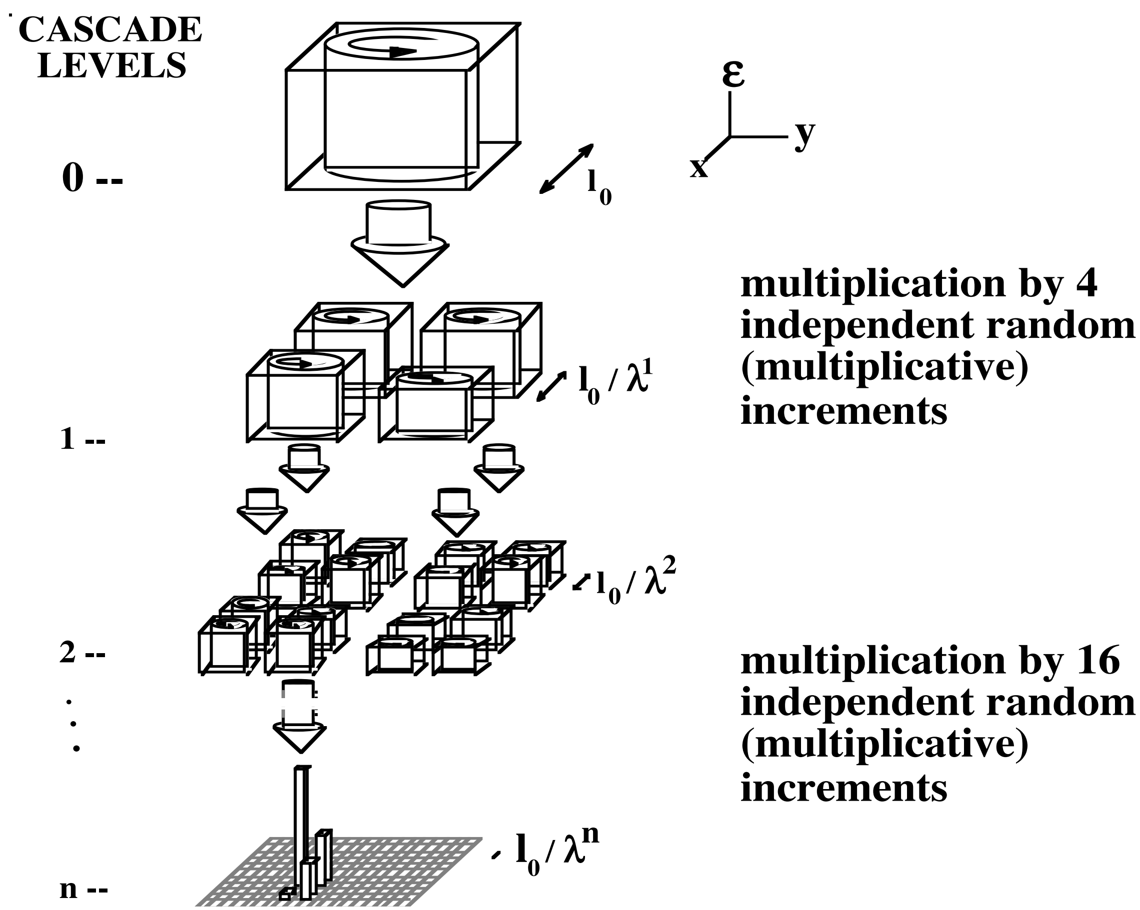
C. K. M. Douglas

writing of observations from aeroplanes remarks: "The upward currents of large cumuli give rise to much turbulence within, below, and around the clouds, and the structure of the clouds is often very complex." One gets a similar impression when making a drawing of a rising cumulus from a fixed point; the details change before the sketch can be completed. We realize thus that: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense.

Thus, because it is not possible to separate eddies into clearly defined classes according to the source of their energy; and as there is no object, for present purposes, in making a distinction based on size

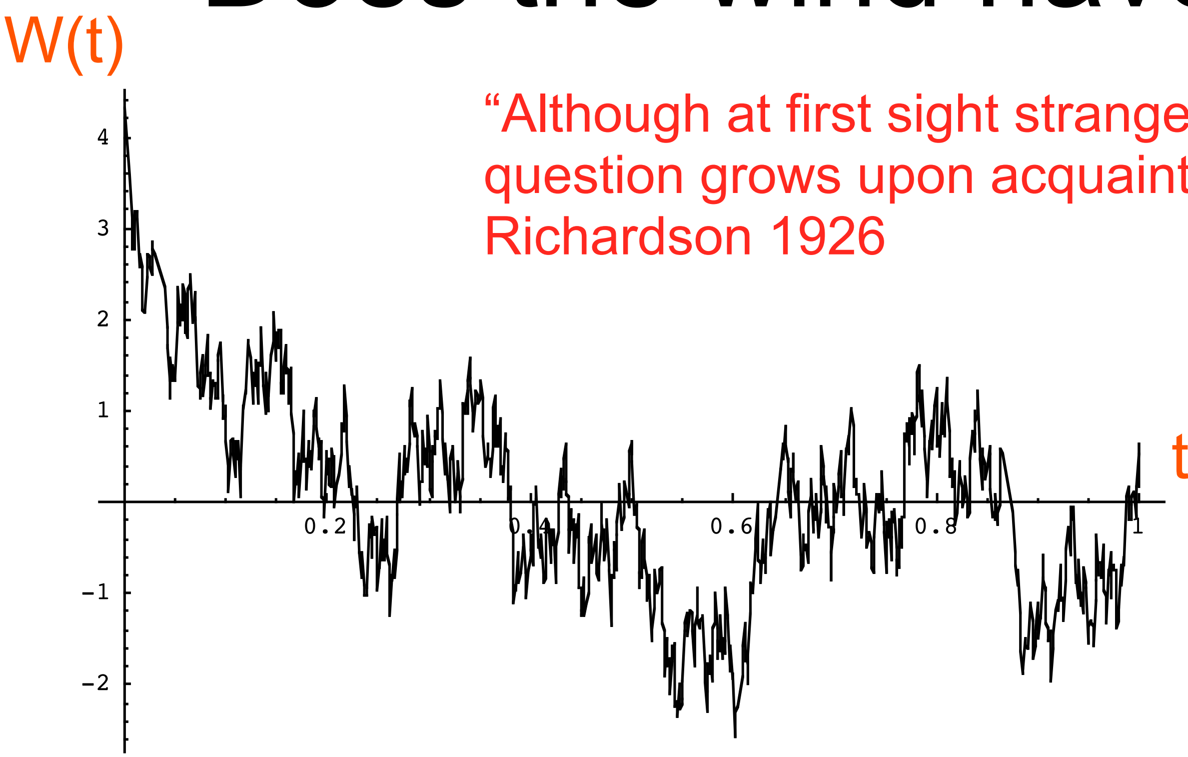
Weather prediction by Numerical Process 1922, p.66

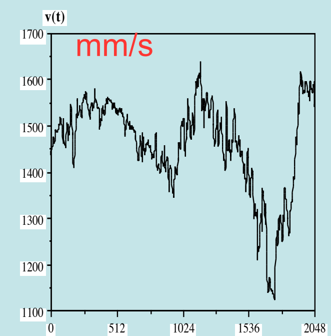
Scale by scale simplicity: cascades



“Does the wind have a velocity?”

“Although at first sight strange, the question grows upon acquaintance…” - Richardson 1926



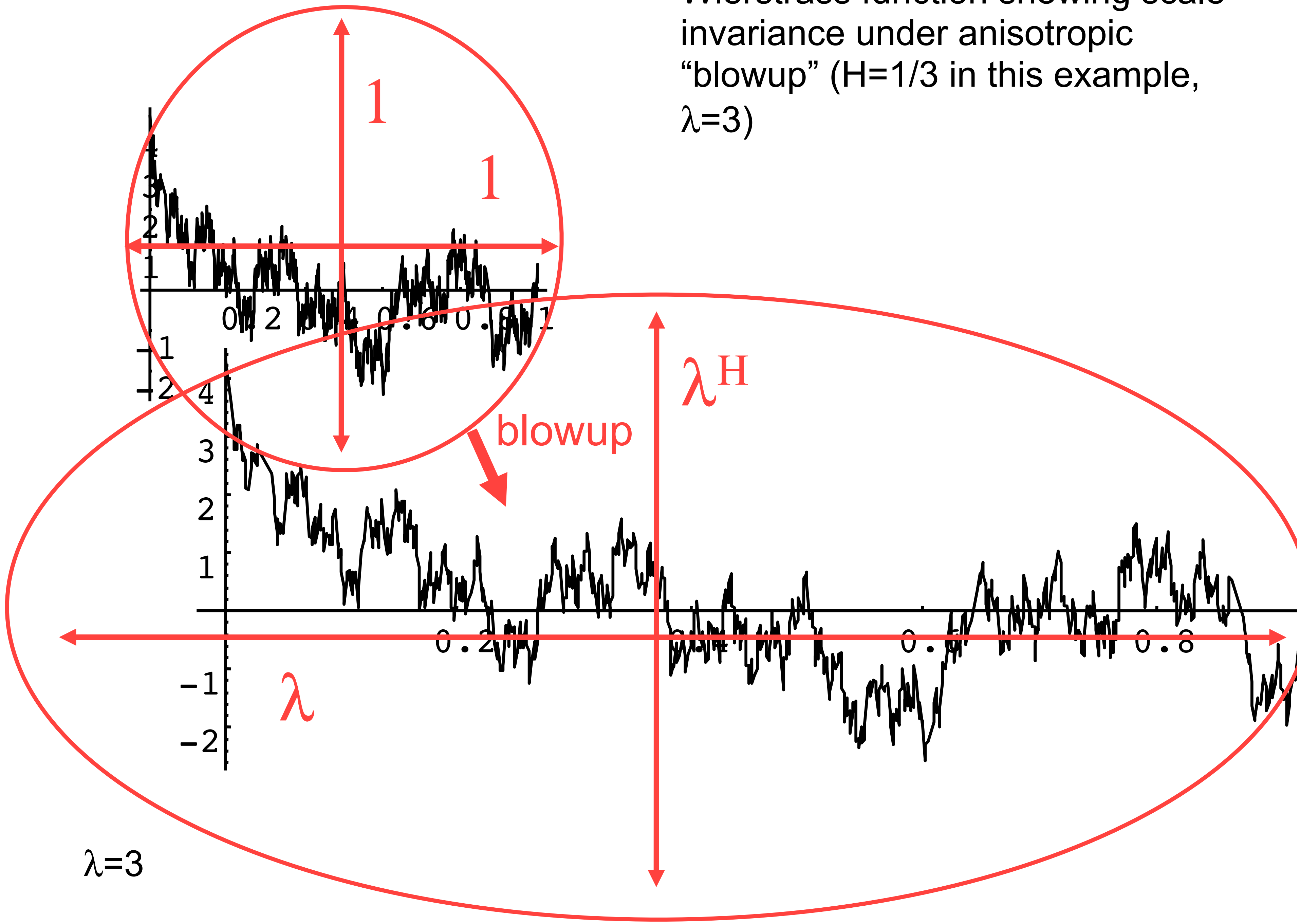


1 second of wind data (roof of Rutherford building, McGill)

Richardson suggested that the trajectory of a particle could be like a Wierstrass function (1872)

Scale invariance and fractals

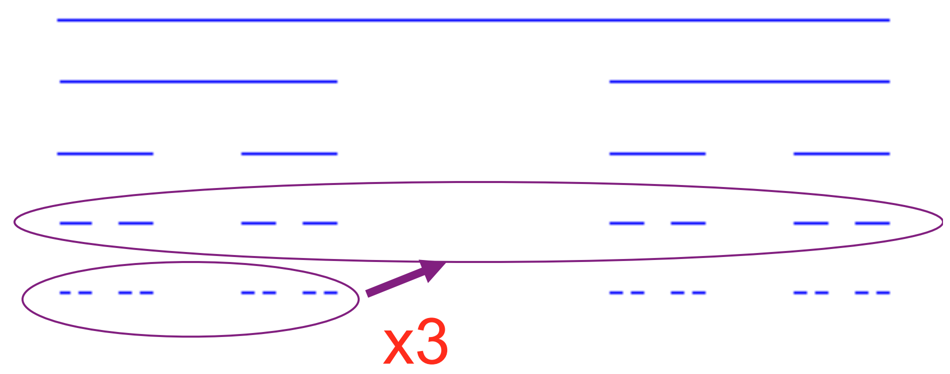
Wierstrass function showing scale invariance under anisotropic "blowup" (H=1/3 in this example, λ=3)



Cantor set

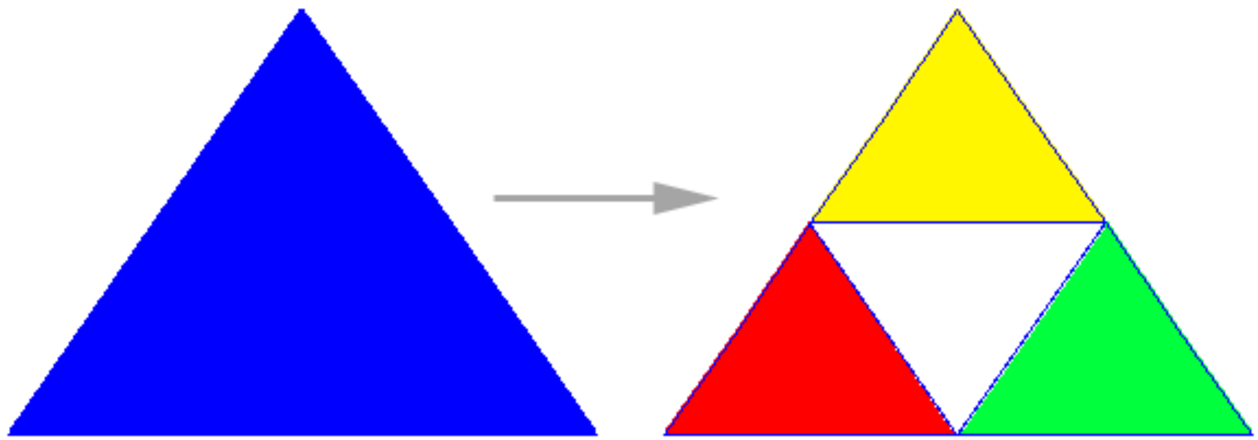
- Let us start with:

and let us iterate:

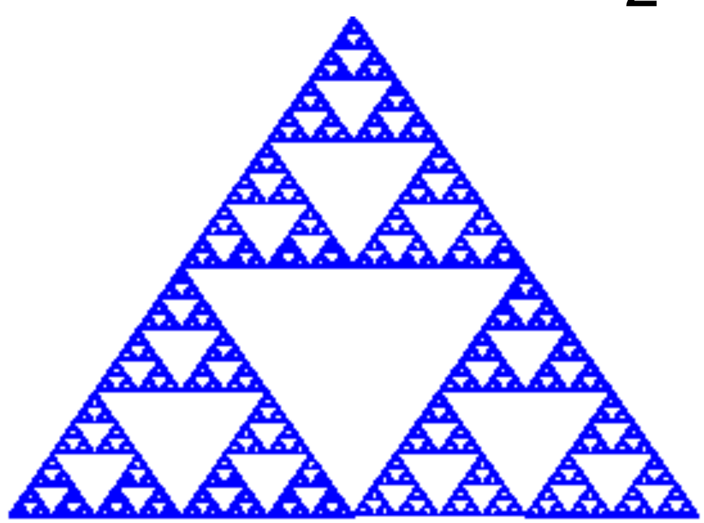


A small part is same as the whole if "blown up" by a factor 3 ("scale invariance", "self-similarity")

Sierpinski Triangle



1 2

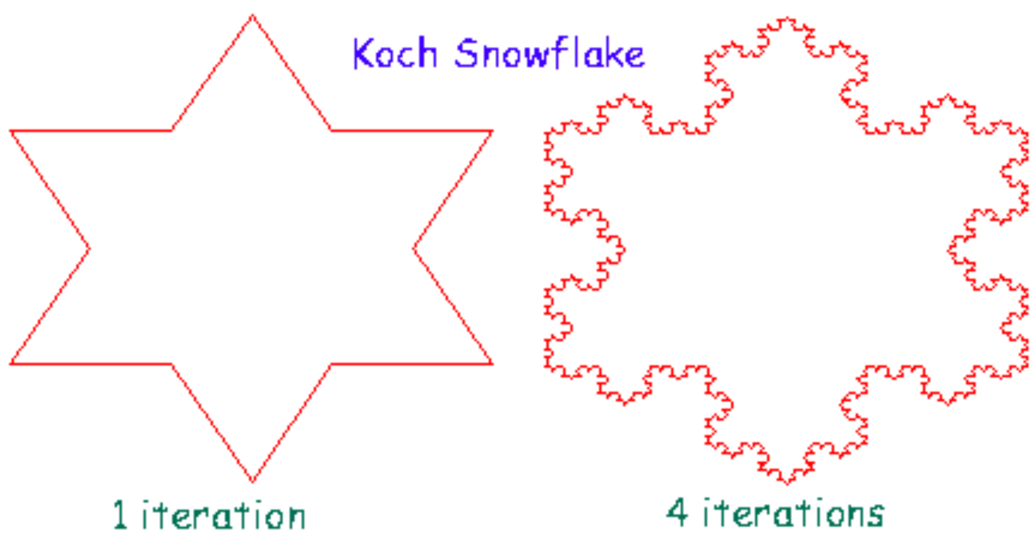


10 iterations

Koch snowflake

Let us start with:

and let us iterate:



Sierpinski Pyramid

- First iteration:

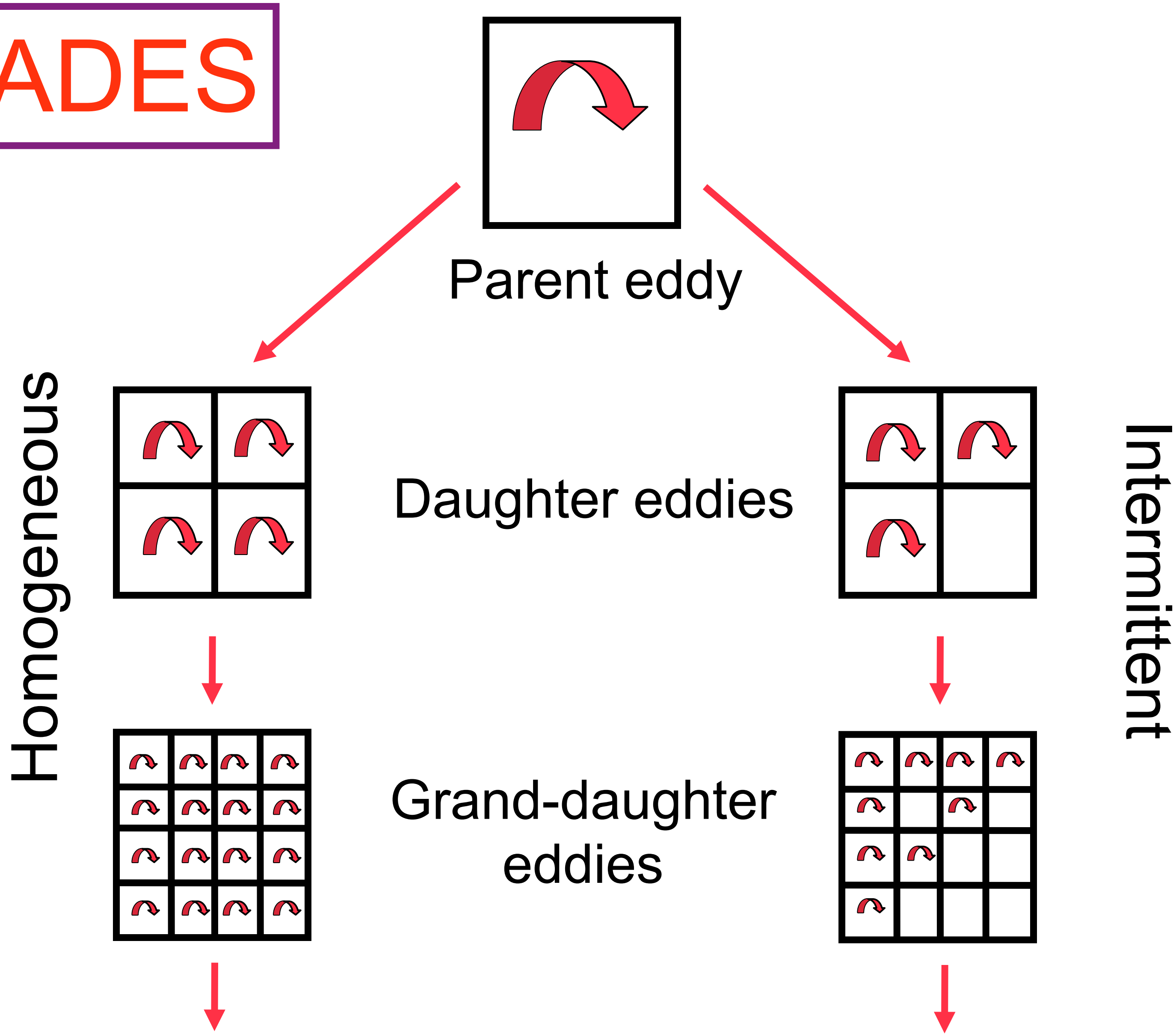
10 th iteration:

Menger Sponge

- motif:

  iterations:

CASCADES



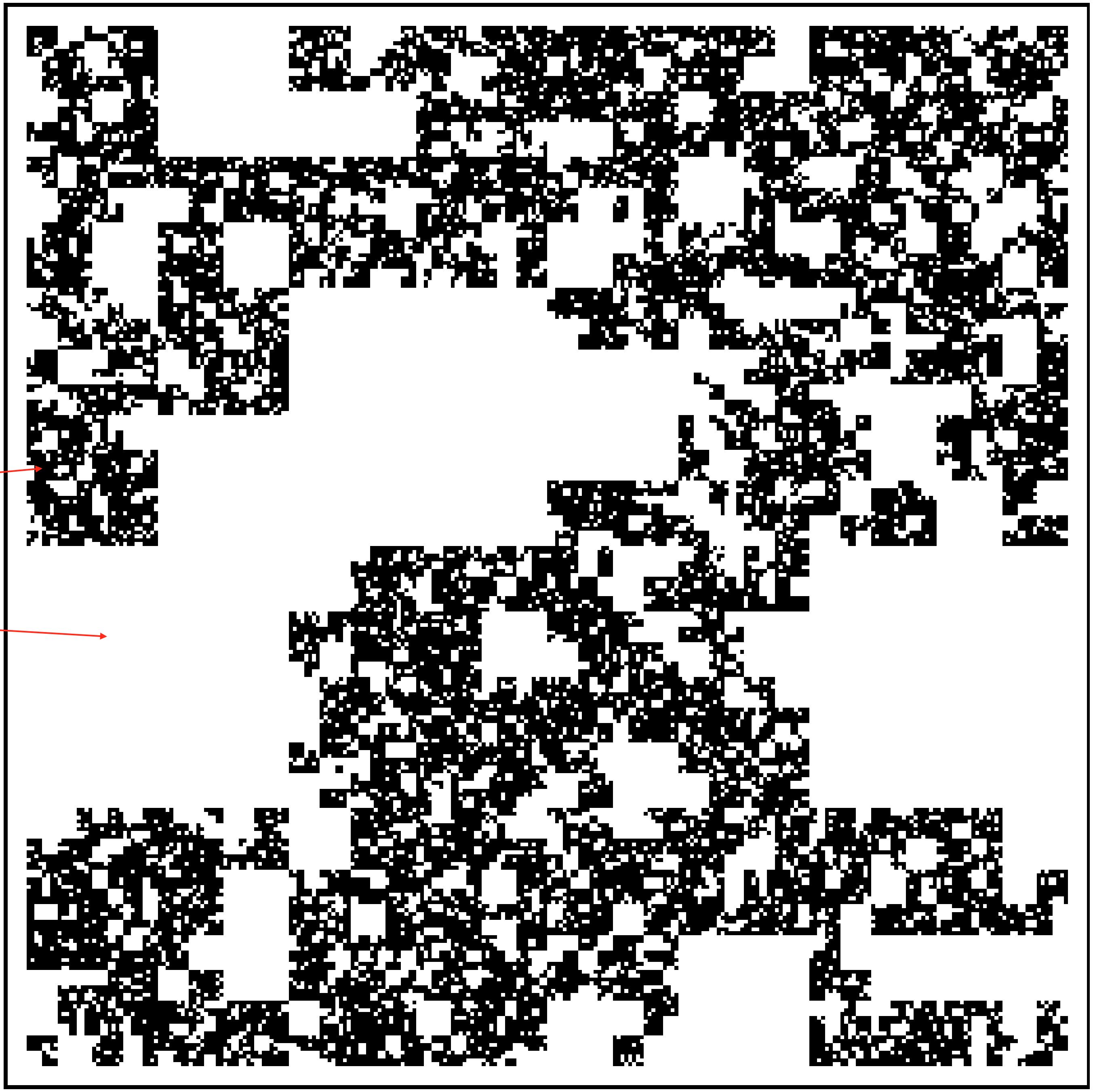
Parent eddy
Homogeneous
Daughter eddies
Intermittent
Grand-daughter eddies

β-model

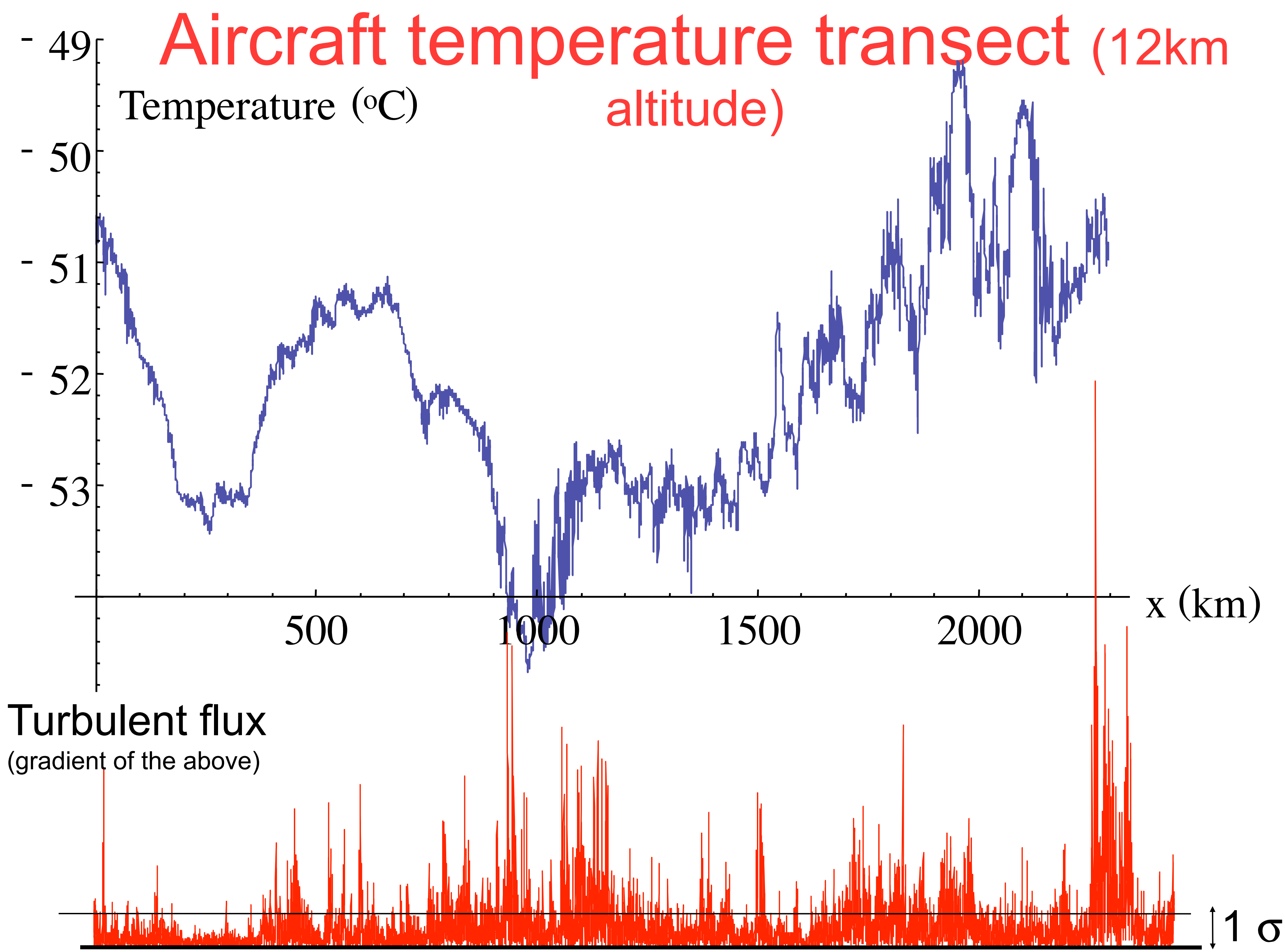
Fractal set

“active”

“calm”



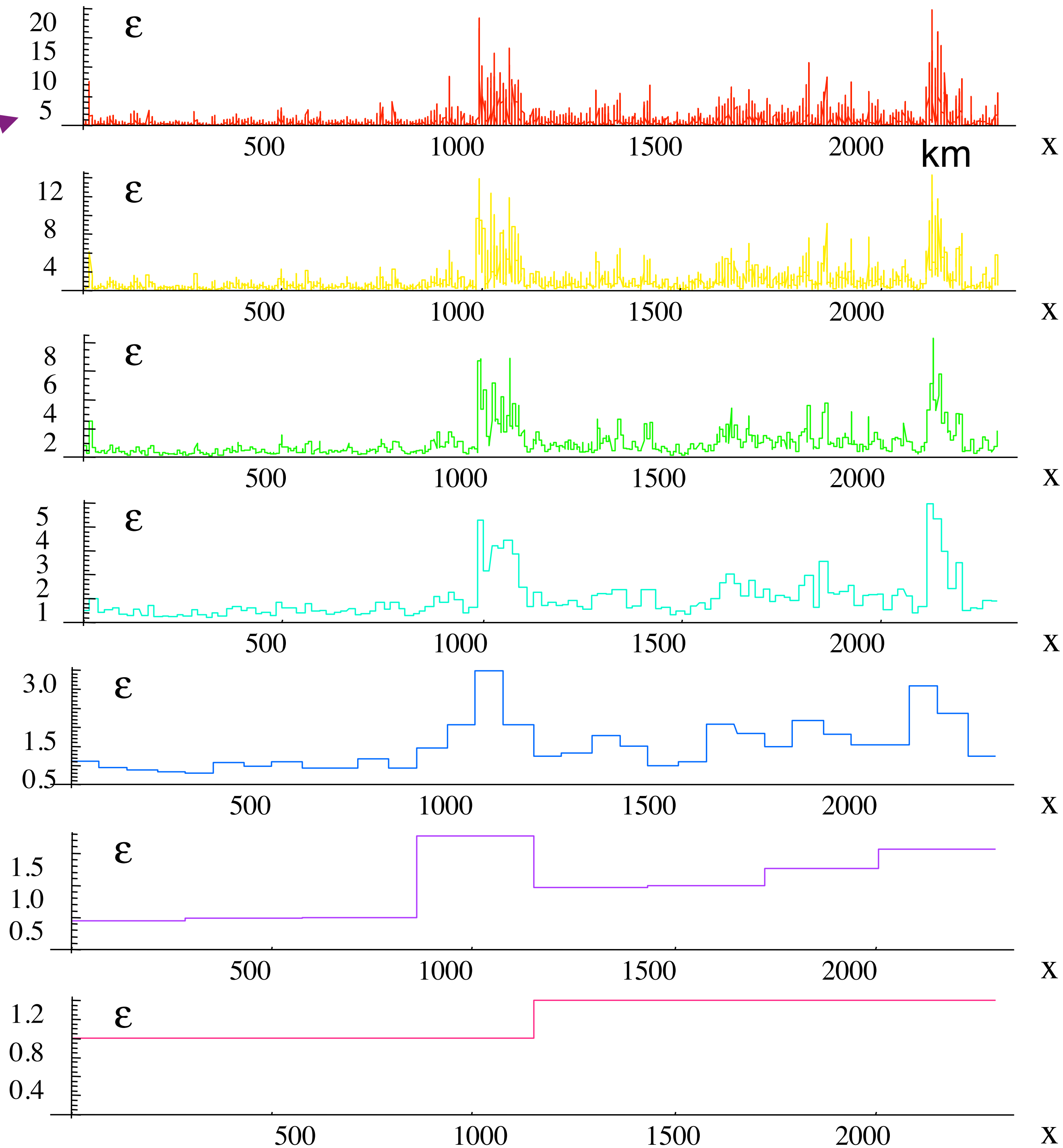
Cascades and Multifractals



Aircraft temperature transect (12km altitude)
Temperature (oC)
- 49
- 50
- 51
- 52
- 53
500
1000
1500
2000
x (km)
Turbulent flux
(gradient of the above)
1 σ

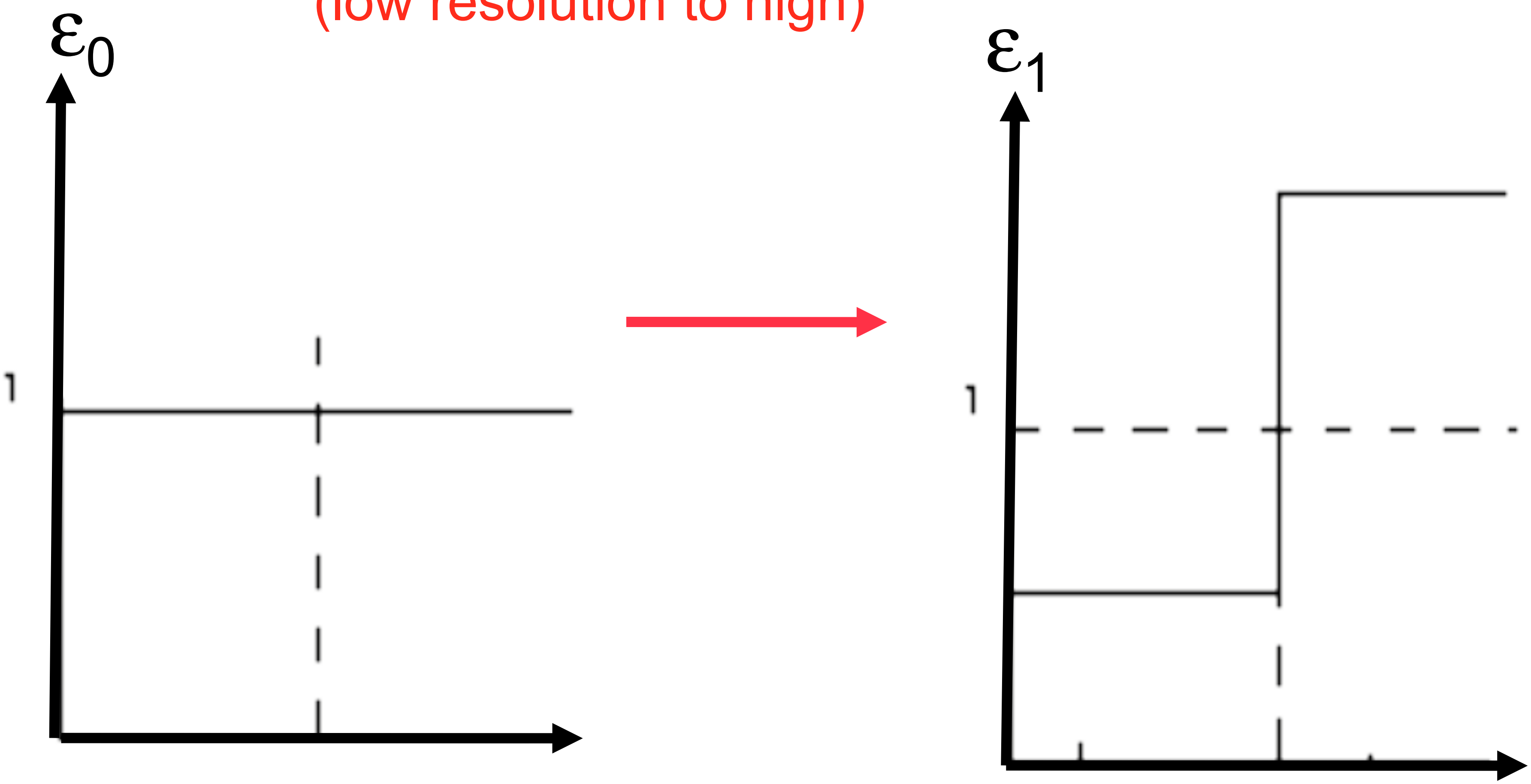
Temperature turbulent flux ε at 280m resolution

High to low Resolution: degrading by factors of 4



Cascades and Multifractals

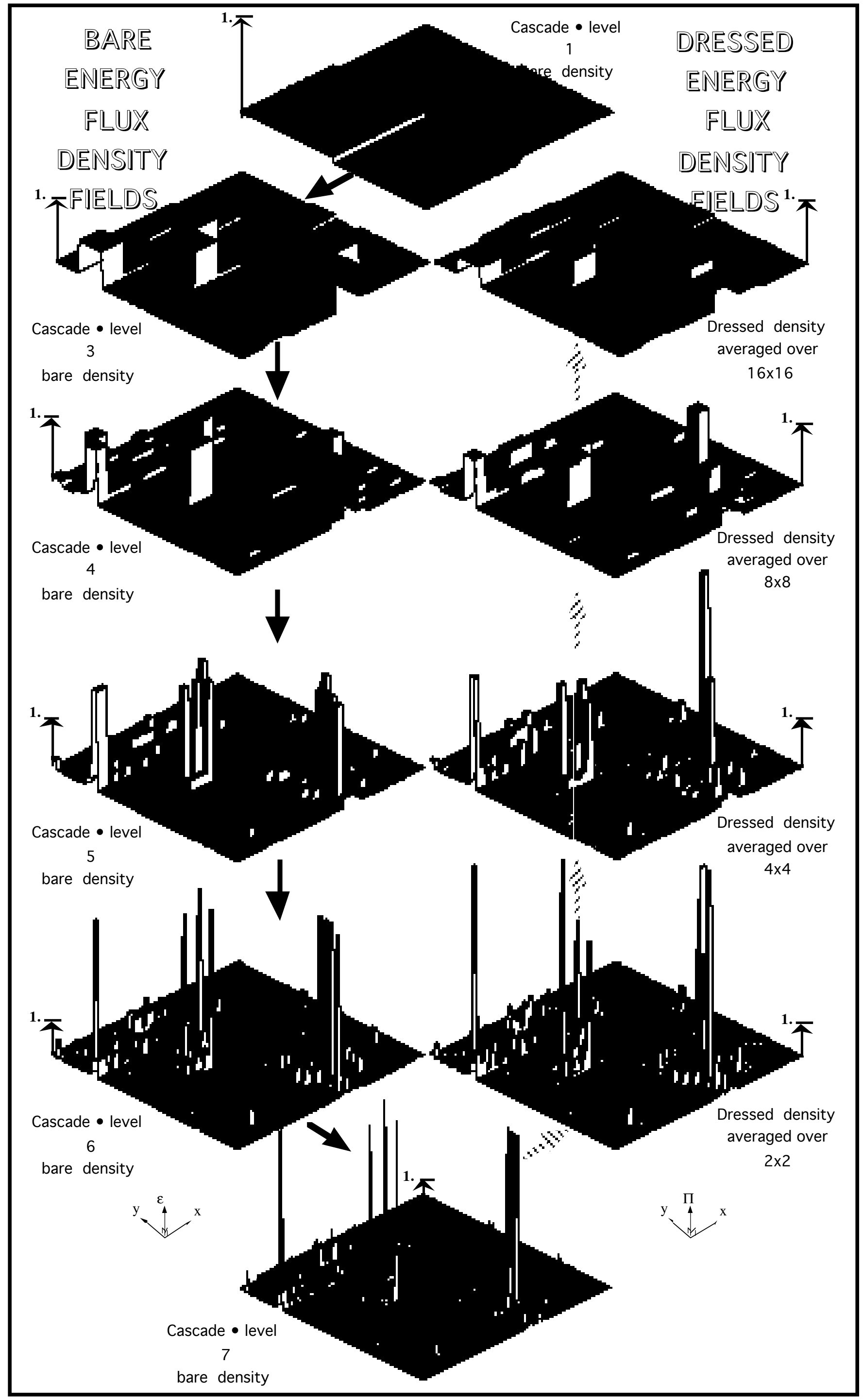
Simulations: adding small scale details
(low resolution to high)

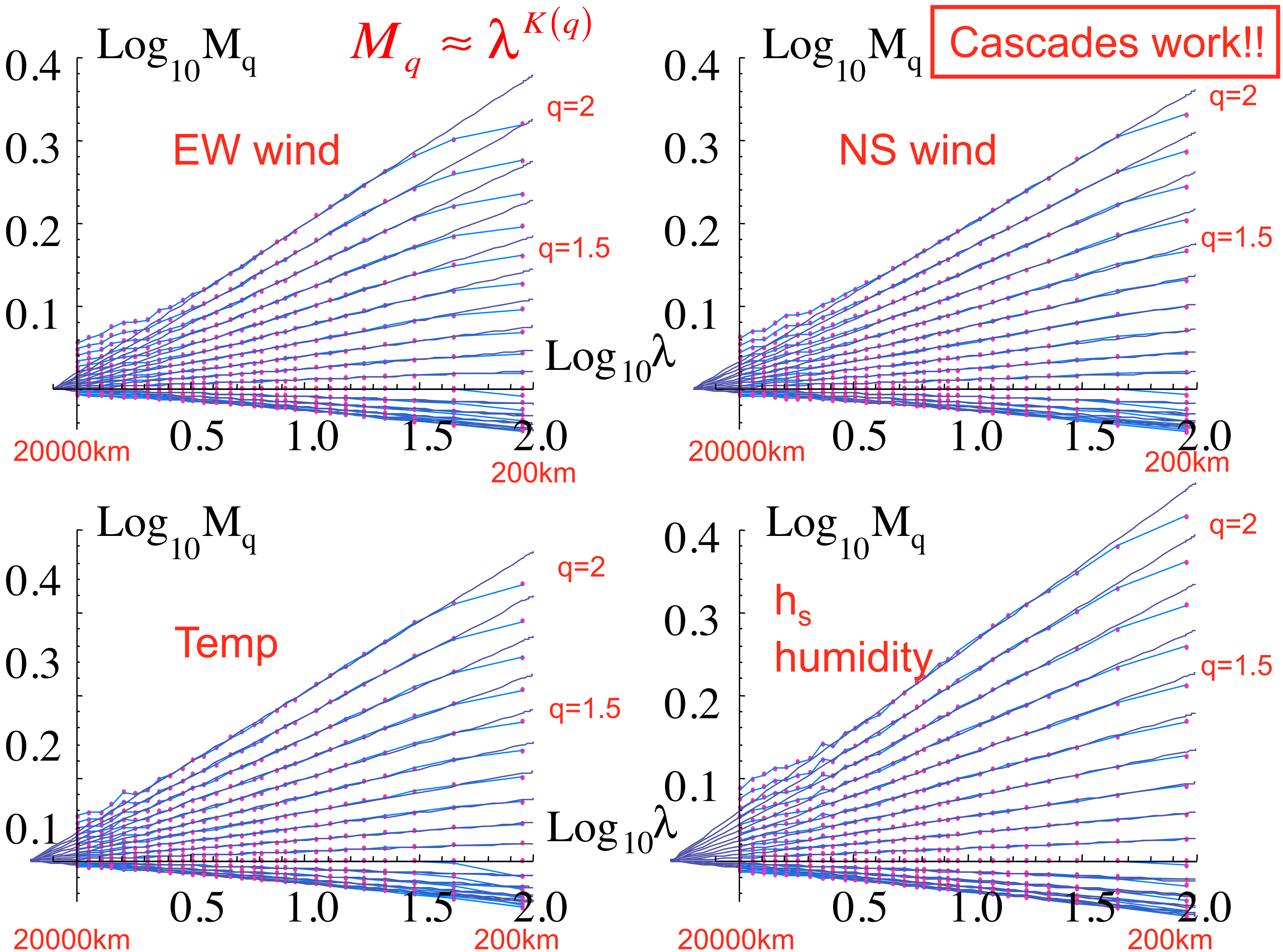


("α model")

Cascades

Multifractal

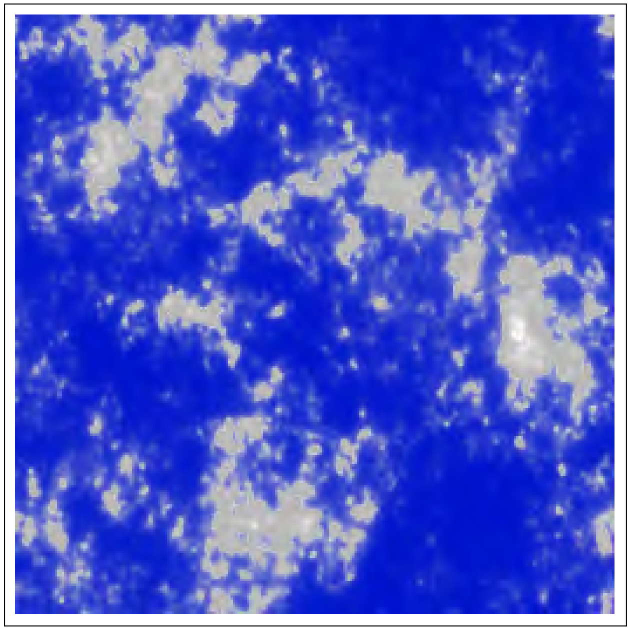


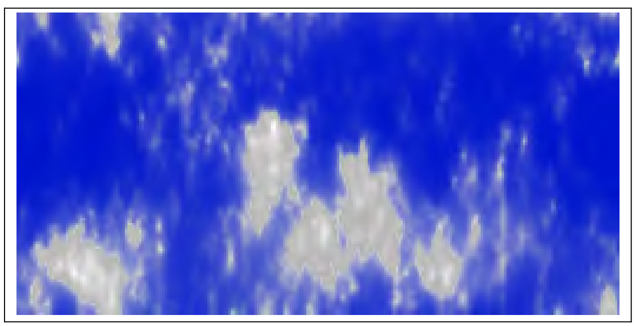


$M_q \approx \lambda^{K(q)}$
Cascades work!!
$\mathrm{Log}_{10}M_q$
EW wind
q=2
q=1.5
0.4
0.3
0.2
0.1
$\mathrm{Log}_{10}\lambda$
0.5
1.0
1.5
2.0
20000km
200km
NS wind
Temp
h_s
humidity

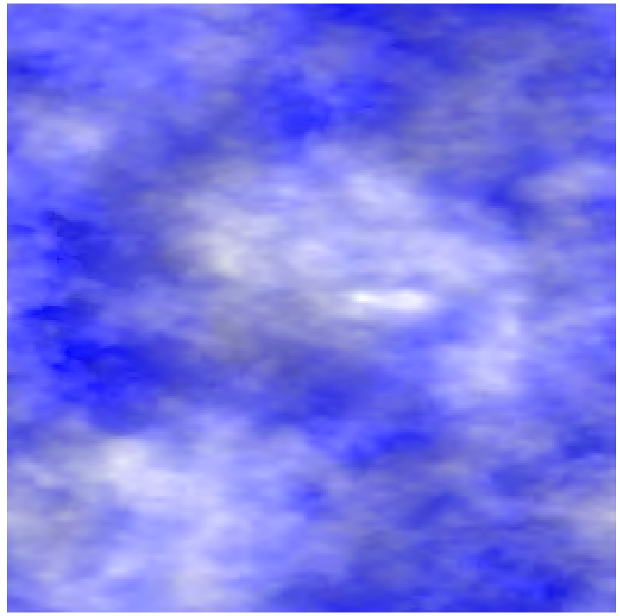
Cascade modeling: clouds and radiative transfer

Cloud liquid water (top)



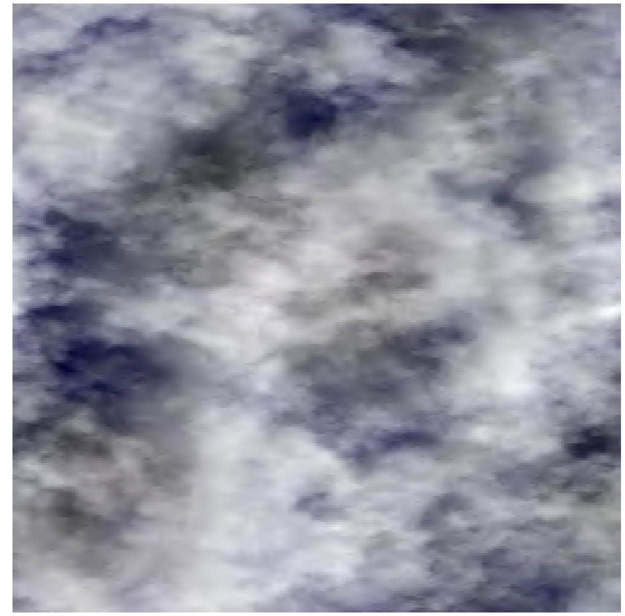


Cloud liquid water (side)

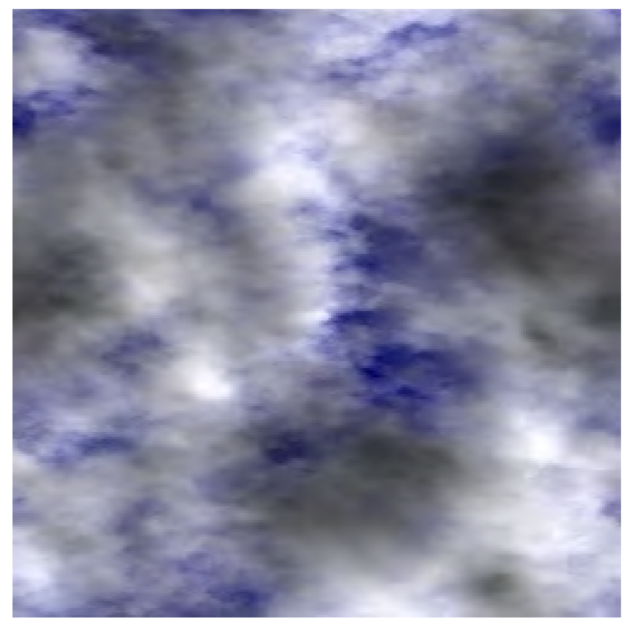


Cloud top, infra red

Cloud top visible



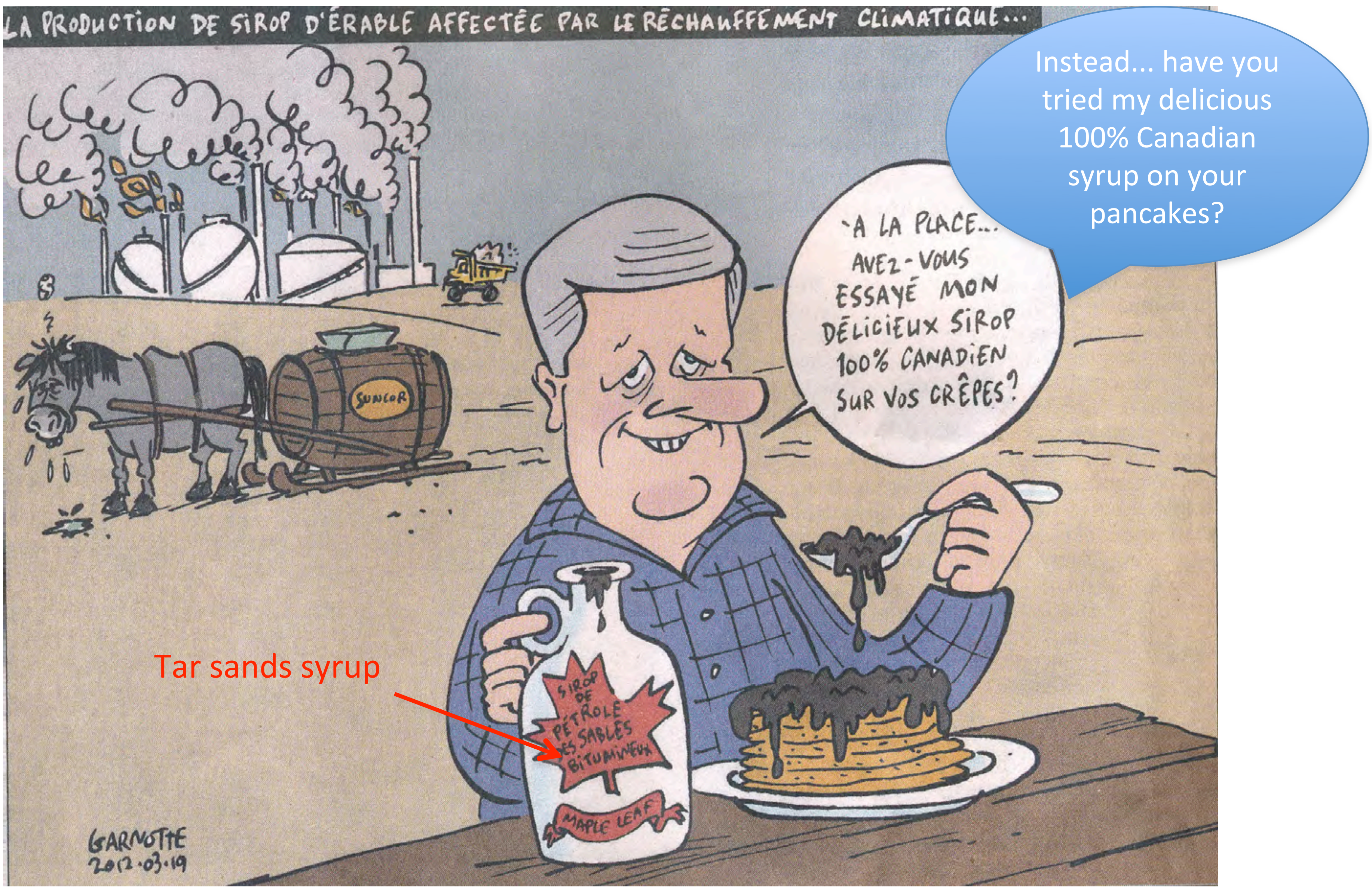
Cloud bottom visible



Cascade Simulations

The Climate

The production of maple syrup is affected by global warming...



What is the climate?

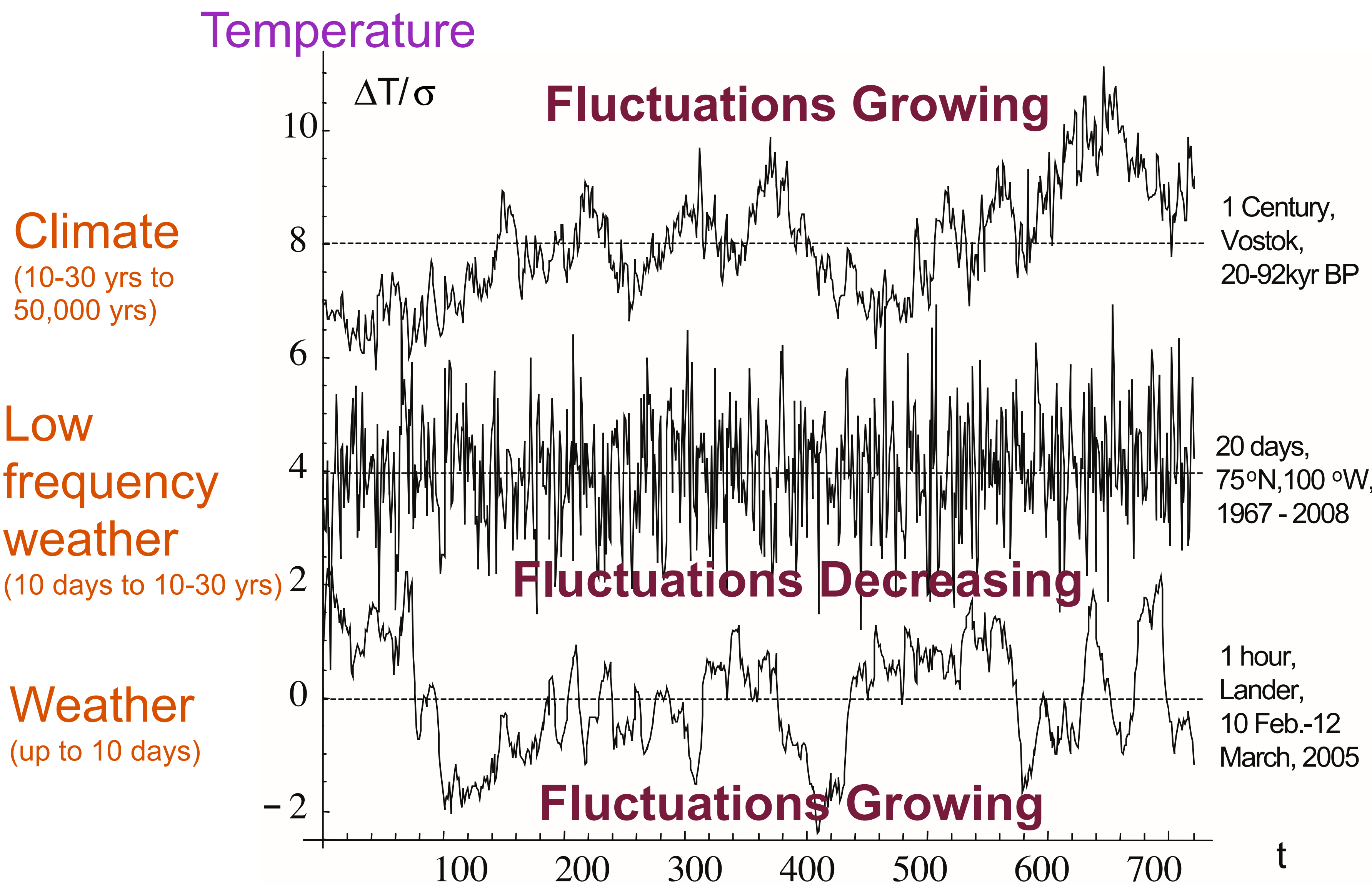
"Climate is what you expect, weather is what you get."

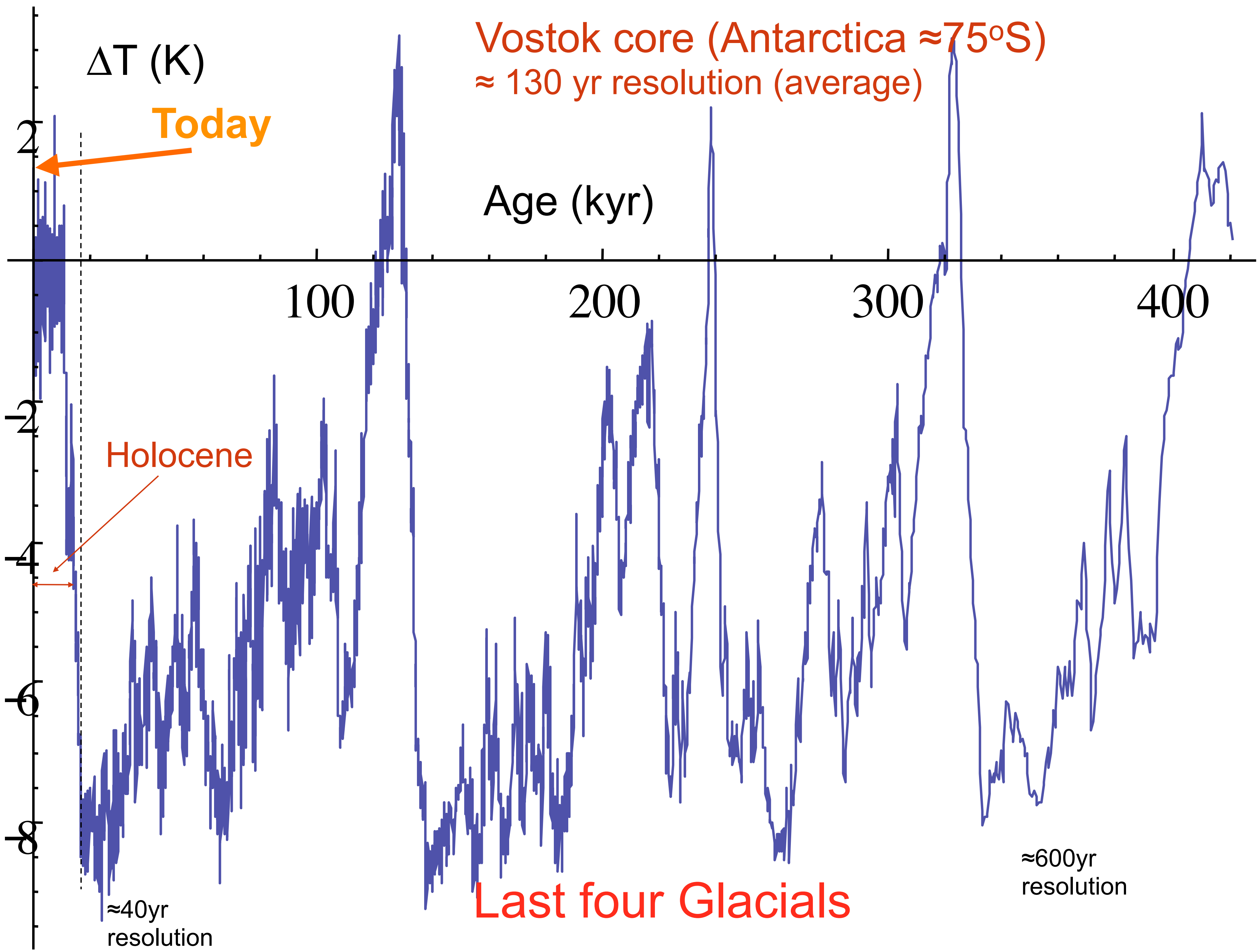
-Farmers Almanac

"Climate is conventionally defined as the long-term statistics of the weather…".

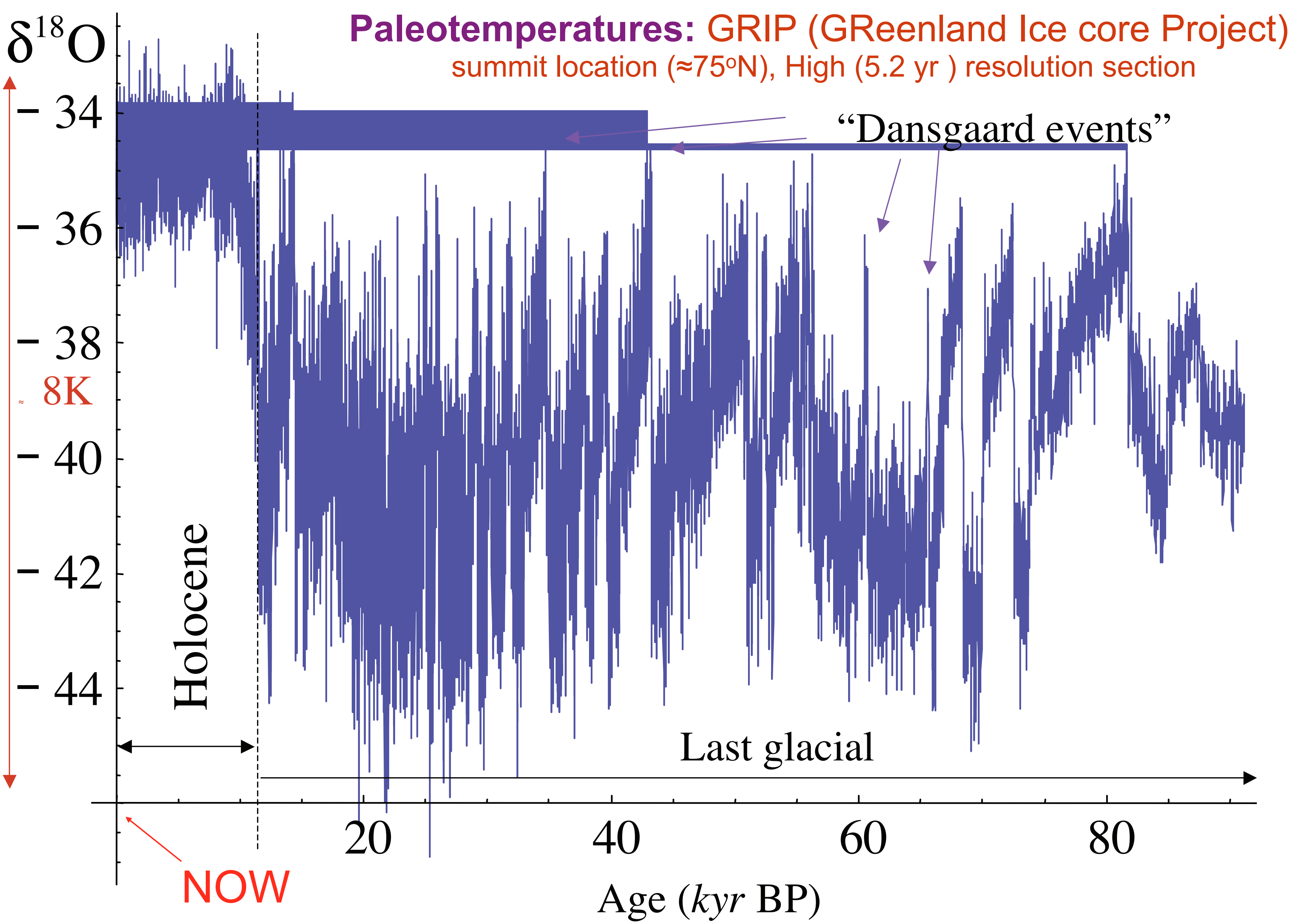
-Committee on Radiative Forcing Effects on Climate, 2005 US National Academy of Science

Three regimes: *three* types of variability: *not* two!



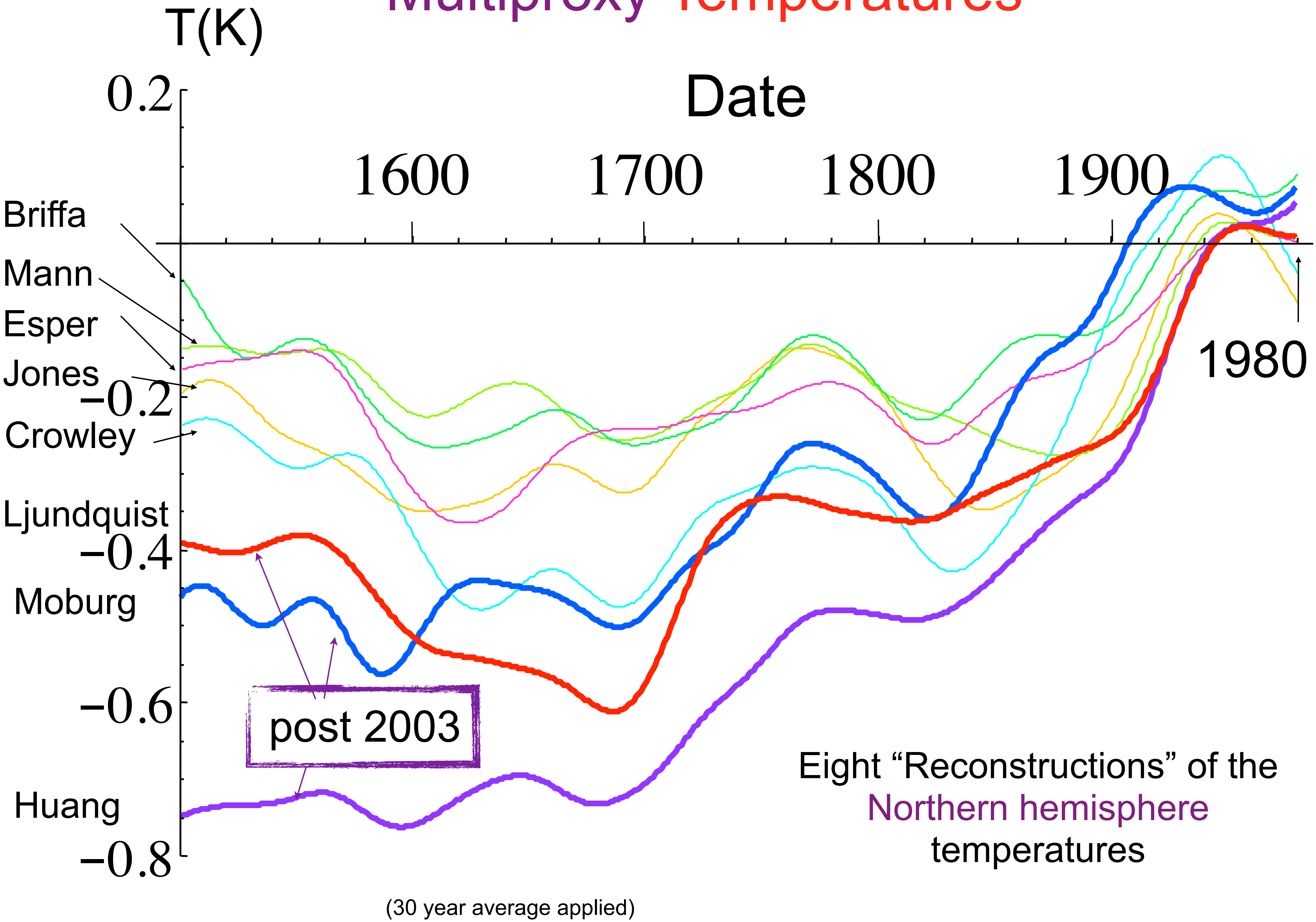


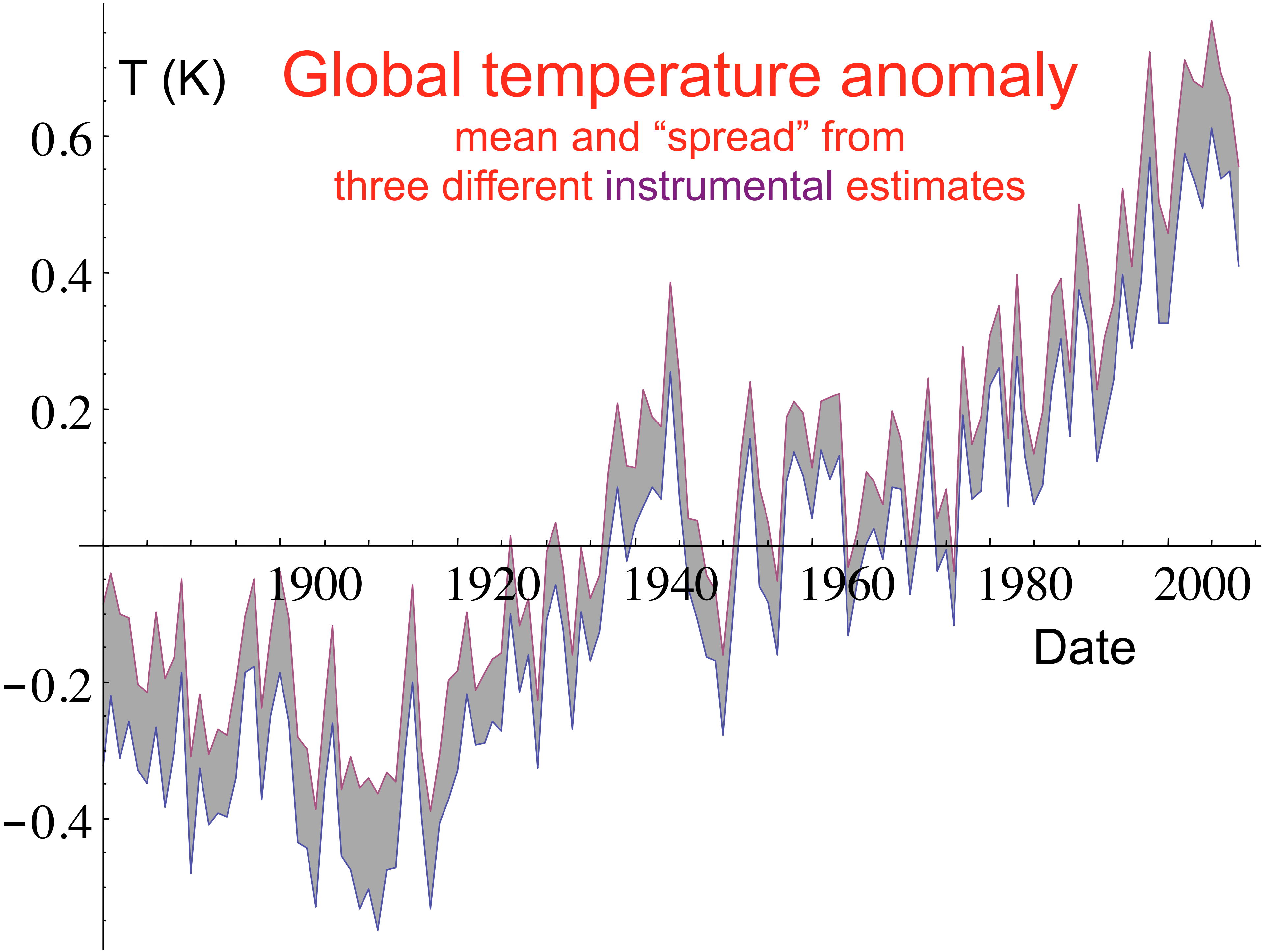
Vostok core (Antarctica ≈75°S)
≈ 130 yr resolution (average)
ΔT (K)
Today
Age (kyr)
2
100
200
300
400
-2
-4
-6
-8
Holocene
≈40yr
resolution
Last four Glacials
≈600yr
resolution



Paleotemperatures: GRIP (GReenland Ice core Project),
summit location (≈75°N), High (5.2 yr) resolution section
δ18O
34
36
38
8K
40
42
44
Holocene
"Dansgaard events"
Last glacial
20
40
60
80
NOW
Age (kyr BP)

Multiproxy Temperatures



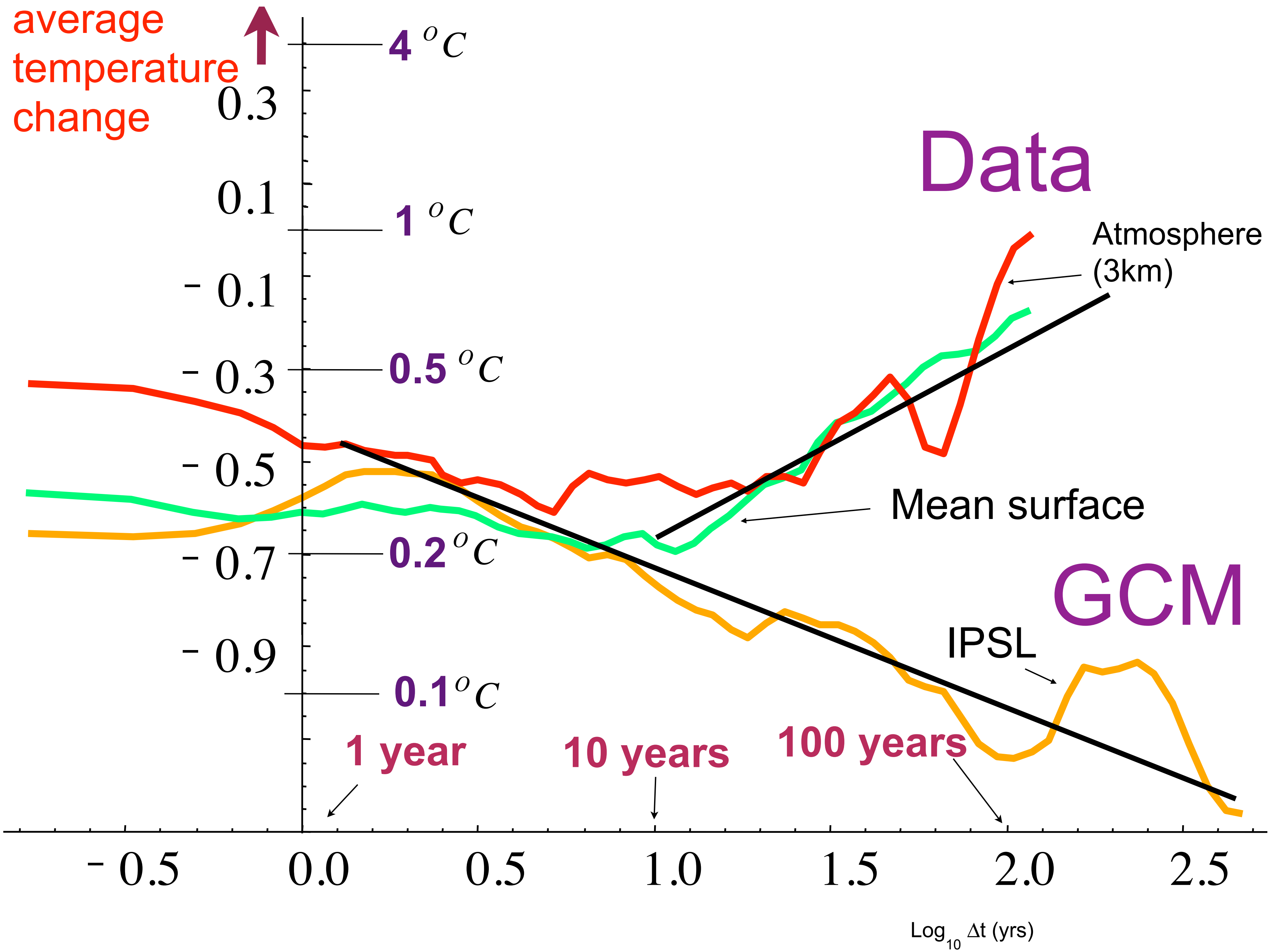


Global temperature anomaly
mean and “spread” from
three different instrumental estimates
T (K)
0.6
0.4
0.2
−0.2
−0.4
1900
1920
1940
1960
1980
2000
Date

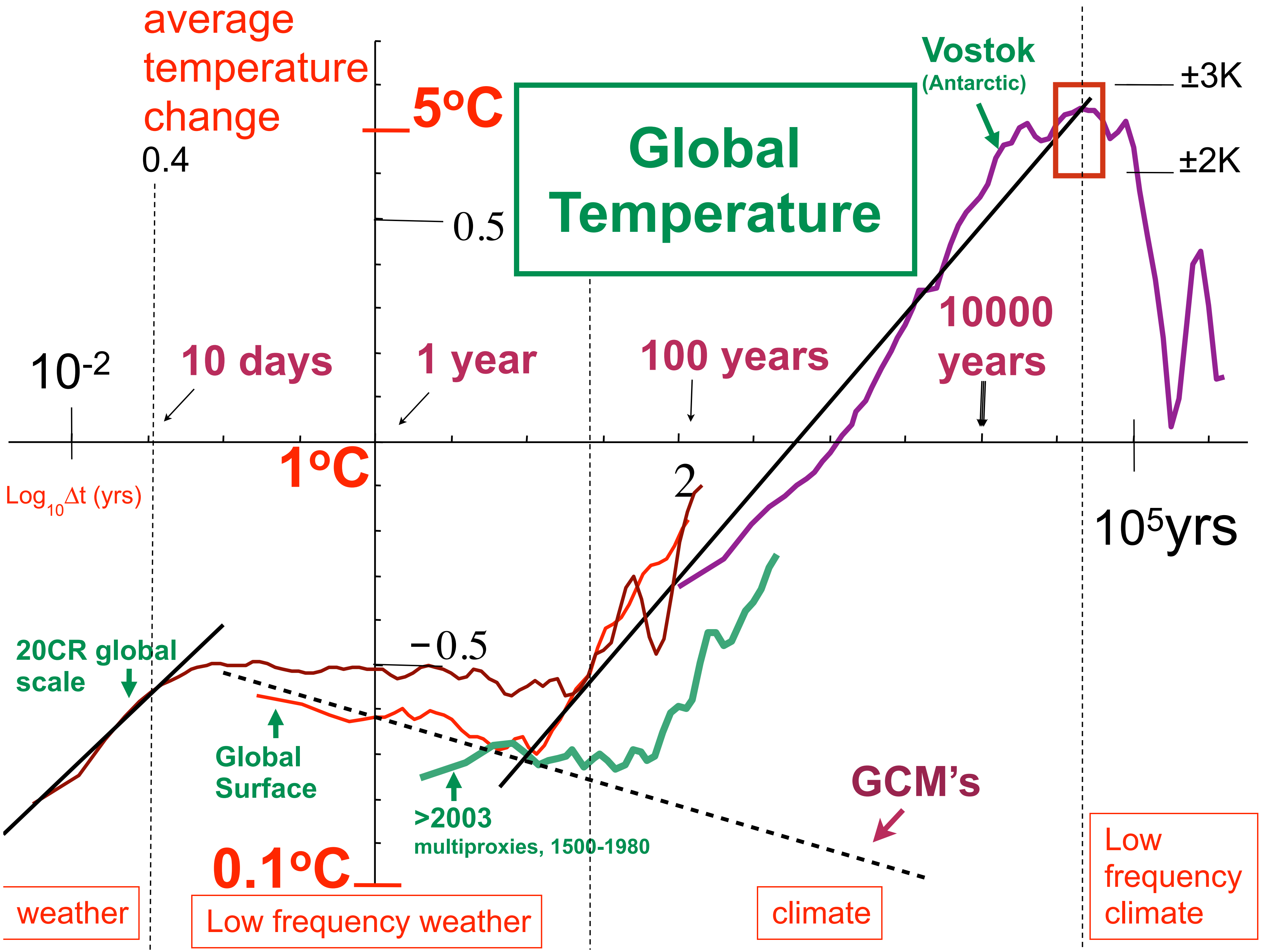
Do Global Climate models predict...

The climate?

...or low frequency weather?



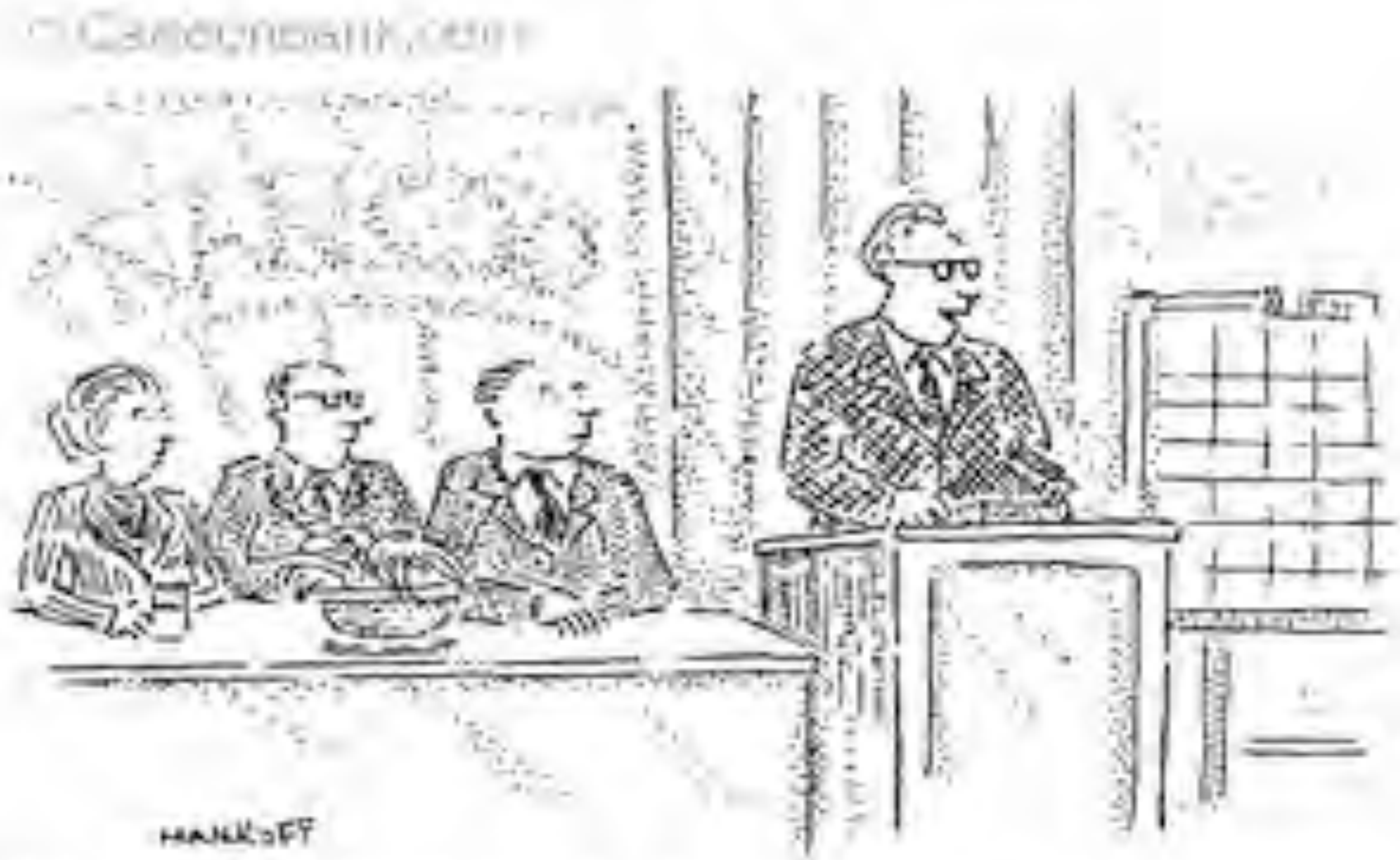
average temperature change
4 ^oC
1 ^oC
0.5 ^oC
0.2 ^oC
0.1 ^oC
0.3
0.1
- 0.1
- 0.3
- 0.5
- 0.7
- 0.9
Data
Atmosphere (3km)
Mean surface
GCM
IPSL
1 year
10 years
100 years
- 0.5
0.0
0.5
1.0
1.5
2.0
2.5
Log_{10} Δt (yrs)



average temperature change
Global Temperature
Vostok
(Antarctic)
±3K
±2K
5°C
0.4
0.5
10000 years
10^{-2}
10 days
1 year
100 years
1°C
Log$_{10}$Δt (yrs)
2
10^{5}yrs
20CR global scale
−0.5
Global Surface
GCM's
>2003
multiproxies, 1500-1980
0.1°C
weather
Low frequency weather
climate
Low frequency climate

Implications for global warming

- By comparing model and natural variability, we found that GCM's seem to be missing a long-time mechanism of internal variability such as land-ice.
- Anthropogenic contributions to 20th warming and 21st C warming scenarios may thus be either over - or under estimated.



"And so, while the end-of-the-world scenario will be rife with unimaginable horrors, we believe that the pre-end period will be filled with unprecedented opportunities for profit."

Conclusions

1. Low level laws: complex (Fluid mechanics)
High level laws simplicity (emergent turbulent laws)

2. Emergent Atmospheric laws are power laws
Fluctuations are scaling, their exponents are scale invariant

3. There are three different regimes:
Weather to ≈ 10 days,
Low frequency weather to ≈ 10-30 yrs,
Climate to ≈ 50- 100kyrs.

4. Without special forcing GCM's produce low frequency weather not climate type variability