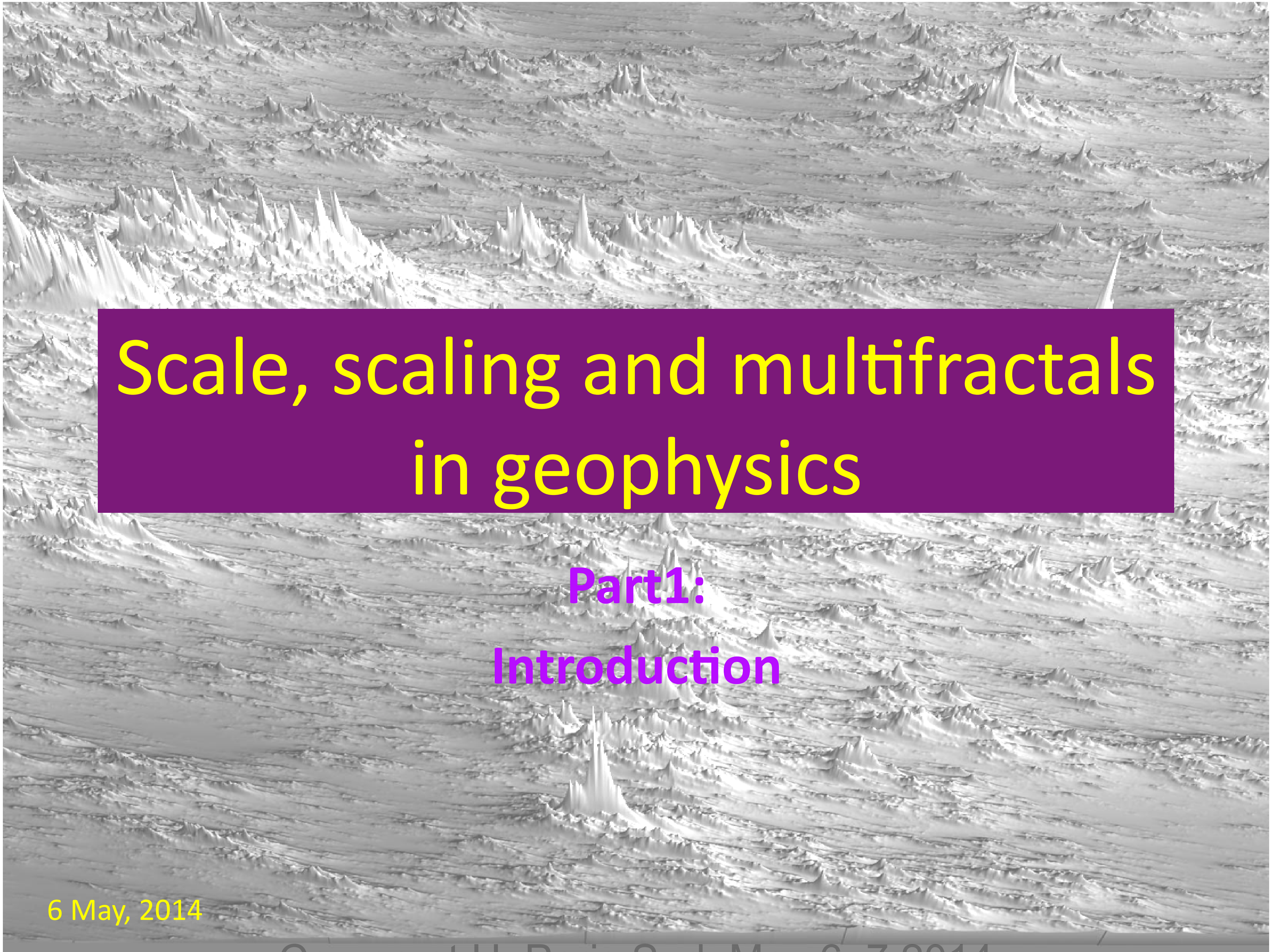


Scale, scaling and multifractals in geophysics

Part1:

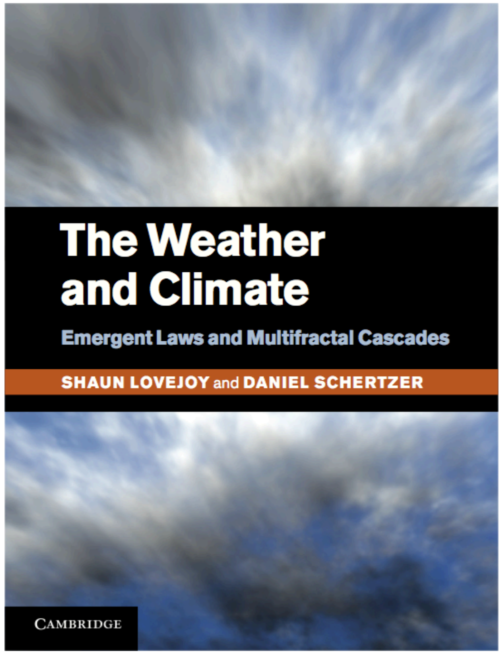
Introduction

6 May, 2014

European Geosciences Union short course, April 28, 2014:

Scale, scaling and multifractals in complex geosystems parts 1, 2 (2x45 minutes, slides available)

Graduate course at McGill: Multifractals and turbulence



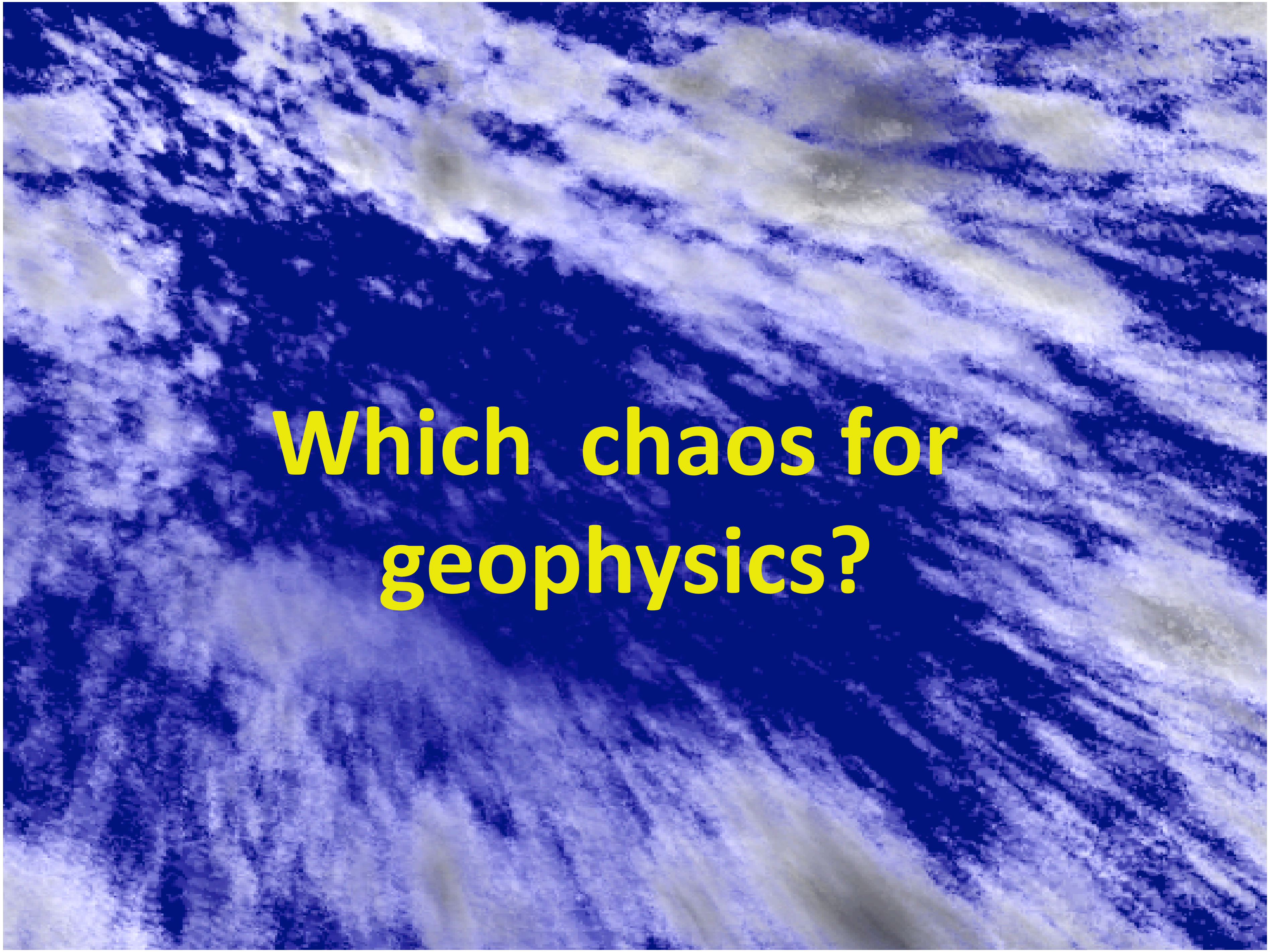
12x2 hours, slides available:

- Lecture 1, Jan. 15, 2014, Introduction: Our multifractal world part 1
- Lecture 2, Jan. 22, 2014, Introduction: Our multifractal world part 2
- Lecture 3, Jan. 29, 2014, Turbulence and spectra
- Lecture 4, Feb. 5, 2014, Spectra, turbulence, fractal sets
- Lecture 5, Feb. 12, 2014, Fractal sets, multifractal cascades
- Lecture 6, Feb. 14, 2014, Multifractals: moments
- Lecture 7, Feb. 19, 2014, Data analysis
- Lecture 8, March 12, 2014, Multifractals: codimensions
- Lecture 9, March 19, 2014, Multifractals: extremes
- Lecture 10: March 26, 2014: Multifractal simulations
- Lecture 11: April 4, 2014: Generalized Scale Invariance: linear
- Lecture 12: April 9, 2014: Generalized Scale Invariance: nonlinear, space-time

Course synopsis

May 6,7

http://www.physics.mcgill.ca/~gang/PHYS616/Multicourses.home.htm



Course at U. Paris Sud, May 6, 7 2014

Deterministic Chaos?

Low Dimensional Nonlinear Dynamics I

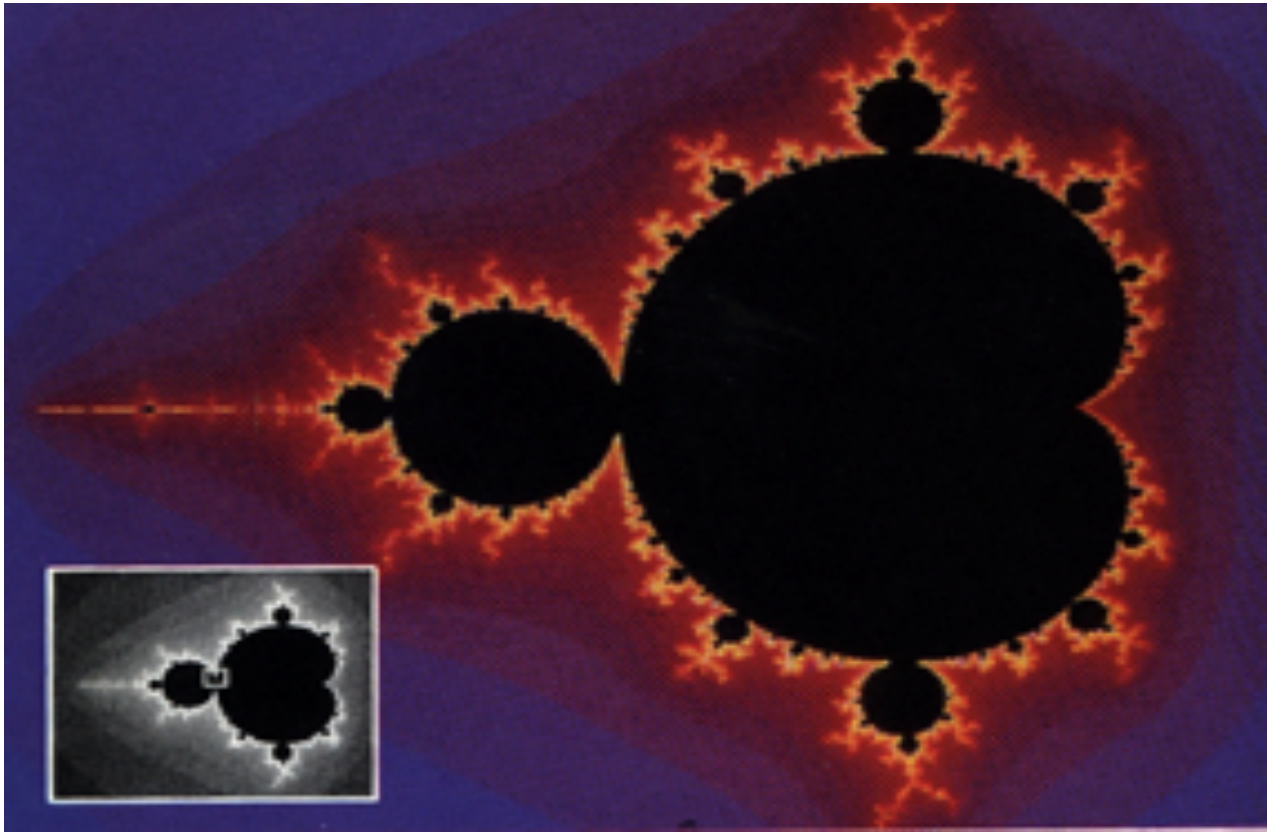
Nonlinear Mappings

Discrete time (=n) evolution of a few variables (x):

Z, C are complex numbers

$$Z_{n+1} = Z_n^2 + C$$

The Mandelbrot set



Low Dimensional Nonlinear Dynamics II

Flows

Continuous time (=t) evolution of a few degrees of freedom (X):

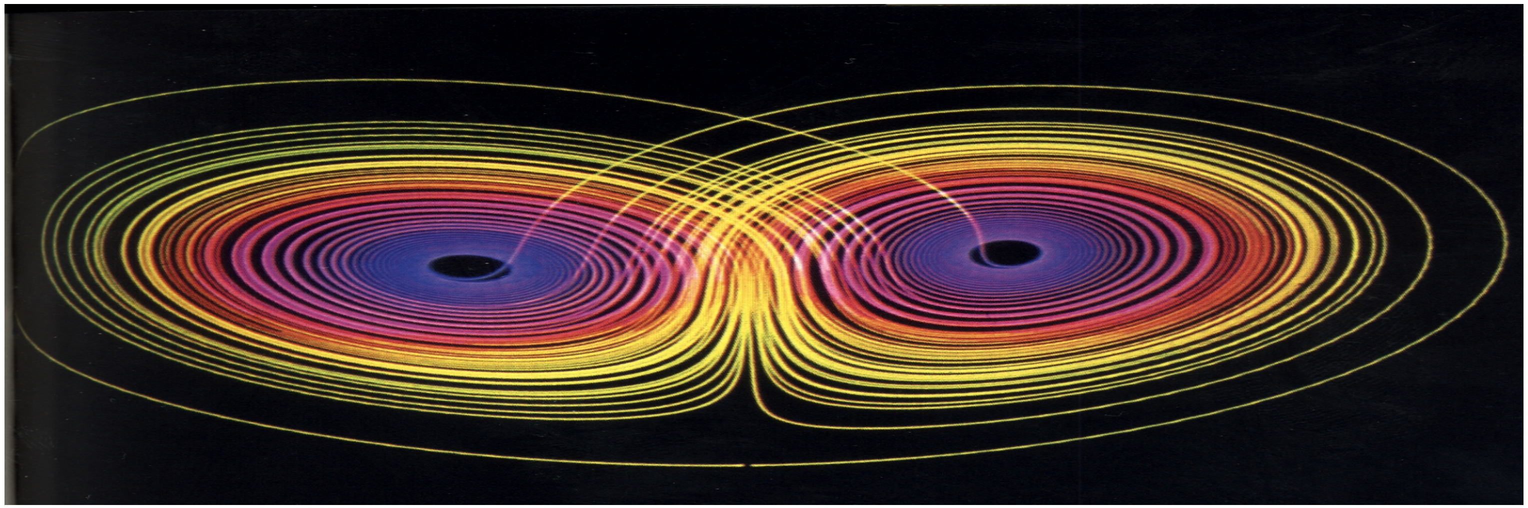
$$\frac{d\underline{X}}{dt} = \underline{F}(\underline{X})$$

Lorenz equations:

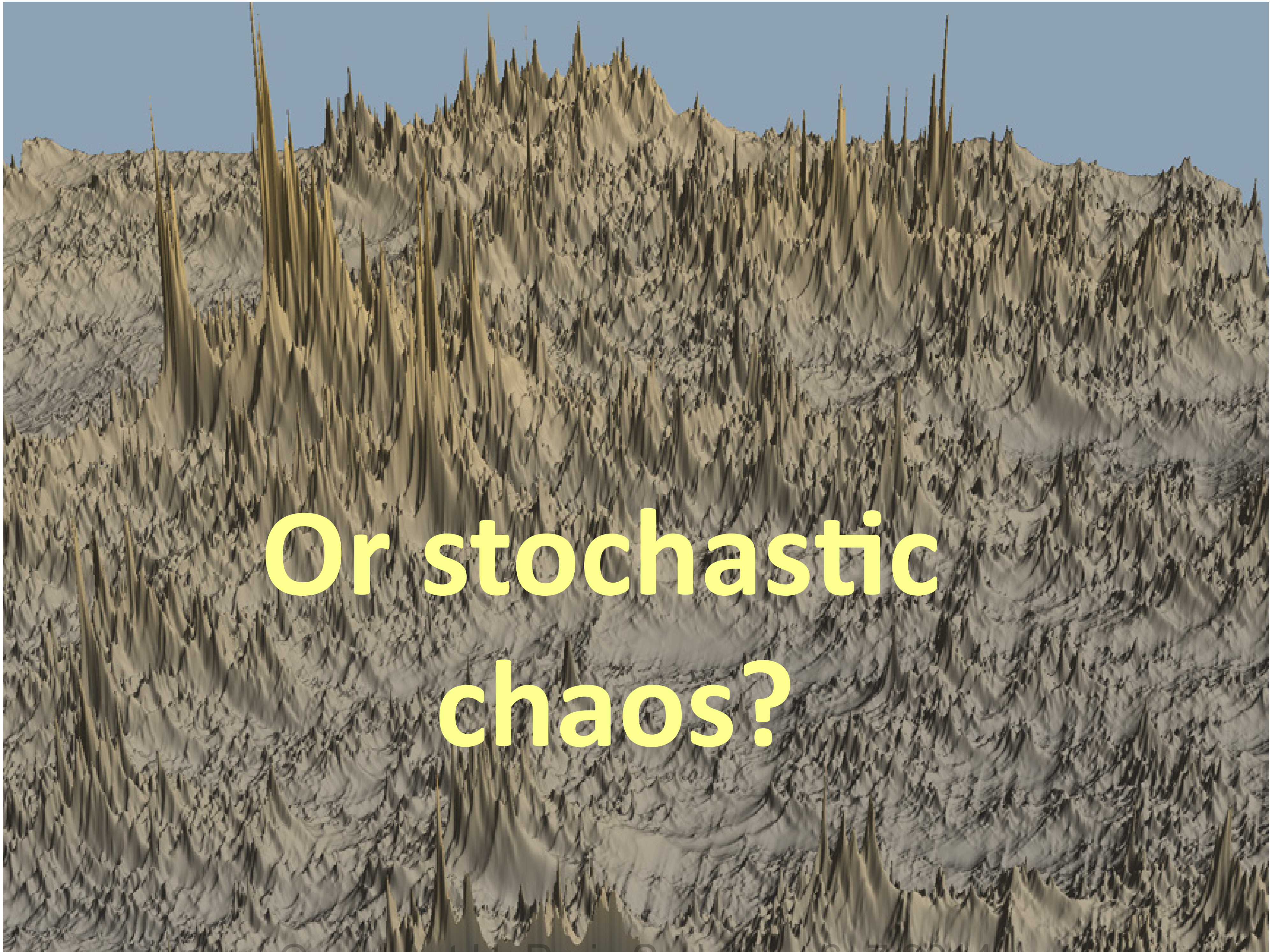
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

where r, b, σ are positive constants.

Few degrees of freedom… few applications



Course at U. Paris Sud, May 6, 7 2014


Or stochastic
chaos?

High Dimensional Nonlinear Dynamics

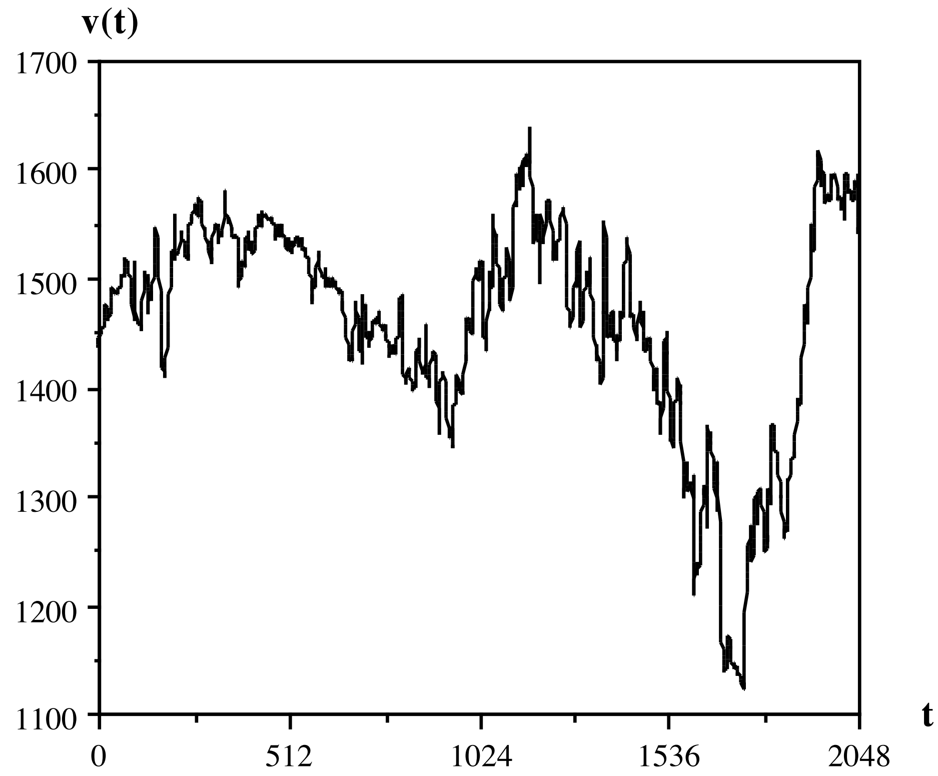
Nonlinear PDE 's

Fields/spatial structures evolving in time
Example: Navier-Stokes Equations:

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla)\,\underline{v} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \underline{v} + \underline{f}$$

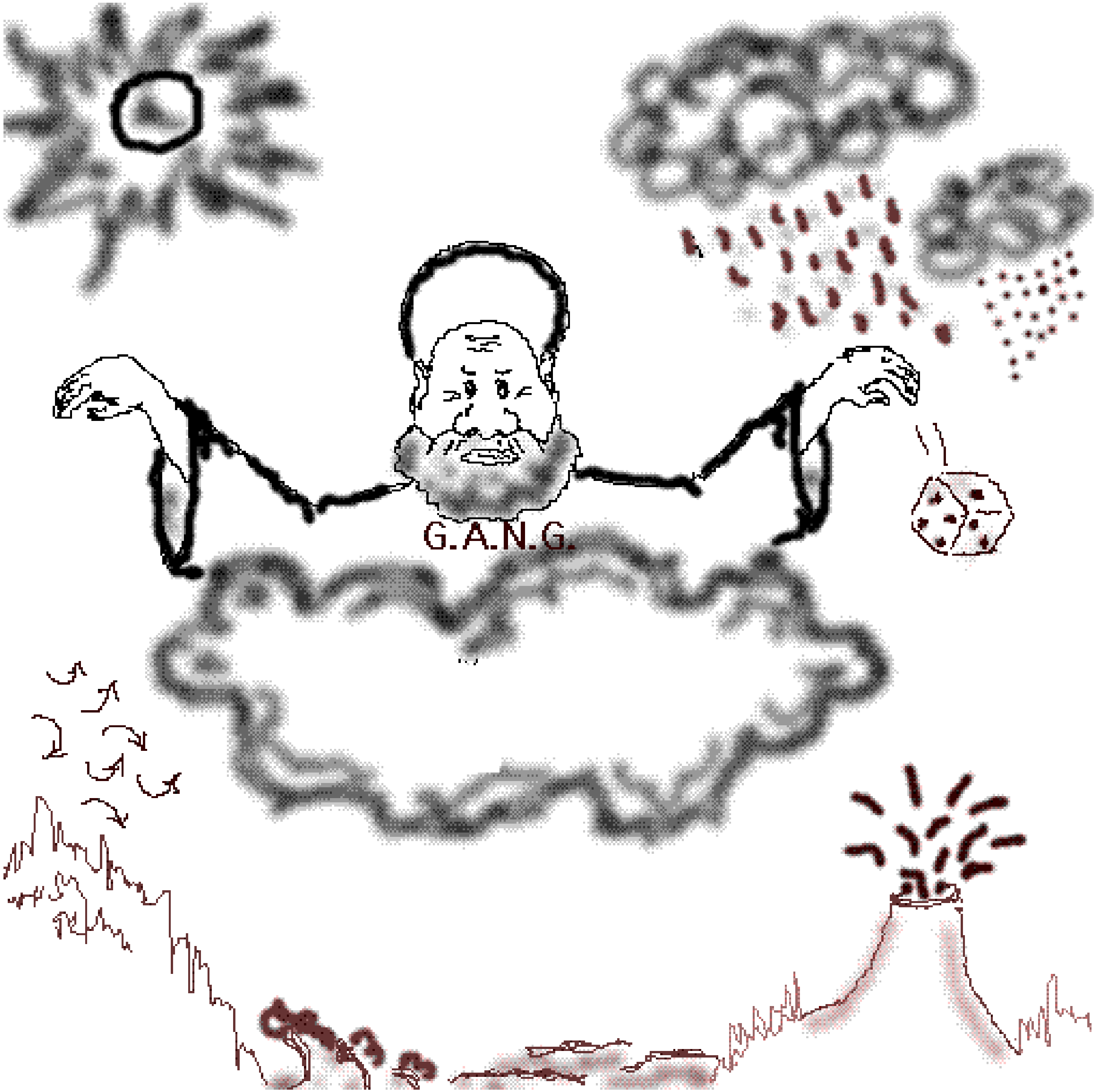
$$\nabla \cdot \underline{v} = 0$$

where $\underline{v}$= velocity, t = time, p = pressure, ρ = density, ν = viscosity, $\underline{f}$= body forces (e.g. stirring, gravity).



1 second of wind data

How does He play Dice?



The Emergence of physical laws

Quantum mechanics → Classical Mechanics

Large scales (usually)

stochastic → deterministic

Statistical mechanics → Continuum mechanics, thermodynamics

Large numbers of particles

stochastic → deterministic

Low level (fundamental) — high level

The emergence of atmospheric dynamics

(Classical)

Continuum mechanics

deterministic

Low level
(fundamental)

Large Re

Laws of turbulence

Classical:

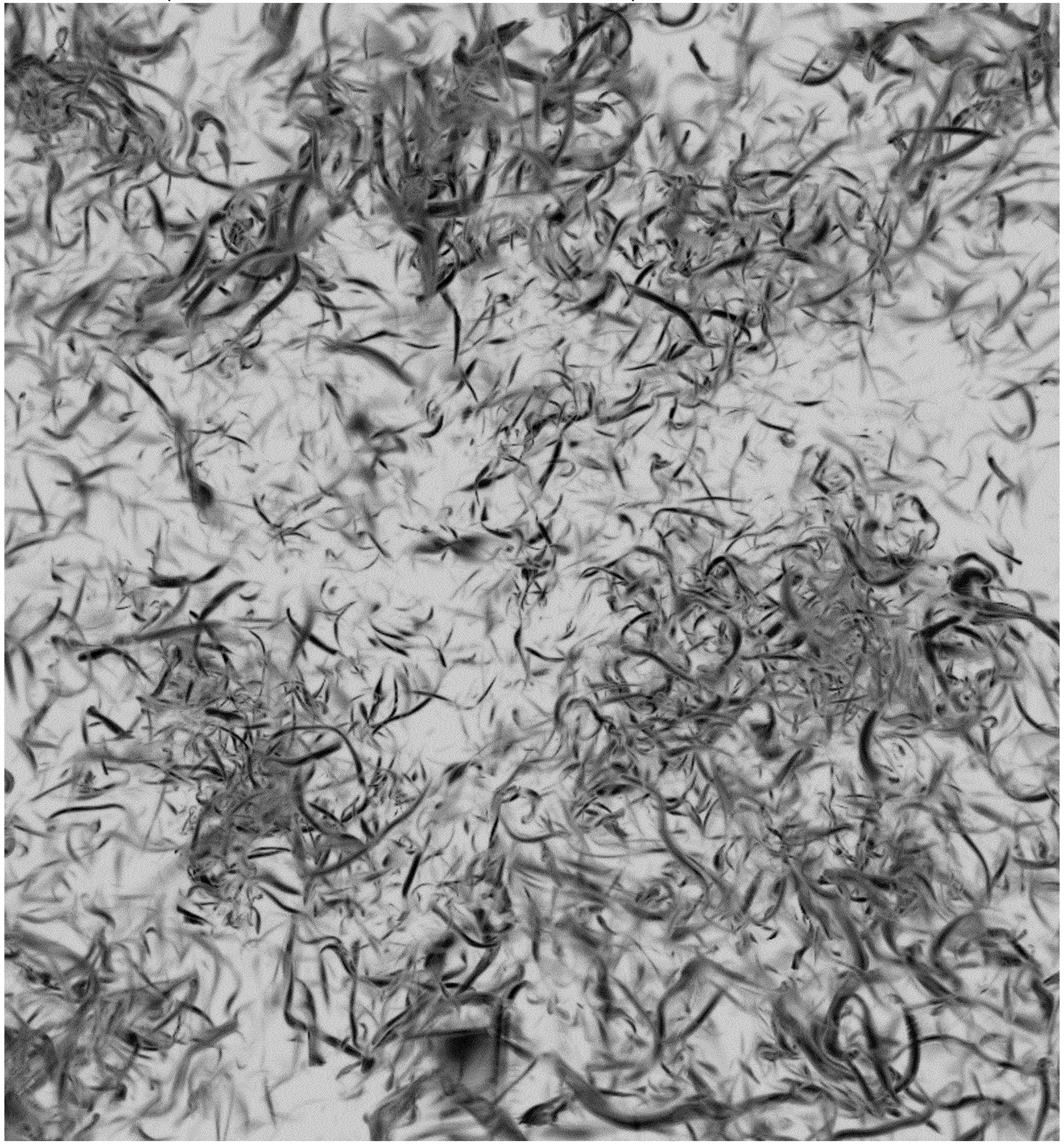
Richardson, Kolmogorov,
Corrsin, Obukhov, Bolgiano

High level

stochastic

Vortices in strongly turbulent fluid

(M. Wiczek, numerical simulation, 2010)



$$\Delta v(\underline{\Delta r}) = \varphi |\underline{\Delta r}|^{H}$$

e.g. Kolmogorov $\varphi = \varepsilon^{1/3}$, $H=1/3$

a) $|\underline{\Delta r}| <\approx 100m$ b) isotropic

c) $\varphi \approx$ constant, quasi Gaussian

Emergent laws and Complexity

The relative simplicity of the high level laws is
due to a
reduction of the complexity
of the system

If all existing emergent laws are used to describe a system, the remaining complexity is *irreducible*

Scale Invariance of the dynamics

Multiscaling of the Navier-Stokes equations

Zoom factor λ $\quad \vec{x} \to \dfrac{\vec{x}}{\lambda}$

Rescaling of the velocity

H is an arbitrary scaling exponent $\quad \vec{v} \to \dfrac{\vec{v}}{\lambda^H}$

$t \to \dfrac{t}{\lambda^{1-H}}$

Rescaling of time, viscosity, forcing follow from dimensional considerations

$\nu \to \dfrac{\nu}{\lambda^{1+H}}$

$\vec{f} \to \dfrac{\vec{f}}{\lambda^{2H-1}}$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{\nabla p}{\rho} + \nu\nabla^2\vec{v} + \vec{f}$$

$$\nabla\cdot\vec{v} = 0$$

(constraint used to eliminate p) where $\vec{v}$ = velocity , t = time , p = pressure , ρ = density , ν = viscosity , $\vec{f}$ = body forces (stirring, gravity)

Kolmogorov's Law:

Considering $\varepsilon = -\dfrac{\partial v^2}{\partial t}$ energy flux to smaller scales to be invariant, we obtain H = 1 / 3, hence for mean shear

$$\Delta\vec{v} \approx \varepsilon^{1/3}\Delta x^{1/3}; \quad E(k) = k^{-5/3}$$

This already leads to singularities:

$$\frac{\partial \vec{v}}{\partial x} = \lim_{\Delta x \to 0}\frac{\Delta v}{\Delta x} \approx \Delta x^{-2/3} \to \infty$$

Emergence of Atmospheric laws

Fluctuations ≈ (turbulent flux) x (scale)^H

Differences, tendencies, wavelet coefficients → Fluctuations

Cascading Turbulent flux → (turbulent flux)

Anisotropic Space-time Scale function → (scale)

Fluctuation /conservation exponent → H

Fourier domain:

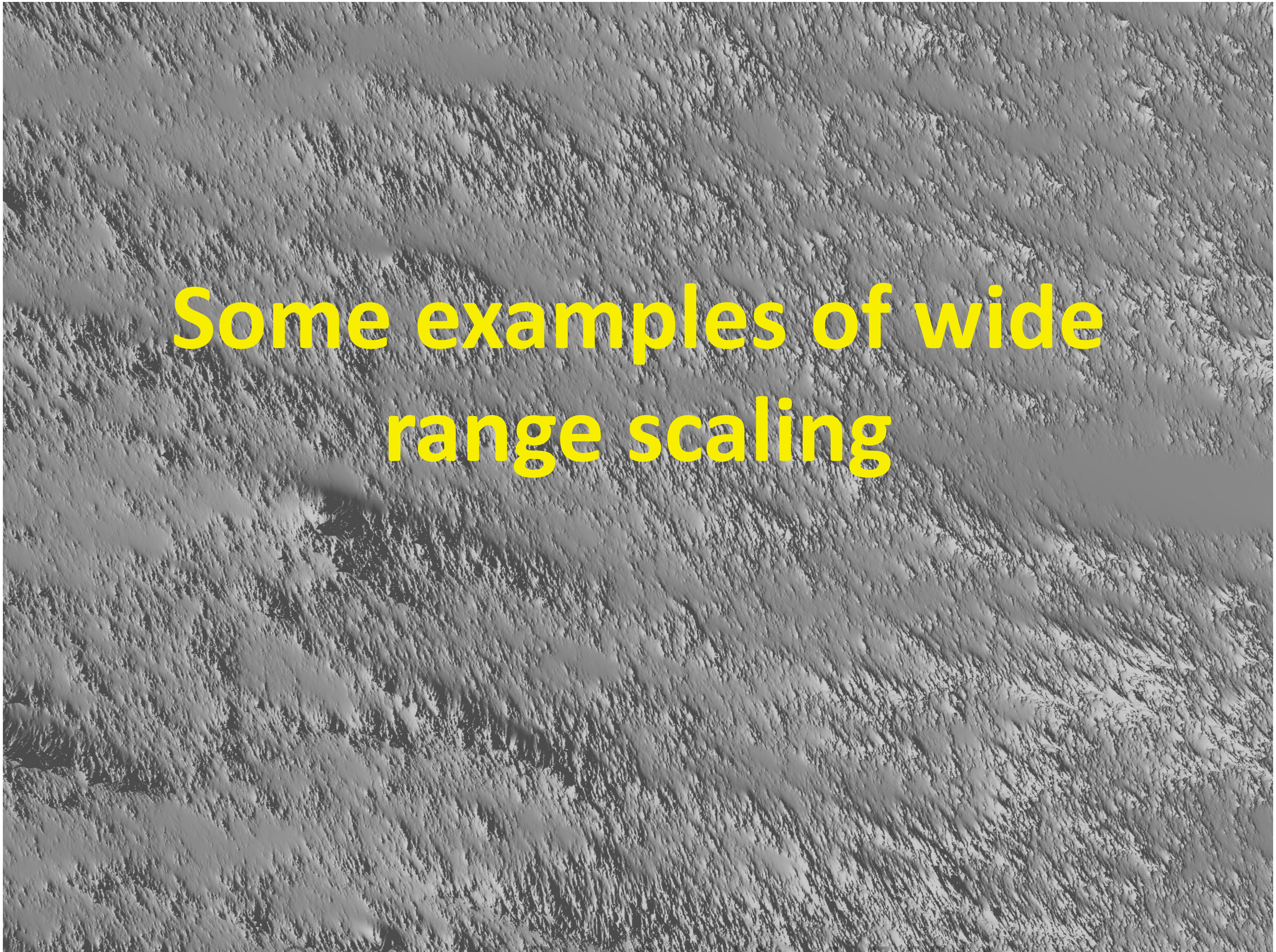
$$\left(\frac{Variance_{observables}}{wavenumber}\right) = \left(\frac{Variance_{flux}}{wavenumber}\right)(wavenumber)^{-2H}$$

$$= (wavenumber)^{-\beta}$$

Space: $E(k) \approx k^{-\beta}$

Time: $E(\omega) \approx \omega^{-\beta}$

Some examples of wide range scaling



Energy Spectra

Scaling geometric sets of points = fractals
Scaling fields=multifractals

$$E(k) \propto k^{-\beta}$$

k=2π/L= wavenumber, β=spectral exponent

Scale invariance

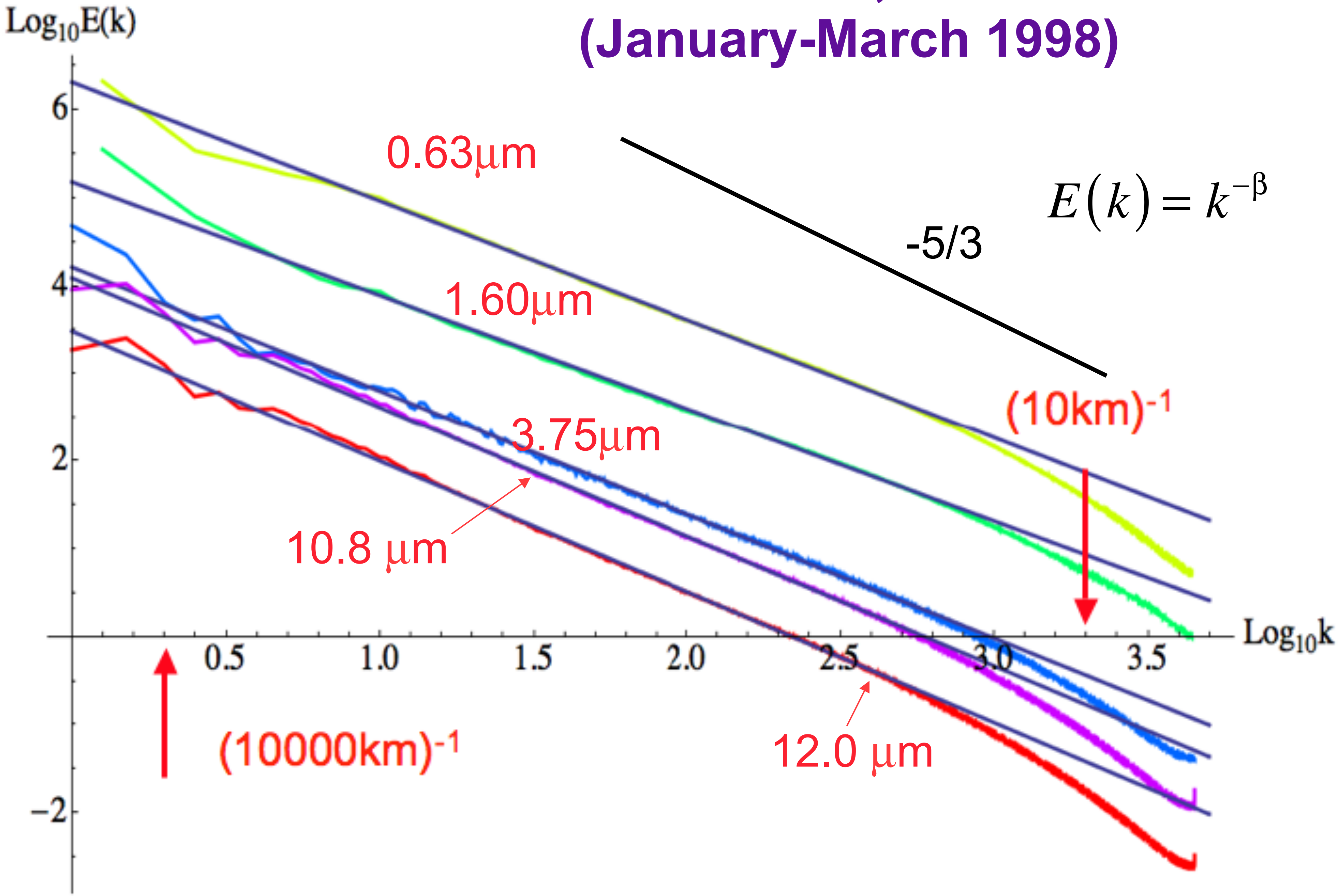
$$E(\lambda^{-1}k) = \lambda^{\beta} E(k)$$

β Invariant under zoom by factor λ in real space.

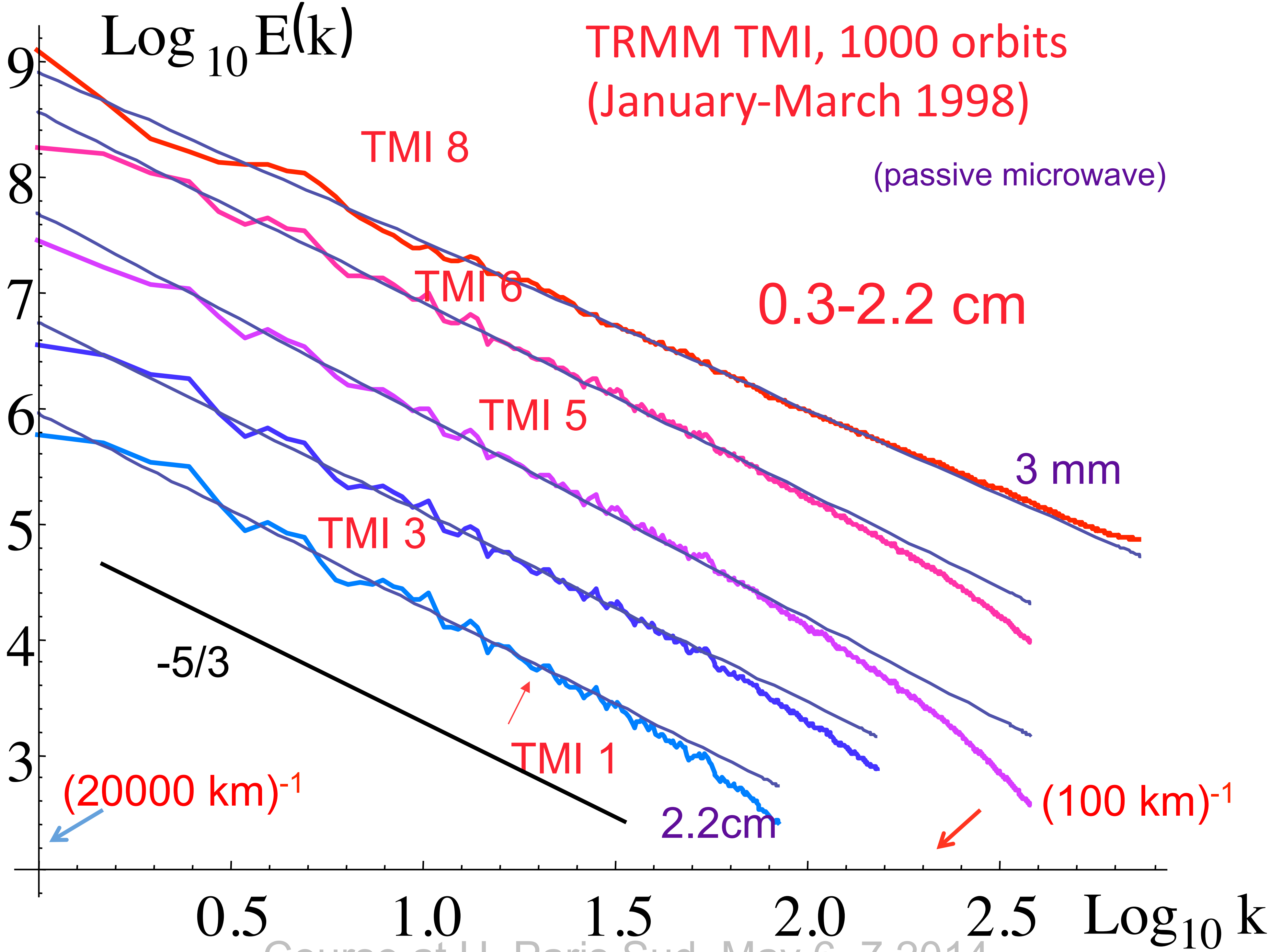
Examples in the spatial domain

The Atmosphere: horizontal

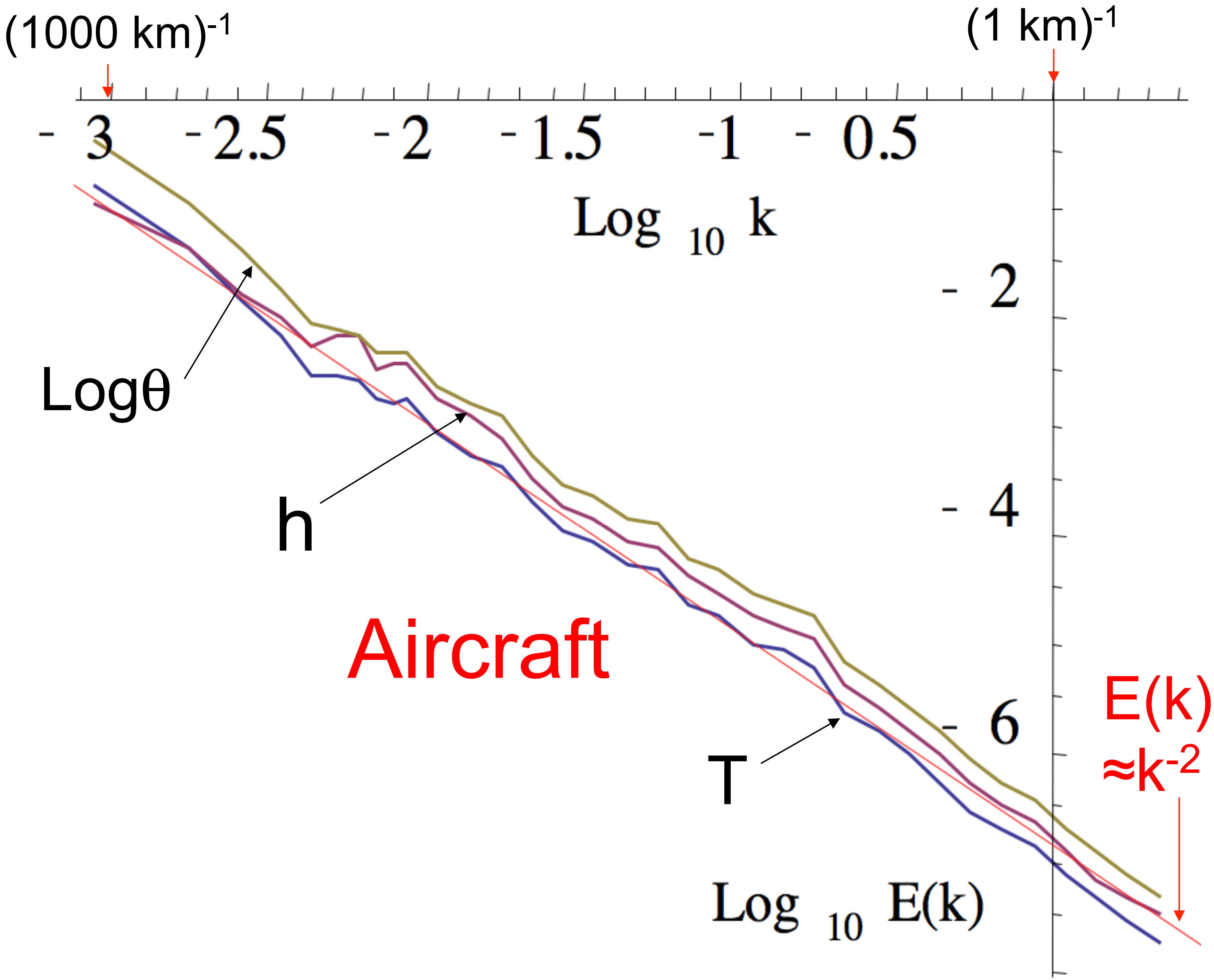
TRMM VIRS, 1000 orbits (January-March 1998)

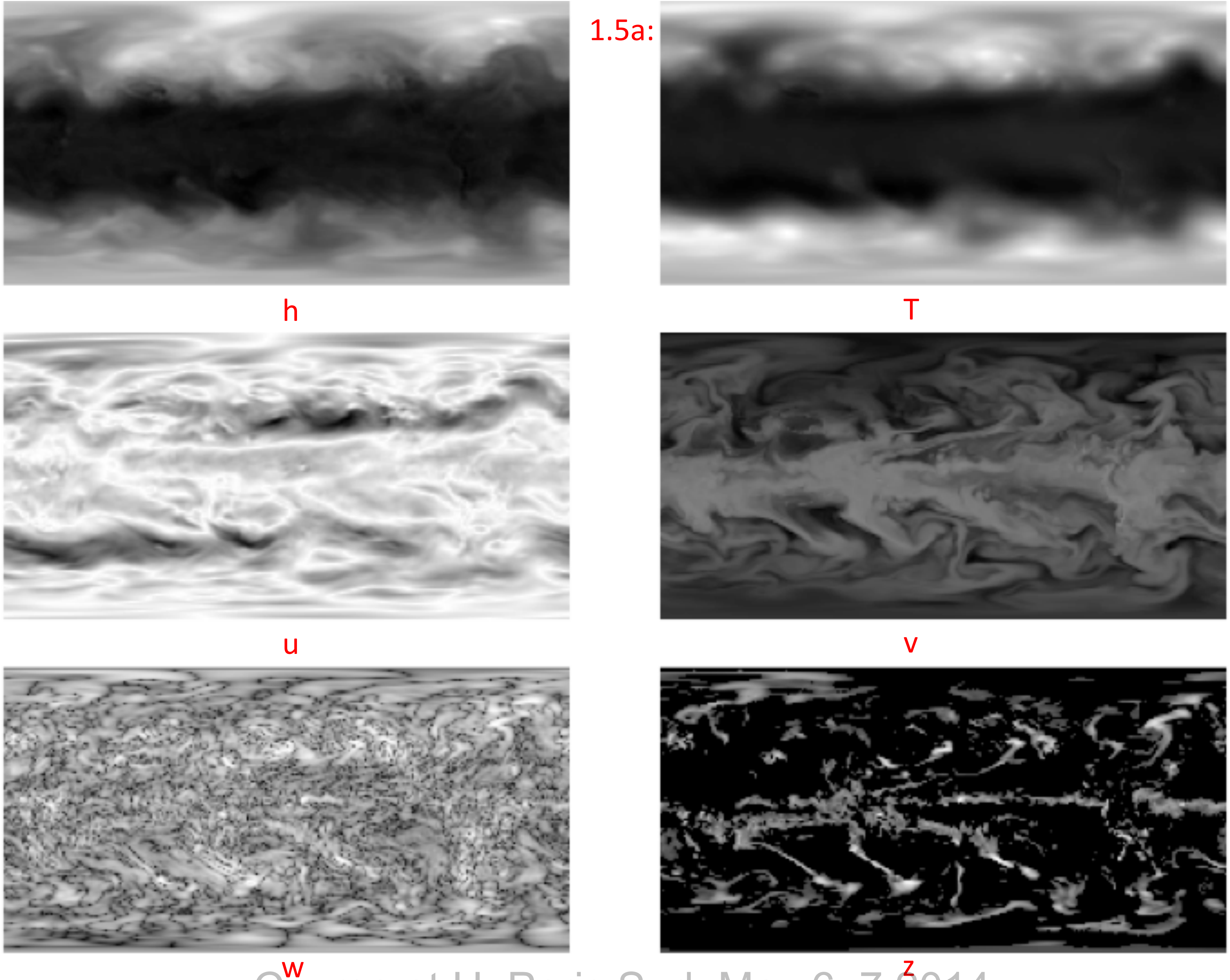


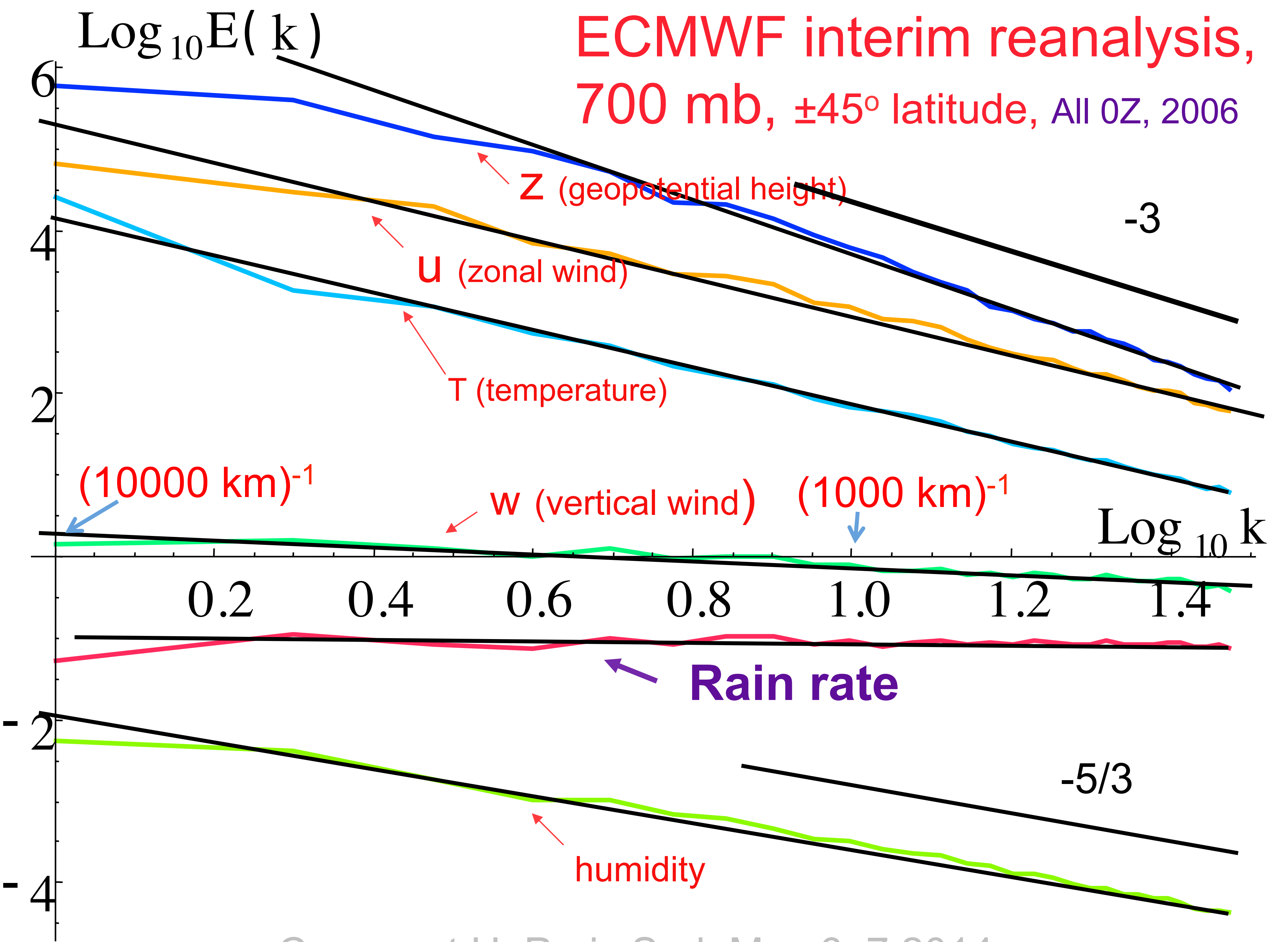
Visible, near infra red, thermal infra red



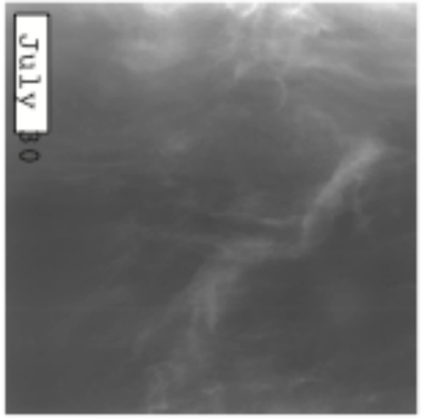
Log 10 E(k)
TRMM TMI, 1000 orbits
(January-March 1998)
(passive microwave)
TMI 8
TMI 6
TMI 5
TMI 3
TMI 1
0.3-2.2 cm
3 mm
2.2cm
-5/3
(20000 km)-1
(100 km)-1
9
8
7
6
5
4
3
0.5
1.0
1.5
2.0
2.5
Log10 k

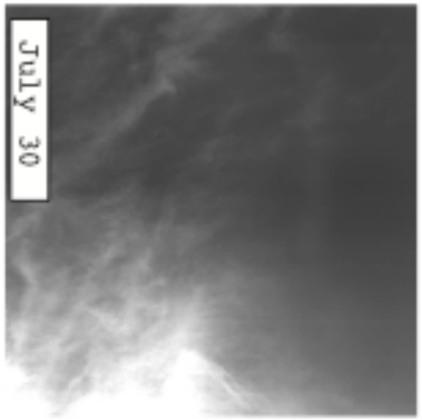

(1000 km)$^{-1}$
(1 km)$^{-1}$
- 3
- 2.5
- 2
- 1.5
-1
- 0.5
Log 10 k
- 2
- 4
- 6
Logθ
h
Aircraft
T
E(k)
≈k^{-2}
Log 10 E(k)

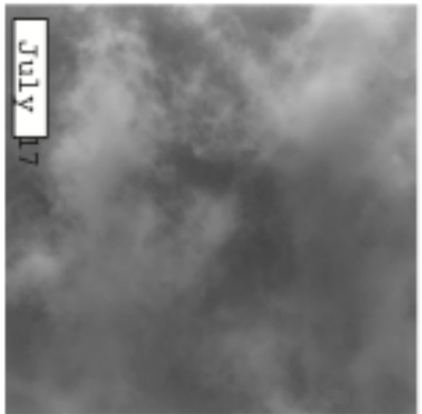

1.5a:
h
T
u
v
w
z


Log 10 E(k)
ECMWF interim reanalysis,
700 mb, ±45° latitude, All 0Z, 2006
Z (geopotential height)
-3
u (zonal wind)
T (temperature)
(10000 km)-1
w (vertical wind)
(1000 km)-1
Log 10 k
Rain rate
-5/3
humidity
6
4
2
- 2
- 4
0.2
0.4
0.6
0.8
1.0
1.2
1.4

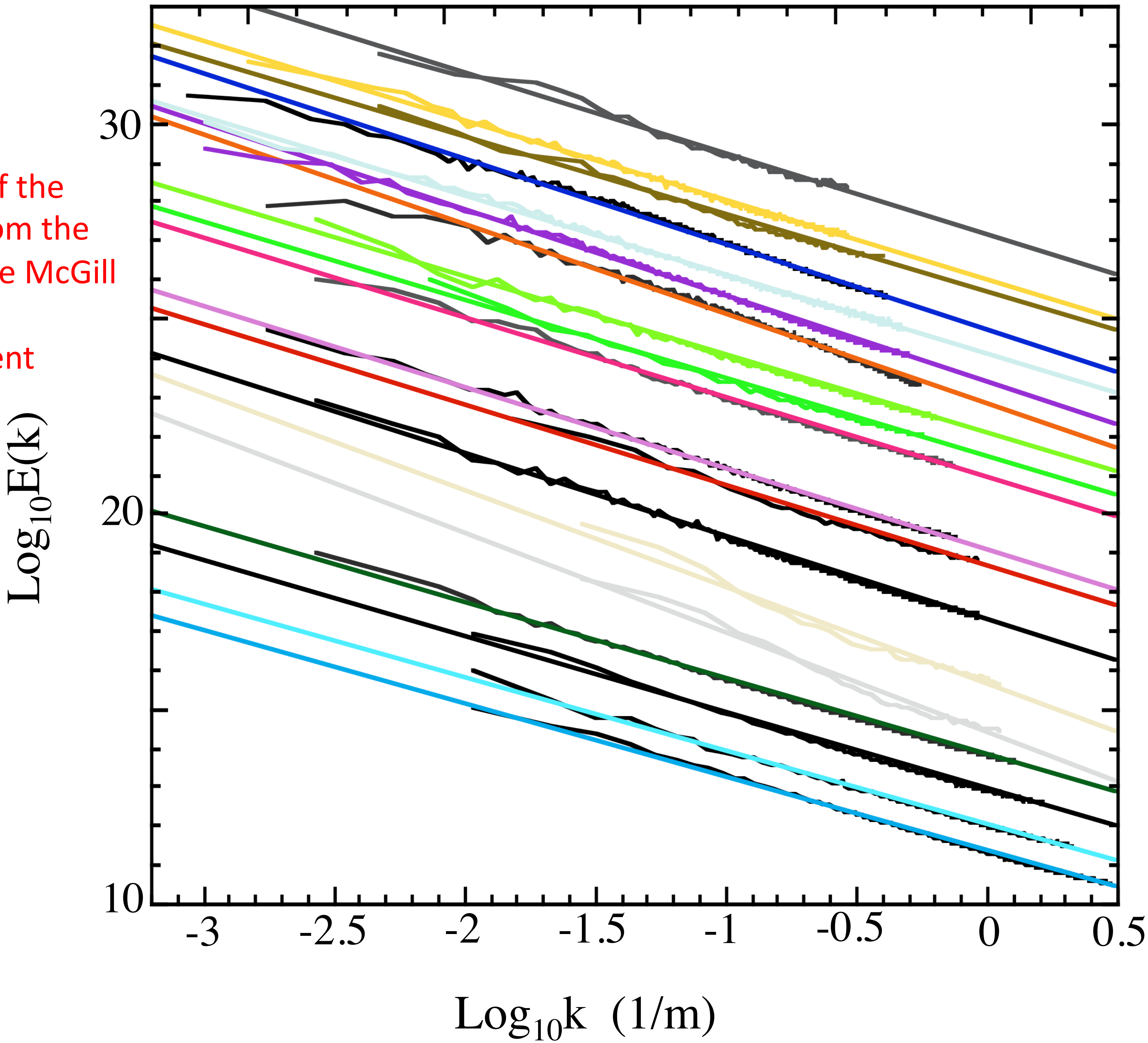
Clouds photographed from the roof of the McGill physics department





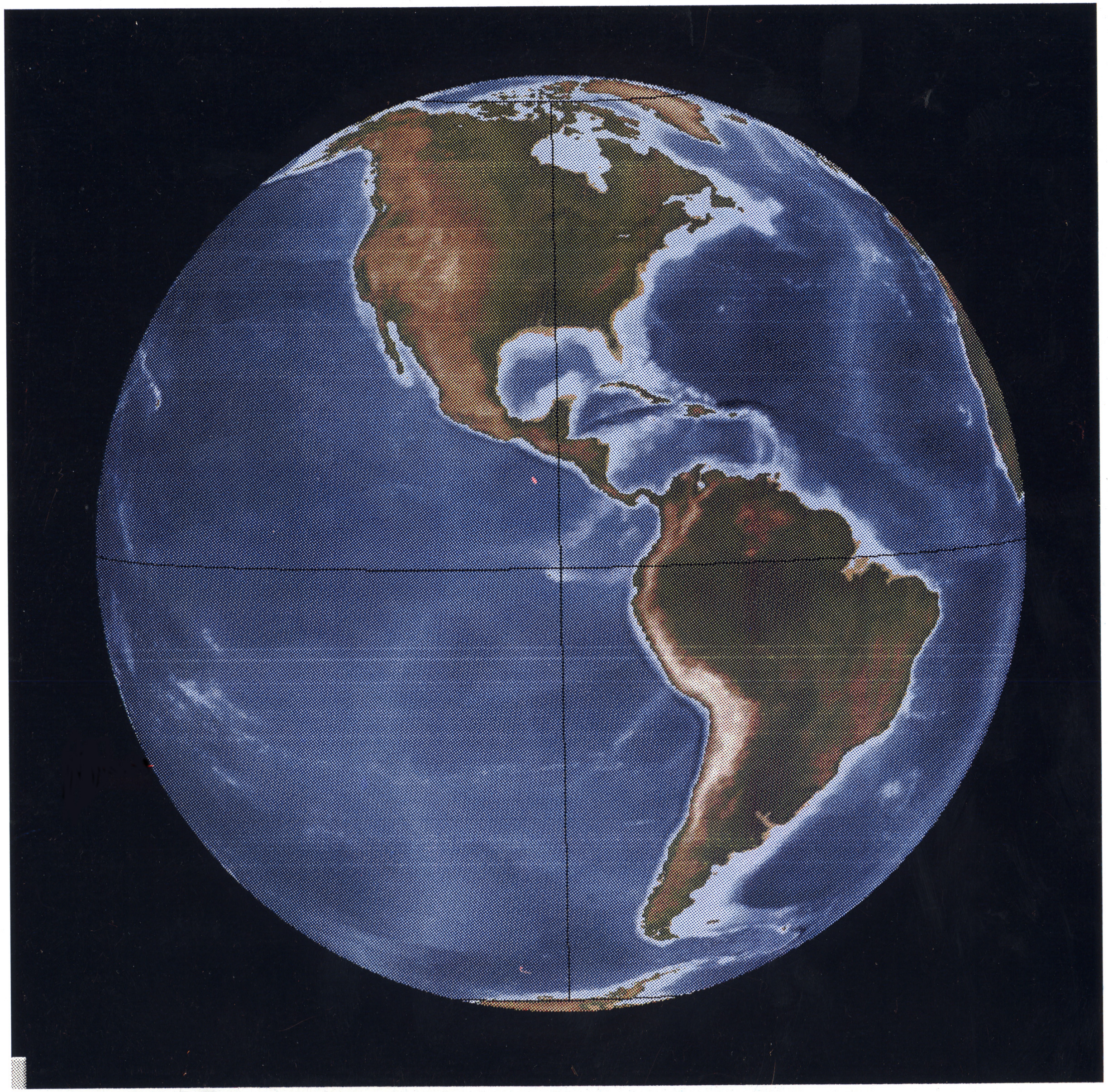


Spectra of the clouds from the roof of the McGill physics department



The earth’s surface, solid earth

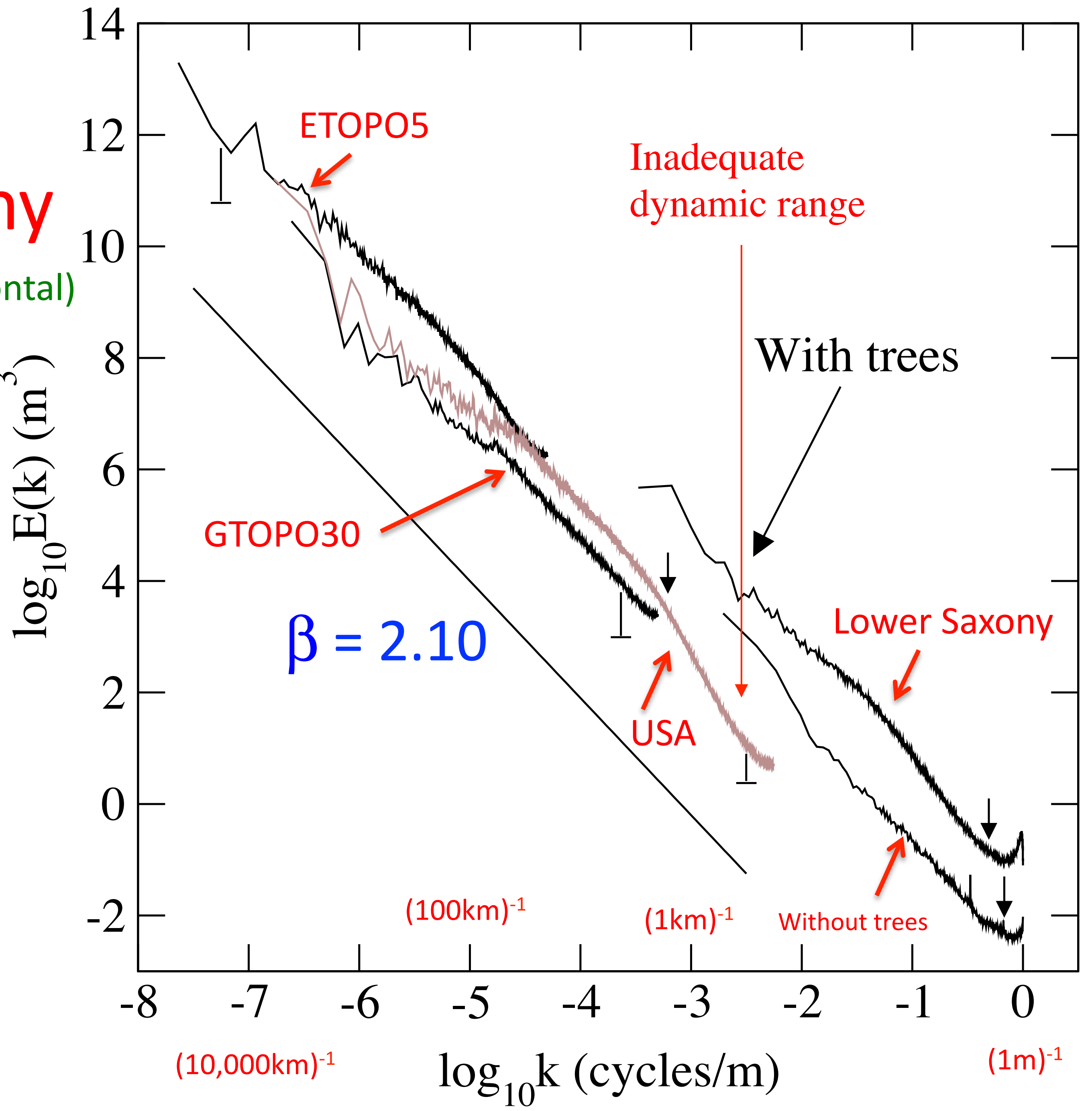
Topography



ETOPO5
altitude data
(5 ’ arc, roughly 10km Resolution)

Topography

(scaling in the horizontal)

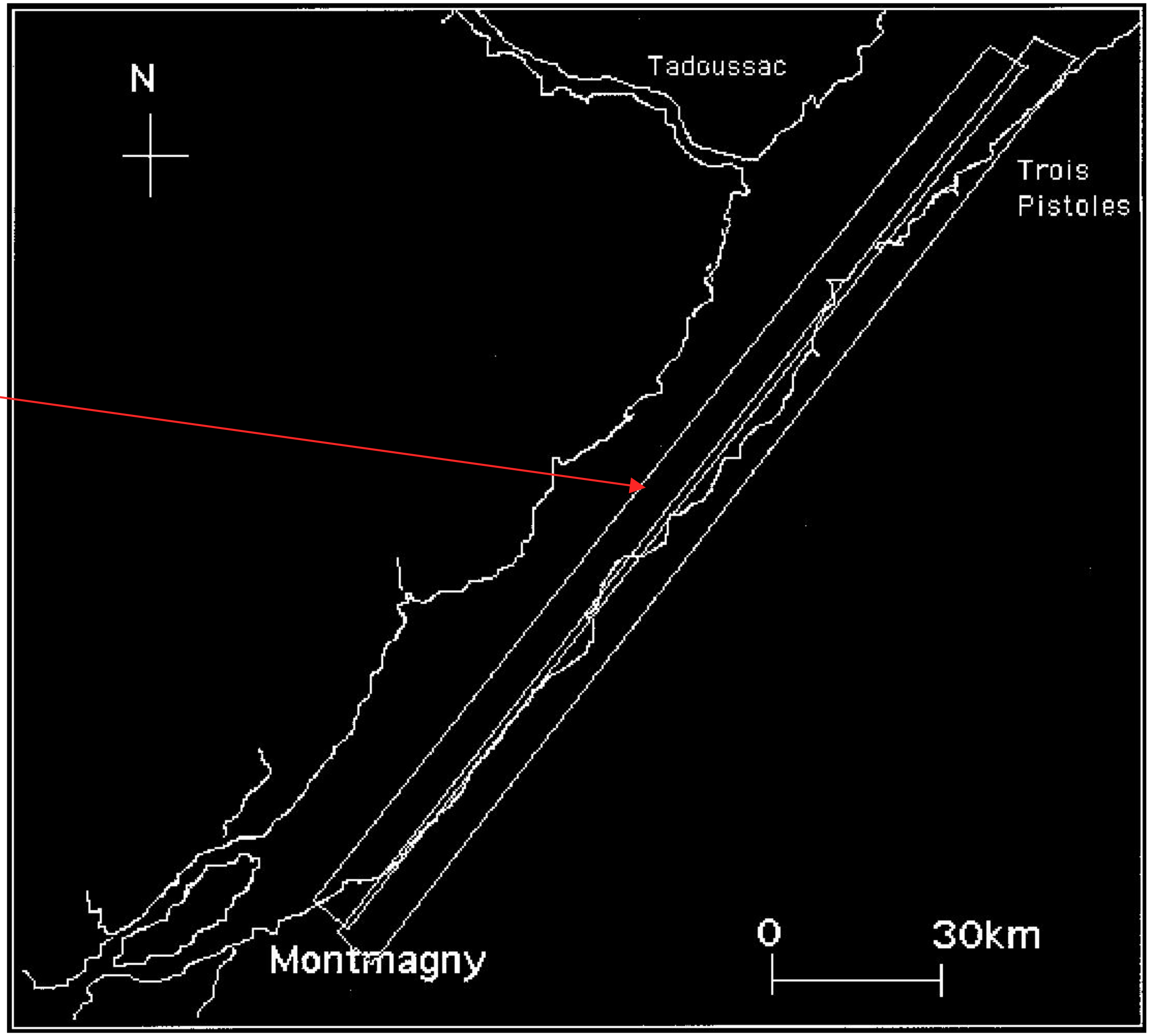


Gagnon, Lovejoy and Schertzer, 2006

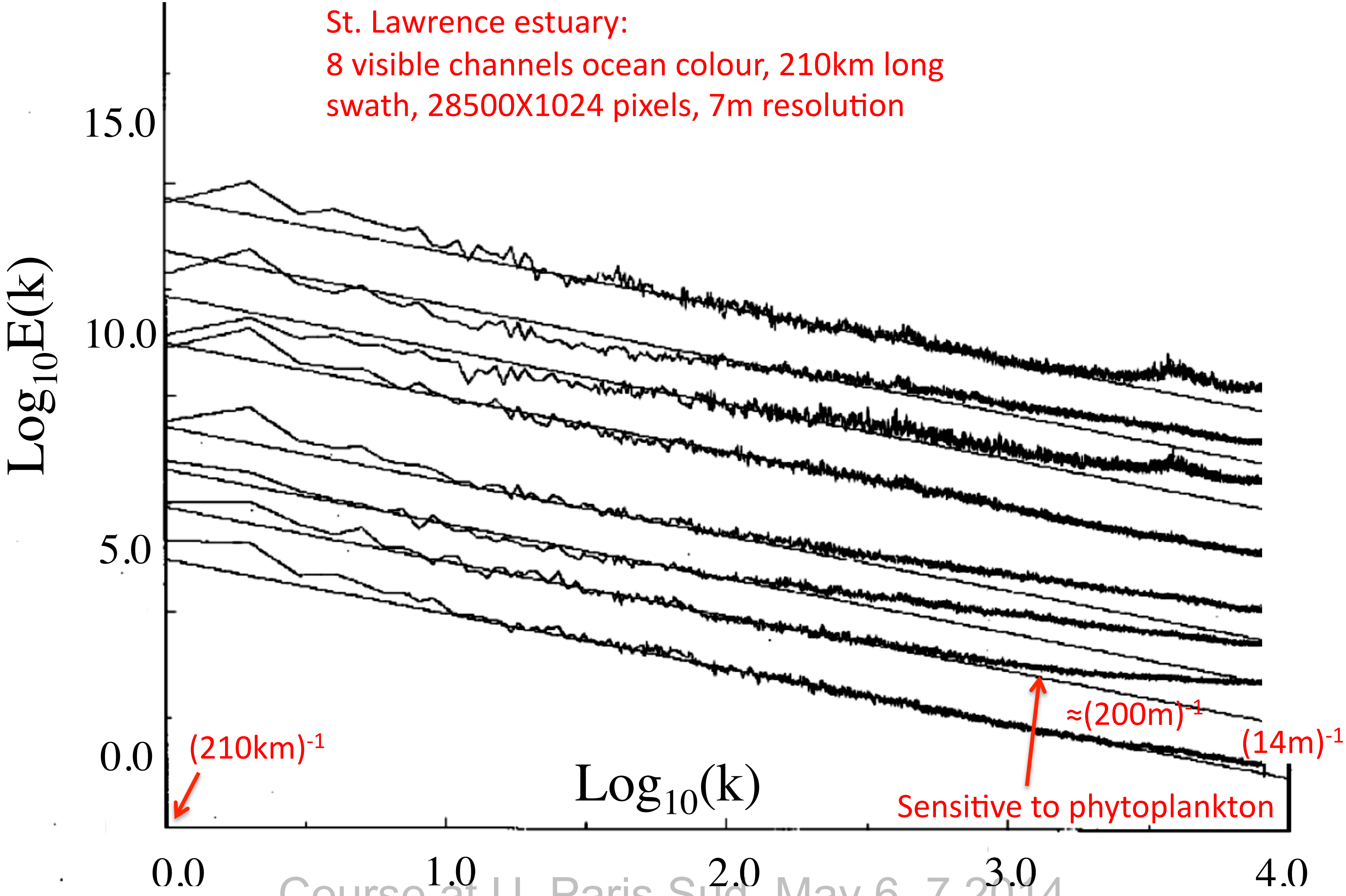
Energy spectra over a scale range of 10^8 Global (ETOPO5, 10km), continental US (GTOPO30: 1km and 90m), Lower Saxony, 20cm).

Ocean Colour:
Mies sensor, experimental region

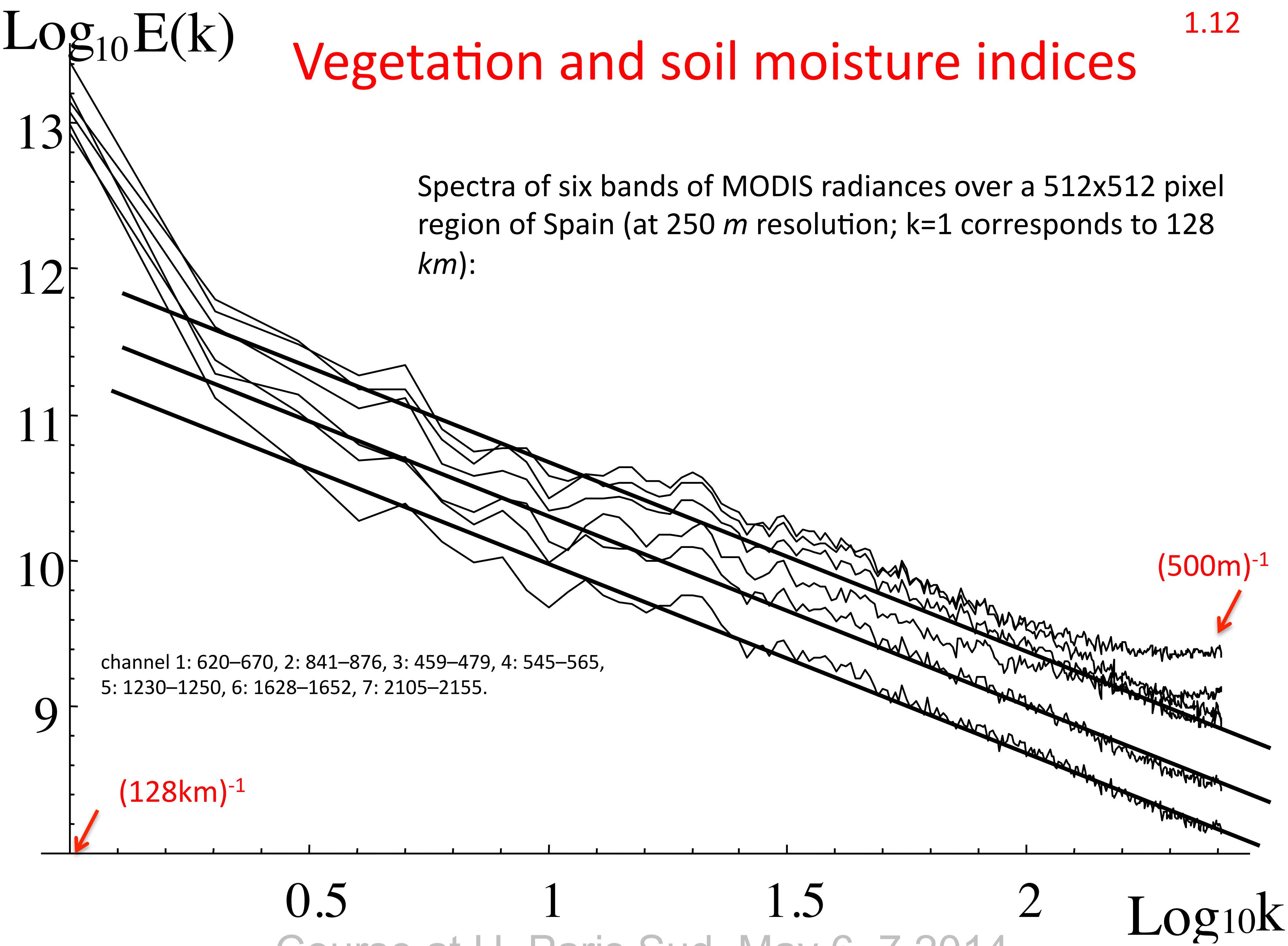
210km long swath,
28500X1024 pixels,
7m resolution,
(8 visible channels)



Ocean surface

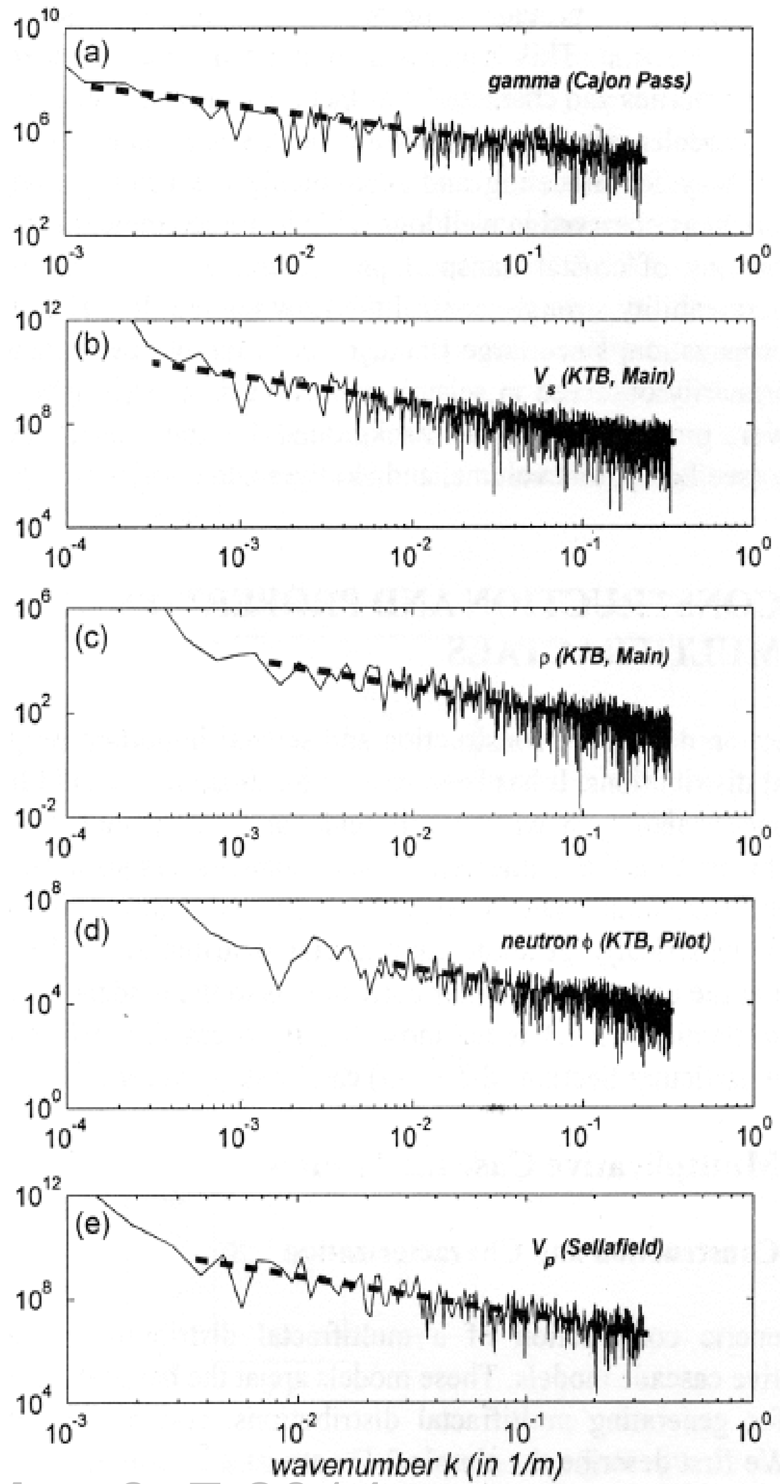


Vegetation and soil moisture indices



The scaling of the KTB borehole

(scaling in the vertical)



(1987-1995) 9.1km deep
Russian Kola: 12.2 km

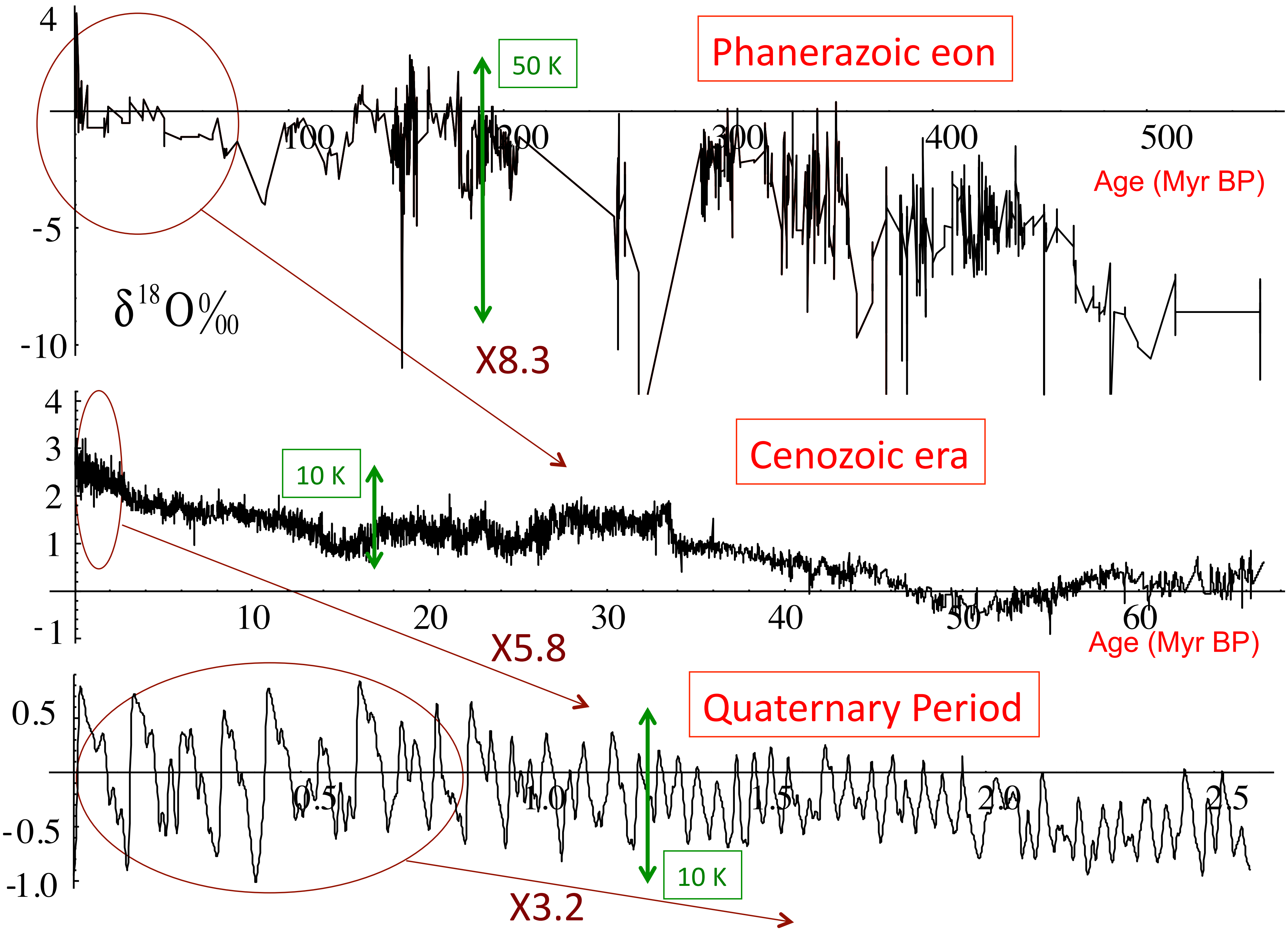
Marsan and Bean (2003)

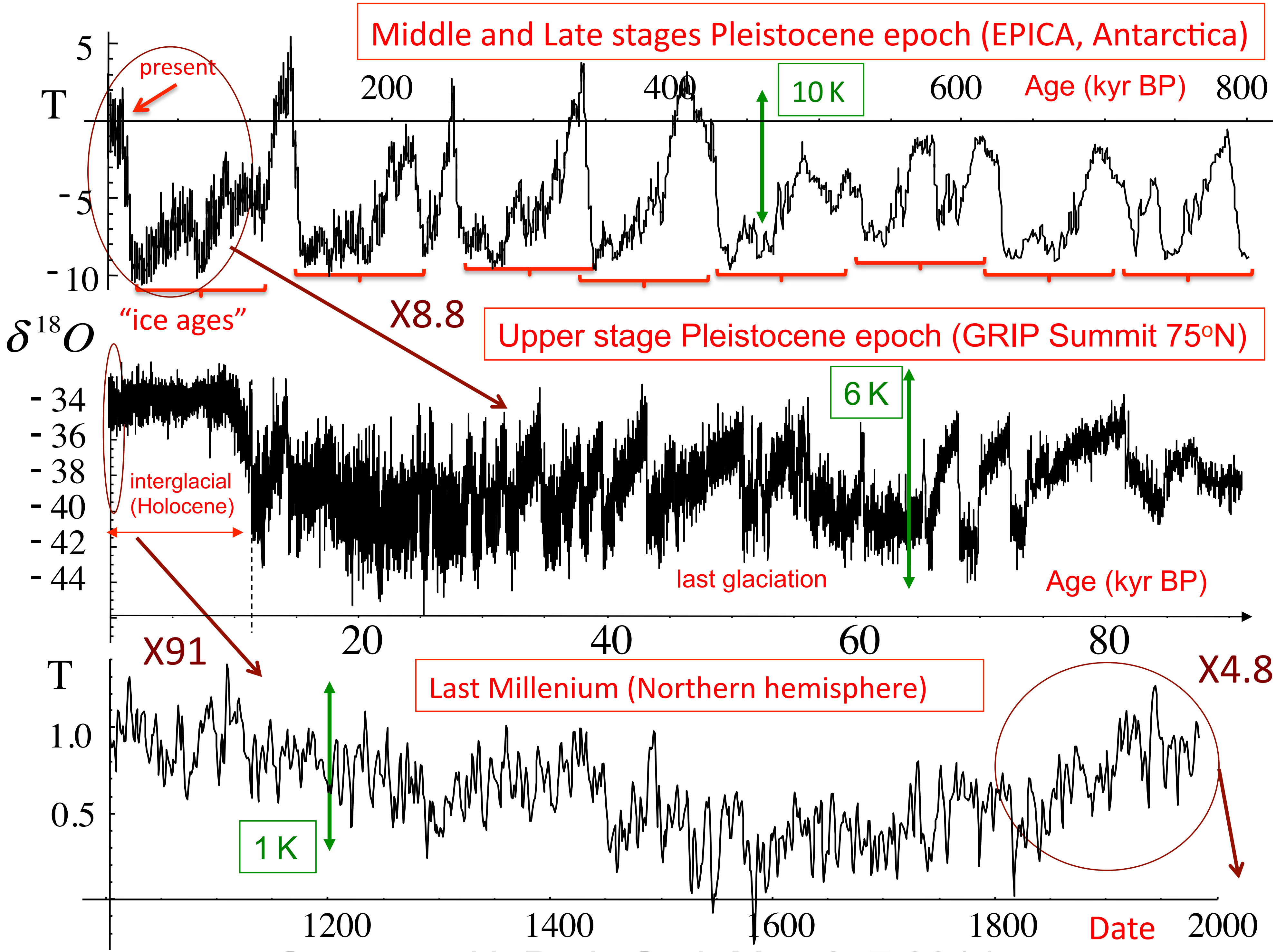
Scaling in time:

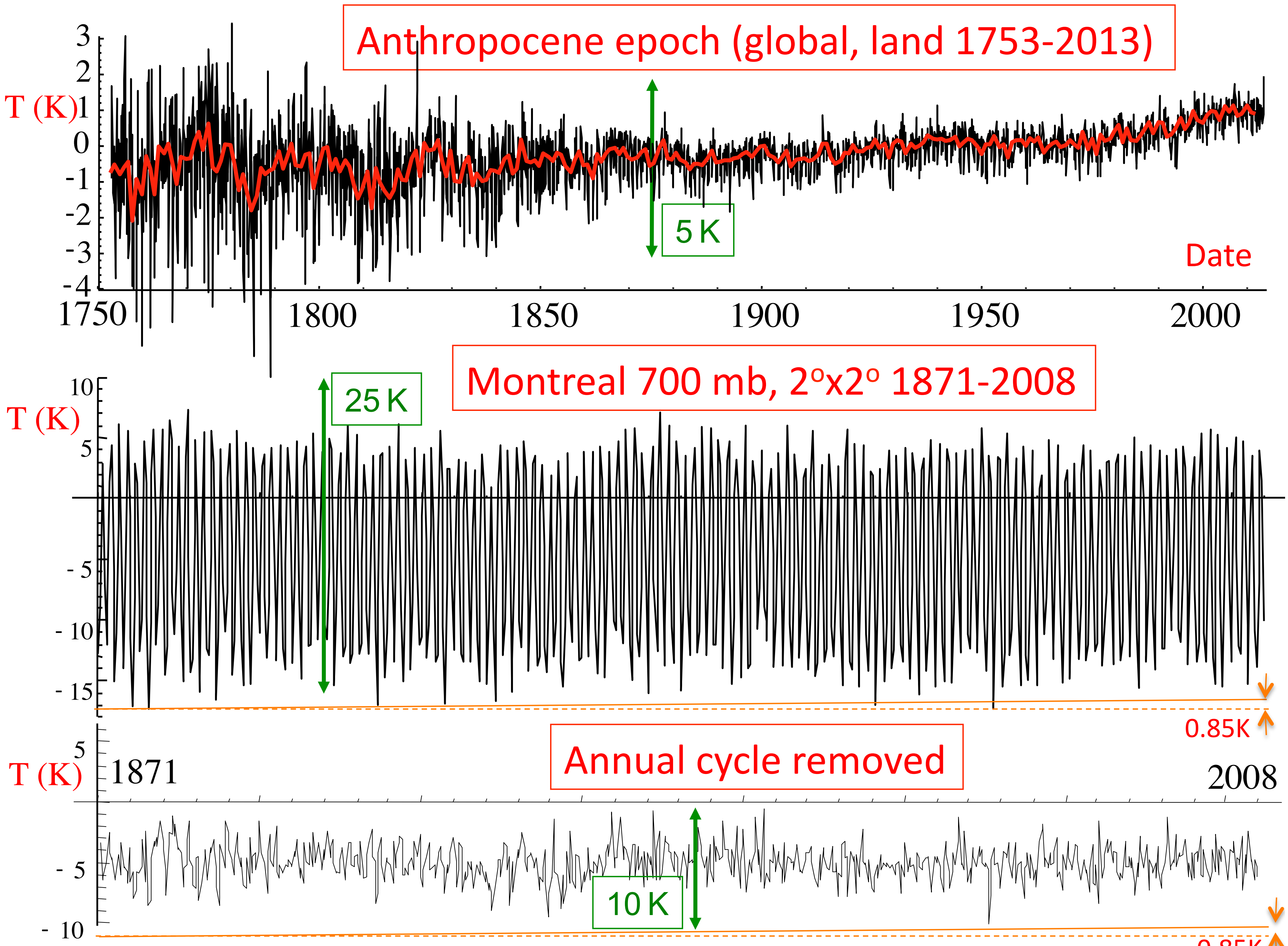
From the age of the earth to the viscous dissipation scale: 4.5×10^9 years - 1 ms:

20 orders of magnitude in time

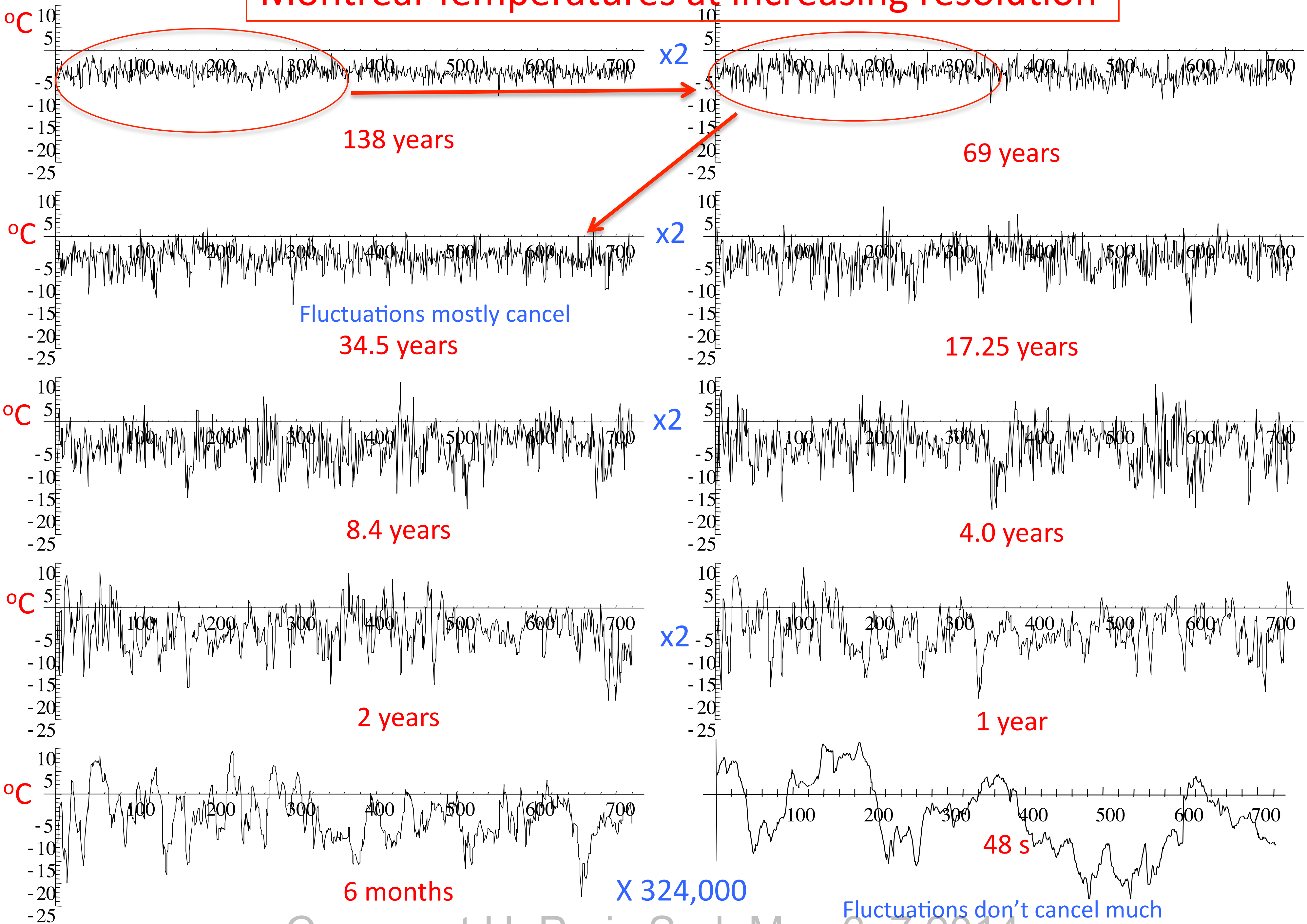
A voyage through scale…


Phanerazoic eon
50 K
Age (Myr BP)
$\delta^{18}O$‰
X8.3
Cenozoic era
10 K
Age (Myr BP)
X5.8
Quaternary Period
10 K
X3.2


Middle and Late stages Pleistocene epoch (EPICA, Antarctica)
T
5
-5
-10
present
200
400
600
800
Age (kyr BP)
10 K
"ice ages"
X8.8
$\delta^{18}O$
Upper stage Pleistocene epoch (GRIP Summit 75ºN)
- 34
- 36
- 38
- 40
- 42
- 44
6 K
interglacial
(Holocene)
last glaciation
Age (kyr BP)
20
40
60
80
X91
X4.8
T
1.0
0.5
Last Millenium (Northern hemisphere)
1 K
1200
1400
1600
1800
Date
2000

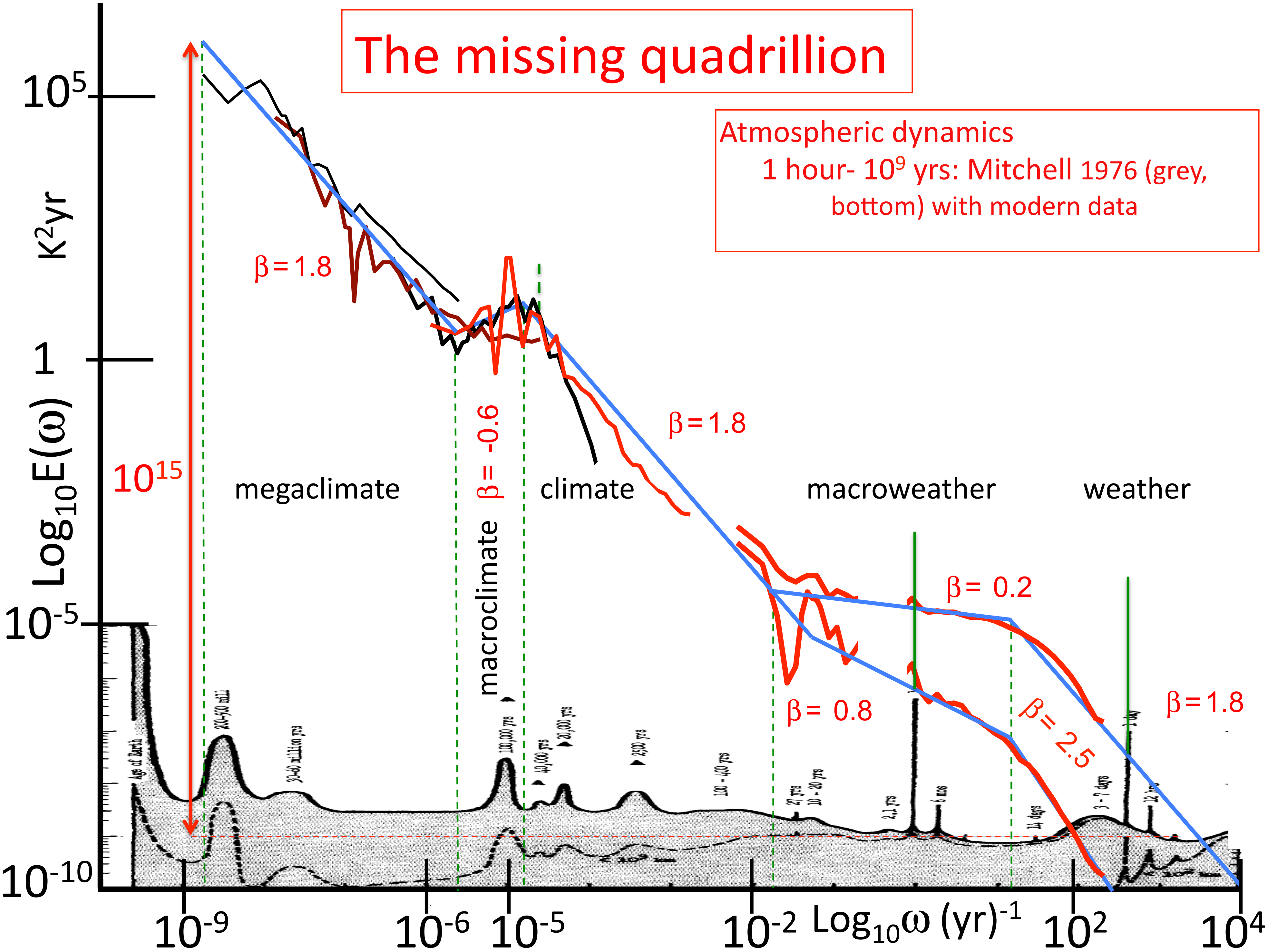

Anthropocene epoch (global, land 1753-2013)
T (K)
3
2
1
0
-1
-2
-3
-4
5 K
Date
1750
1800
1850
1900
1950
2000
Montreal 700 mb, 2°x2° 1871-2008
T (K)
10
5
- 5
- 10
- 15
25 K
0.85K
Annual cycle removed
T (K)
5
1871
2008
- 5
- 10
10 K
0.85K

Montreal Temperatures at increasing resolution

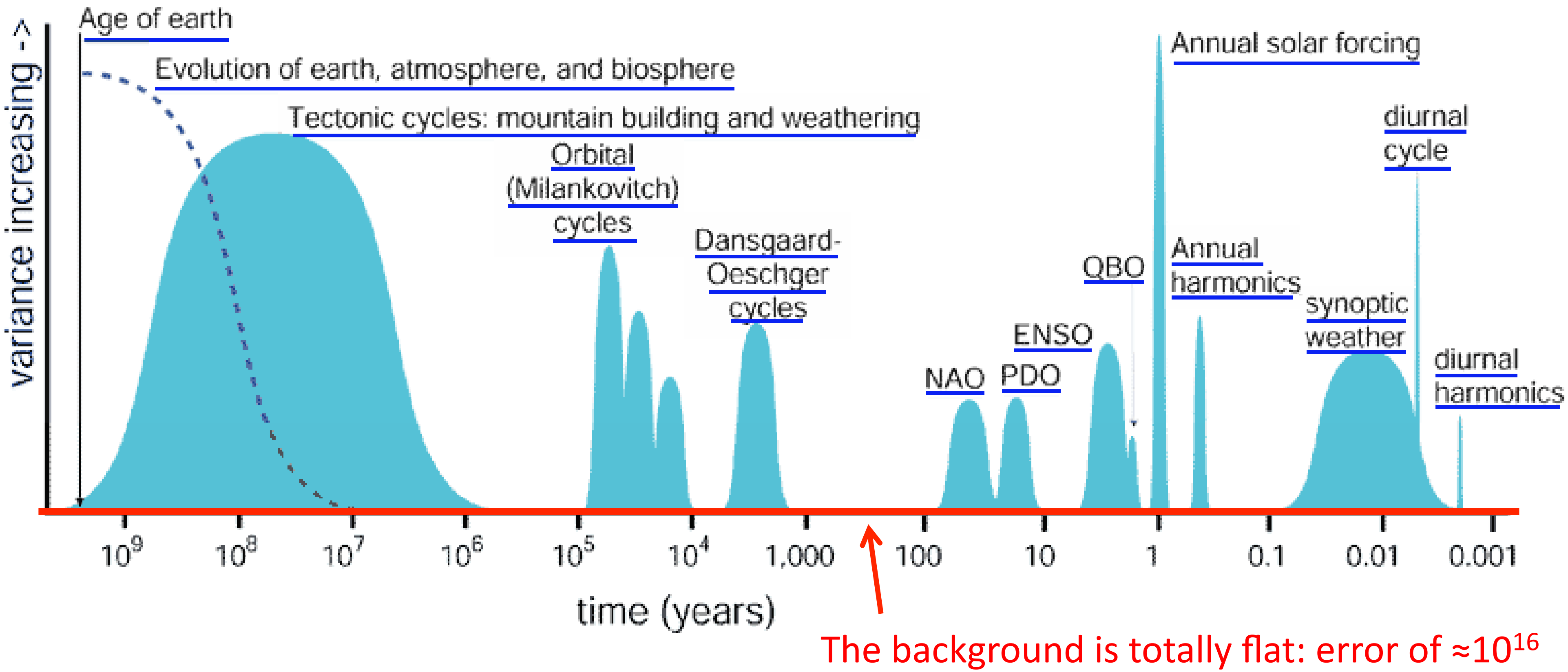


The missing quadrillion

Atmospheric dynamics
1 hour- 10^9 yrs: Mitchell 1976 (grey, bottom) with modern data



The NOAA NCDC Paleoclimate data site graph (inspired by Mitchell)



The explanation of the figure:

"… figure is intended as a mental model to provide a general "powers of ten" overview of climate variability, and to convey the basic complexities of climate dynamics for a general science savvy audience."

The site assures us that just "because a particular phenomenon is called an oscillation, it does not necessarily mean there is a particular oscillator causing the pattern. Some prefer to refer to such processes as variability."

How to understand the variability?

Answer #1:

Scale bound thinking

Scale bound thinking

Antonie van Leeuwenhoek

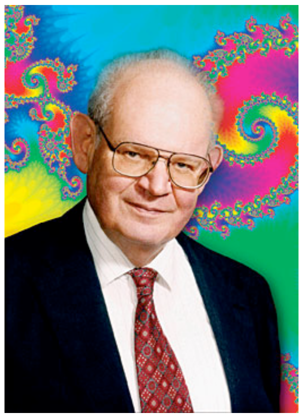
(1632–1723)

A new world in a drop of water

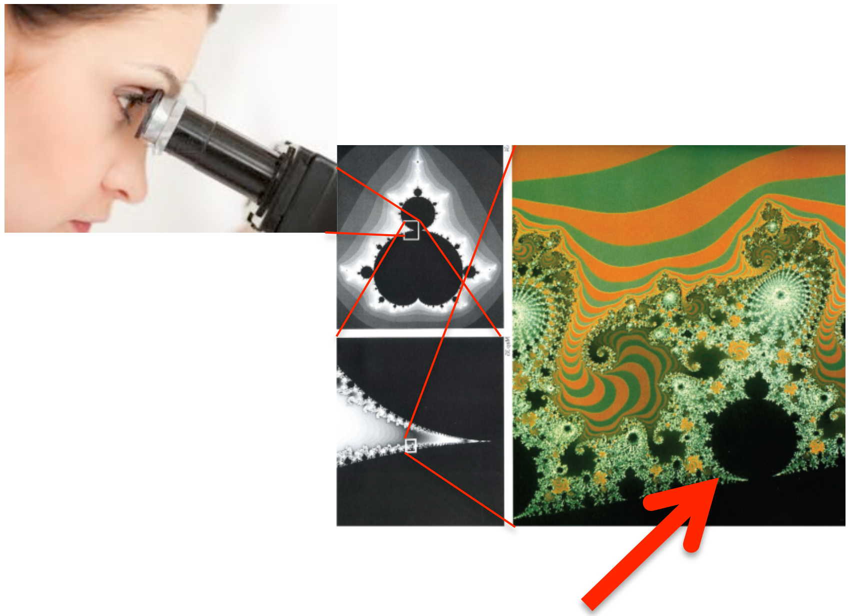
…..the discovery of micro-organisms

"Animalcules," described in depth by Leeuwenhoek, c1695–1698. By Anton van Leeuwenhoek

Pure, (self-similar) Fractal thinking



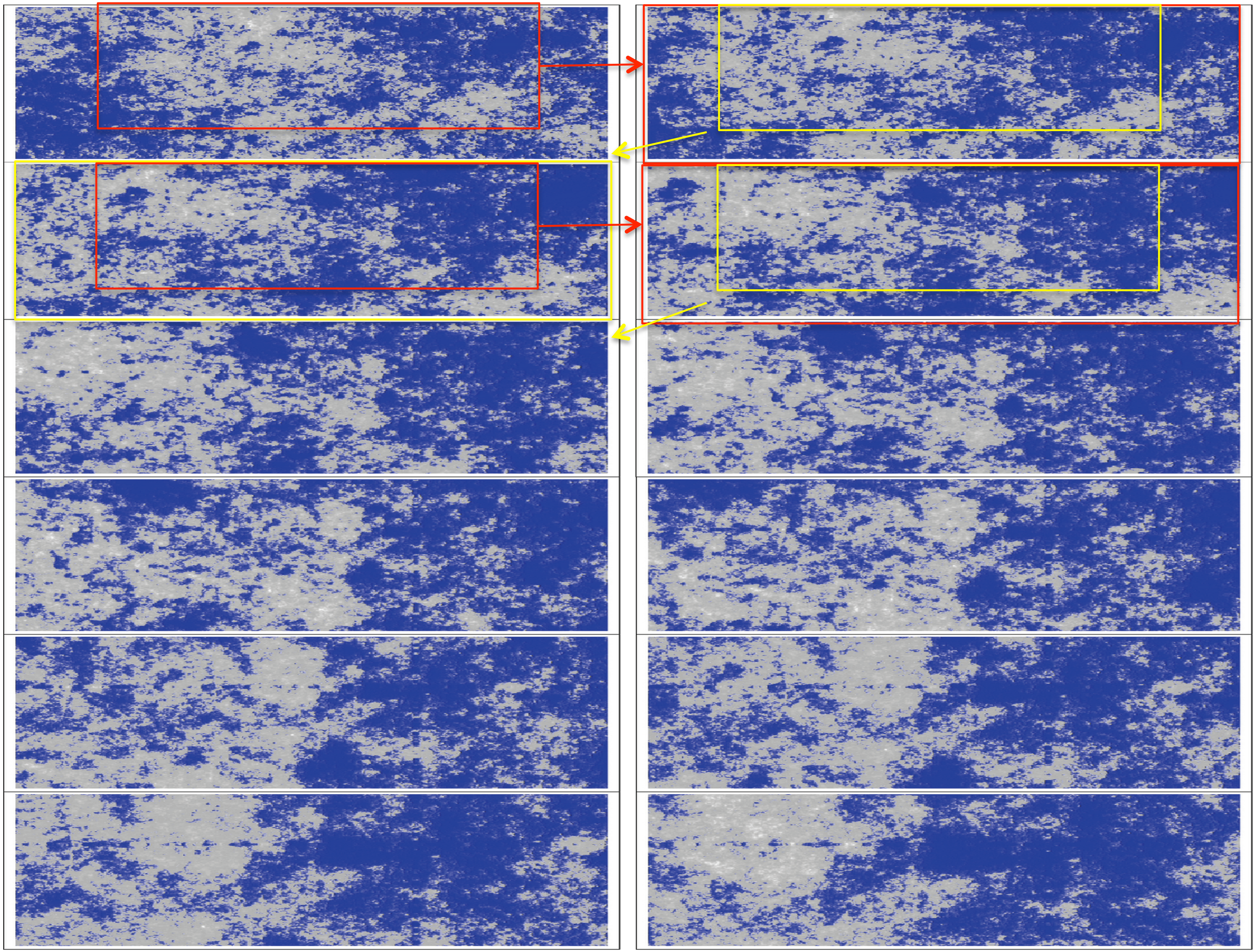
Mandelbrot 1924-2010



The same!!!

(the Mandelbrot set)

Clouds..... Zooming in by factors of 1.7

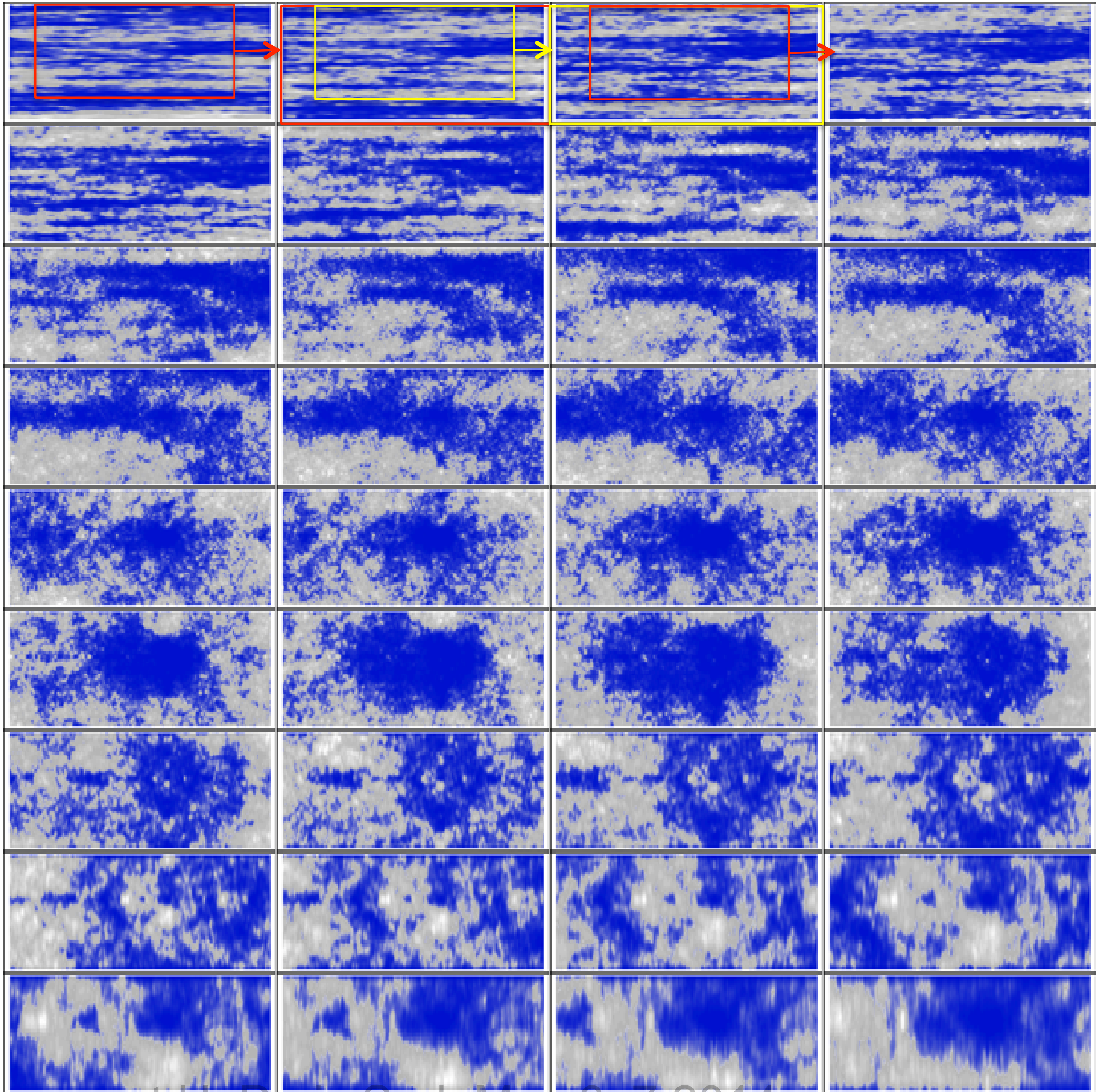


But not here!

Need Scale invariant thinking!

(Zoom factor 1000)

Vertical cross-section of the atmosphere

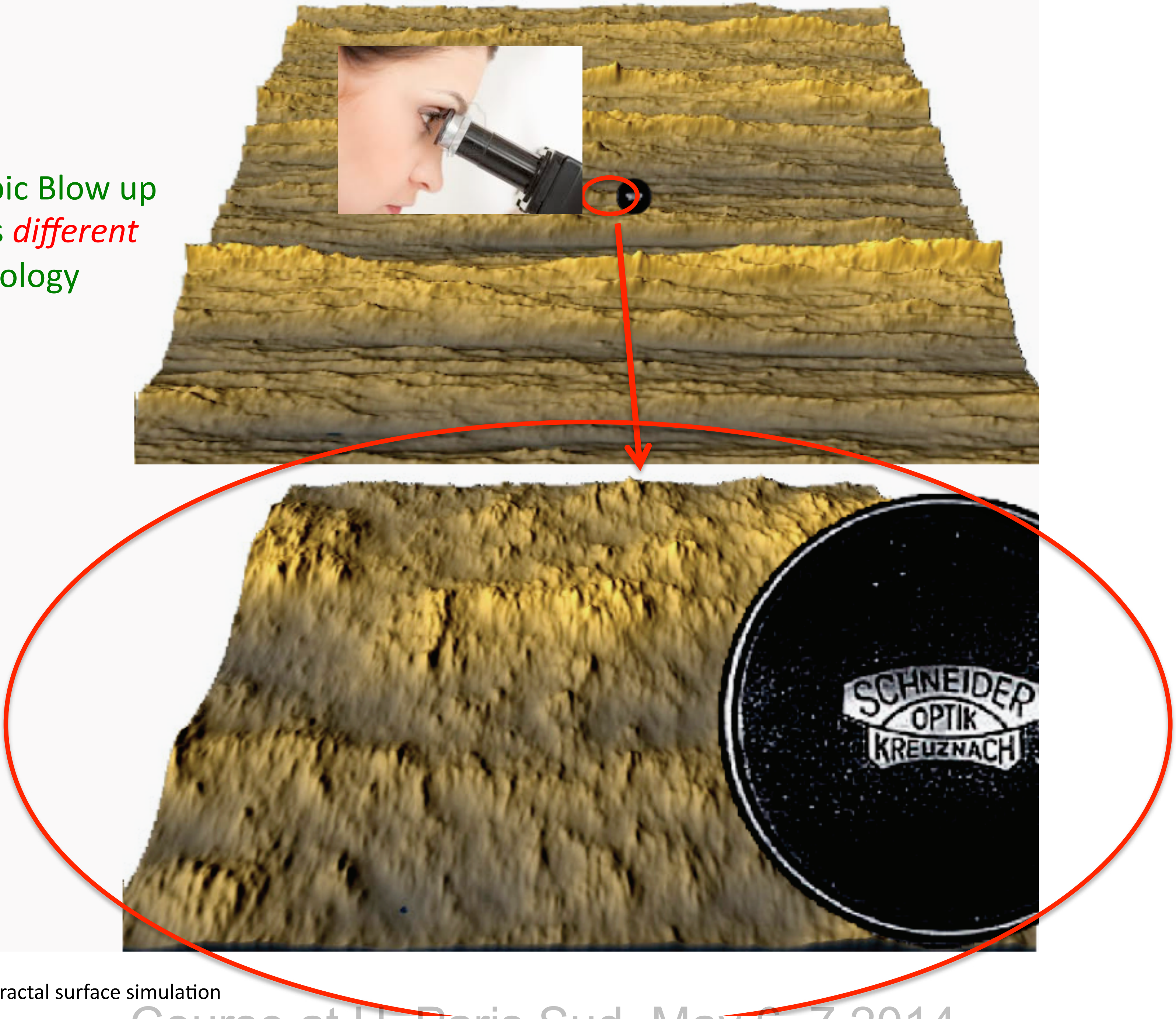


Scale invariance and the Phenomenological Fallacy

1) Morphology not dynamics is taken as fundamental

2) Scaling is reduced to the isotropic (self-similar) special case

Isotropic Blow up reveals *different* morphology



Anisotropic multifractal surface simulation