Ideal Hydrodynamics

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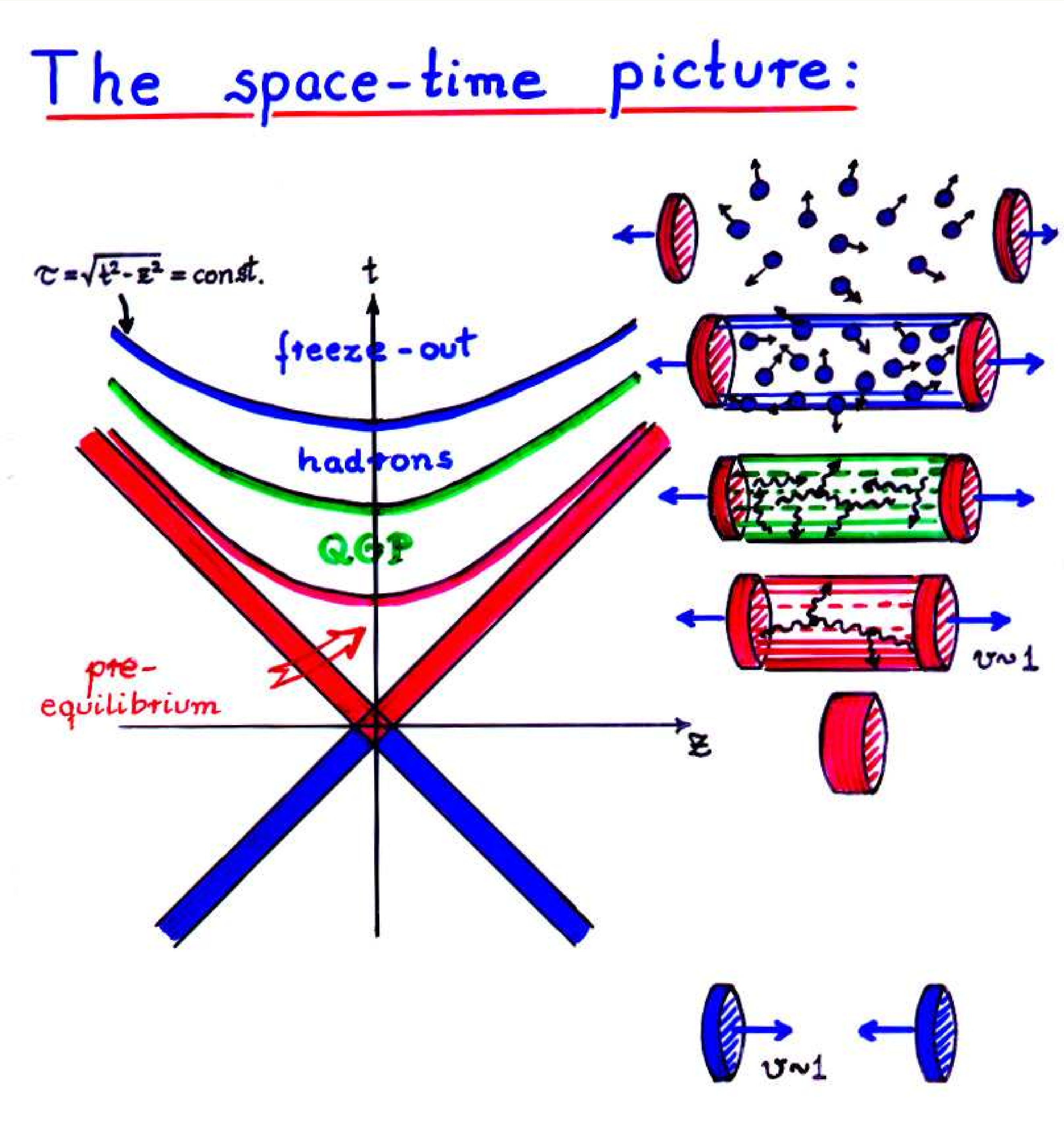


Transient matter

- **lifetime**
  $t \sim 10\,\mathrm{fm/c}$
  $\sim 10^{-23}$ **seconds**
- **small size**
  $r \sim 10\,\mathrm{fm}$
  $\sim 10^{-14}\,\mathrm{m}$
- **rapid expansion**

Multiplicity @ LHC
~ 15000

Conservation laws

Conservation of energy and momentum:

$$\partial_\mu T^{\mu\nu}(x) = 0$$

Conservation of charge:

$$\partial_\mu N^{\mu}(x) = 0$$

Local conservation of particle number and energy-momentum.

$$\Longleftrightarrow$$ **Hydrodynamics!**

This can be generalized to multicomponent systems and systems with several conserved charges:

$$\partial_\mu N_i^{\mu} = 0,$$

$i =$ **baryon number, strangeness, charge. . .**

Consider only **baryon number conservation**, $i = B$.

$\Rightarrow$ **5 equations contain 14 unknowns!**

$\Rightarrow$ **The system of equations does not close.**

$\Rightarrow$ **Provide 9 additional equations or Eliminate 9 unknowns.**

So what are the components of $T^{\mu\nu}$ and N^{μ}?

- N^{μ} and $T^{\mu\nu}$ can be decomposed with respect to arbitrary, normalized, time-like 4-vector u^{μ},

$$u_\mu u^\mu = 1$$

- Define a projection operator

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, \quad \Delta^{\mu\nu} u_\nu = 0,$$

which projects on the 3-space orthogonal to u^{μ}.

- **Then**

$$\boxed{N^\mu = nu^\mu + \nu^\mu}$$

where

$n = N^\mu u_\mu$	is (baryon) charge density in the frame where $u = (1, \mathbf{0})$, local rest frame, LRF
$\nu^\mu = \Delta^{\mu\nu} N_\nu$	is charge flow in LRF,

and

$$\boxed{T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu}}$$

$\epsilon \equiv u_\mu T^{\mu\nu} u_\nu$ energy density in LRF
$P \equiv -\frac{1}{3}\Delta^{\mu\nu} T_{\mu\nu}$ isotropic pressure in LRF
$q^\mu \equiv \Delta^{\mu\alpha} T_{\alpha\beta} u^\beta$ energy flow in LRF
$\pi^{\mu\nu} \equiv [\frac{1}{2}(\Delta^\mu{}_\alpha \Delta^\nu{}_\beta + \Delta^\nu{}_\beta \Delta^\mu{}_\alpha) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}] T^{\alpha\beta}$
(trace-free) stress tensor in LRF

- **The 14 unknowns in 5 equations:**

$$\left.\begin{array}{ll} N^\mu & 4 \\ T^{\mu\nu} & 10 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{cc} n,\ \epsilon,\ P & 3 \\ q^\mu & 3 \\ \nu^\mu & 3 \\ \pi^{\mu\nu} & 5 \end{array}\right.$$

- So far u^μ is arbitrary. It attains a physical meaning by relating it to N^μ or $T^{\mu\nu}$:

1. Eckart frame:
$$u_E^\mu \equiv \frac{N^\mu}{\sqrt{N_\nu N^\nu}}$$
u^μ is 4-velocity of charge flow, $\nu^\mu = 0$.
The 14 unknowns are n, ϵ, P, q^μ, $\pi^{\mu\nu}, u^\mu$.
2. Landau frame:
$$u_L^\mu \equiv \frac{T^{\mu\nu} u_\nu}{\sqrt{u_\alpha T^{\alpha\beta} T_{\beta\gamma} u^\gamma}}$$
u^μ is 4-velocity of energy flow, $q^\mu = 0$.
The 14 unknowns are n, ϵ, P, ν^μ, $\pi^{\mu\nu}, u^\mu$.

- In general, the hydrodynamical equations are not closed and cannot be solved uniquely.

Ideal hydrodynamics

Suppose particles are in **local thermodynamical equilibrium**, i.e., single particle phase space distribution function is given by:

$$f_i(x,k) = \frac{g}{(2\pi)^3}\left[\exp\left(\frac{k_\mu u^\mu(x) - \mu(x)}{T(x)}\right) \pm 1\right]^{-1}$$

where

$T(x)$ and $\mu(x)$: local temperature and chemical potential
$u(x)^\mu$: local 4-velocity of fluid flow.

Then kinetic theory definitions give

$$N^\mu(x) \equiv \sum_i q_i \int \frac{\mathrm{d}^3\mathbf{k}}{E}\, k^\mu f_i(x,k) = n(T,\mu)\, u^\mu$$

$$T^{\mu\nu}(x) \equiv \sum_i \int \frac{\mathrm{d}^3\mathbf{k}}{E}\, k^\mu k^\nu f_i(x,k)$$

$$= (\epsilon(T,\mu) + P(T,\mu))\, u^\mu u^\nu - P(T,\mu)\, g^{\mu\nu}$$

where

$$n(T,\mu) = \sum_i q_i \int \mathrm{d}^3\mathbf{k}\, f_i(x,E)$$ **is local charge density,**

$$\epsilon(T,\mu) = \sum_i \int \mathrm{d}^3\mathbf{k}\, E f_i(x,E)$$ **is local energy density and**

$$P(T,\mu) = \sum_i \int \mathrm{d}^3\mathbf{k}\, \frac{\mathbf{k}^2}{3E} f_i(x,E)$$ **is local pressure.**

Note! $f(x,E)$ **is distribution in local rest frame:** $u^\mu = (1,\mathbf{0})$**.**

$\rightarrow$ **Local thermodynamical equilibrium implies there is no viscosity:**

$$\boxed{\nu^\mu = q^\mu = \pi^{\mu\nu} = 0.}$$

Ideal fluid approximation:

$$N^\mu = nu^\mu$$
$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\mu}$$

- **Local equilibrium** $\Rightarrow$ **no viscosity**: $\nu^\mu = q^\mu = \pi^{\mu\nu} = 0$.
- Now N^μ and $T^{\mu\nu}$ contain **6 unknowns**, ϵ, P, n and u^μ, but there are still only **5 equations**!
- In thermodynamical equilibrium ϵ, P and n are not independent! They are specified by two variables, T and μ.
- The **equation of state** (EoS), $P(T,\mu)$ eliminates one unknown!
- Any **equation of state** of the form

$$P = P(\epsilon, n)$$

closes the system of hydrodynamic equations and makes it **uniquely solvable** (given initial conditions).

Remark: $P = P(\epsilon, n)$ **is not a complete equation of state in a thermodynamical sense.**

A complete equation of state allows to compute all thermodynamic variables.

For example, $s = s(\epsilon, n)$**:** $\mathrm{d}s = 1/T\mathrm{d}\epsilon - \mu/T\mathrm{d}n$ **(1st law of thermod.)**

$$\frac{1}{T} = \frac{\partial s}{\partial \epsilon}|_n, \qquad \frac{\mu}{T} = -\frac{\partial s}{\partial n}|_\epsilon, \qquad P = Ts + \mu n - \epsilon$$

$P = P(\epsilon, n)$ **does not work!**

$$\frac{\partial P}{\partial \epsilon}|_n = ? \qquad \frac{\partial P}{\partial n}|_\epsilon = ?$$

However, $P = P(T, \mu)$ **does work!**

$$\mathrm{d}P = s\mathrm{d}T + n\mathrm{d}\mu \quad \Rightarrow \quad s = \frac{\partial P}{\partial T}|_\mu, \qquad n = \frac{\partial P}{\partial \mu}|_T$$

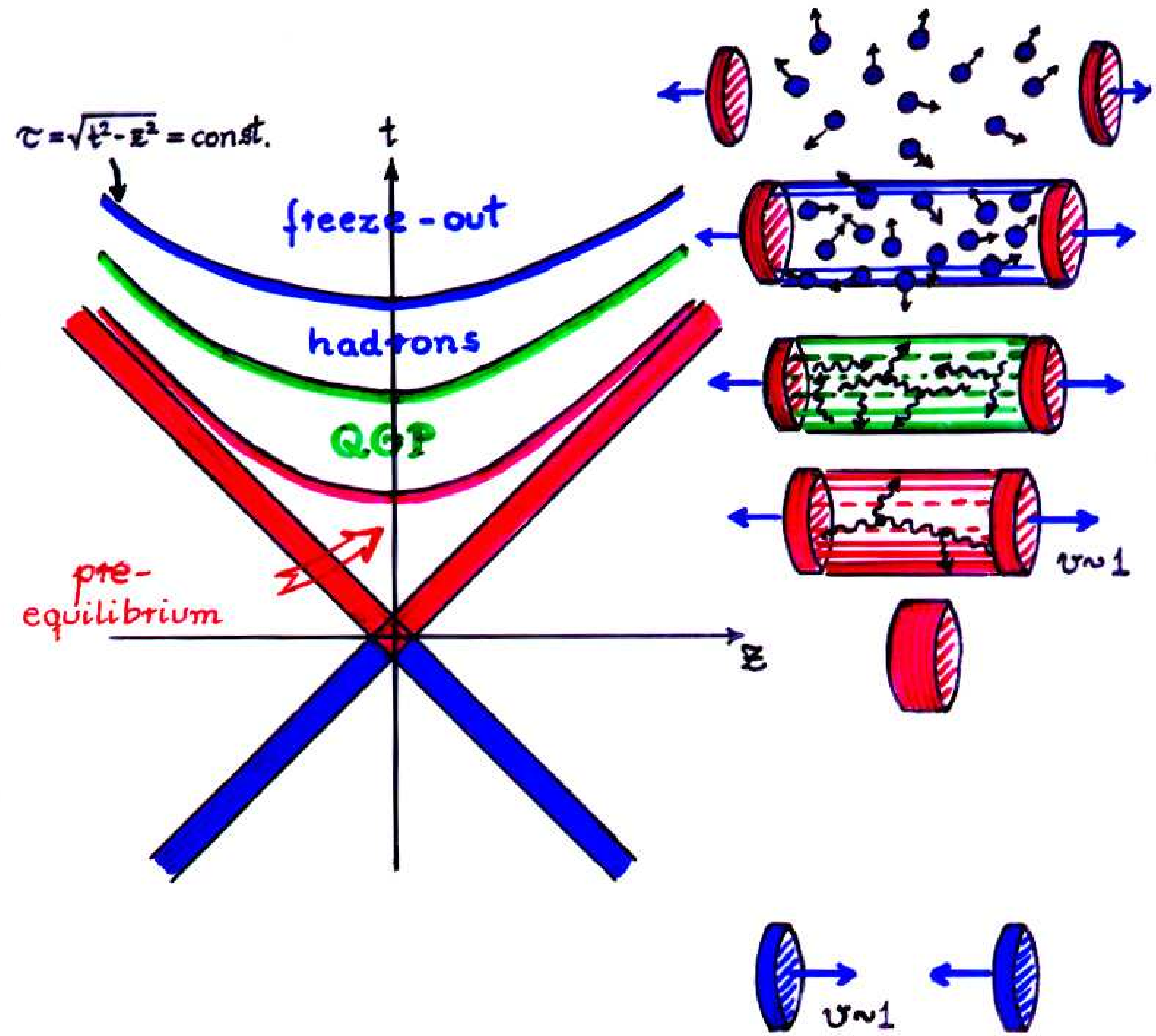
Entropy in ideal fluid

is conserved!

$$\partial_\mu S^\mu = 0$$

where $S^\mu = su^\mu$**.**

The space-time picture:



Usefulness of hydro?

- Initial state: unknown
- Equation of state: unknown
- Transport coefficients: unknown
- Freeze-out: unknown

$\Rightarrow$ Predictive power?

– *"Hydro doesn't know where to start nor where to end"* (M. Prakash)

Usefulness of hydro?

- **Initial state:** **unknown**
- **Equation of state:** **want to study**
- **Transport coefficients:** **want to study**
- **Freeze-out:** **unknown**

$\Rightarrow$ **Predictive power?**

$\Longrightarrow$ **Need More Constraints!**

"Hydrodynamical method"

1. Use another model to fix unknowns (and add new assumptions. . .)
   - initial: color glass condensate or pQCD+saturation
   - initial and/or final: hadronic cascade
   - etc.

2. Use data to fix parameters:

Principle		Example @ RHIC
• use one set of data	$\Longleftrightarrow$	$\left.\frac{\mathrm{d}N}{\mathrm{d}y\, p_T\, \mathrm{d}p_T}\right\|_{b=0}$ and $\frac{\mathrm{d}N}{\mathrm{d}y}(b)$
• fix parameters to fit it	$\Longleftrightarrow$	$\begin{cases} \epsilon_{0,\mathrm{max}} = 29.6\,\mathrm{GeV/fm}^3 \\ \tau_0 = 0.6\,\mathrm{fm}/c \\ T_{\mathrm{fo}} = 130\,\mathrm{MeV} \end{cases}$
• predict another set of data	$\Longleftrightarrow$	HBT, photons & dileptons, elliptic flow. . .

Equations of motion

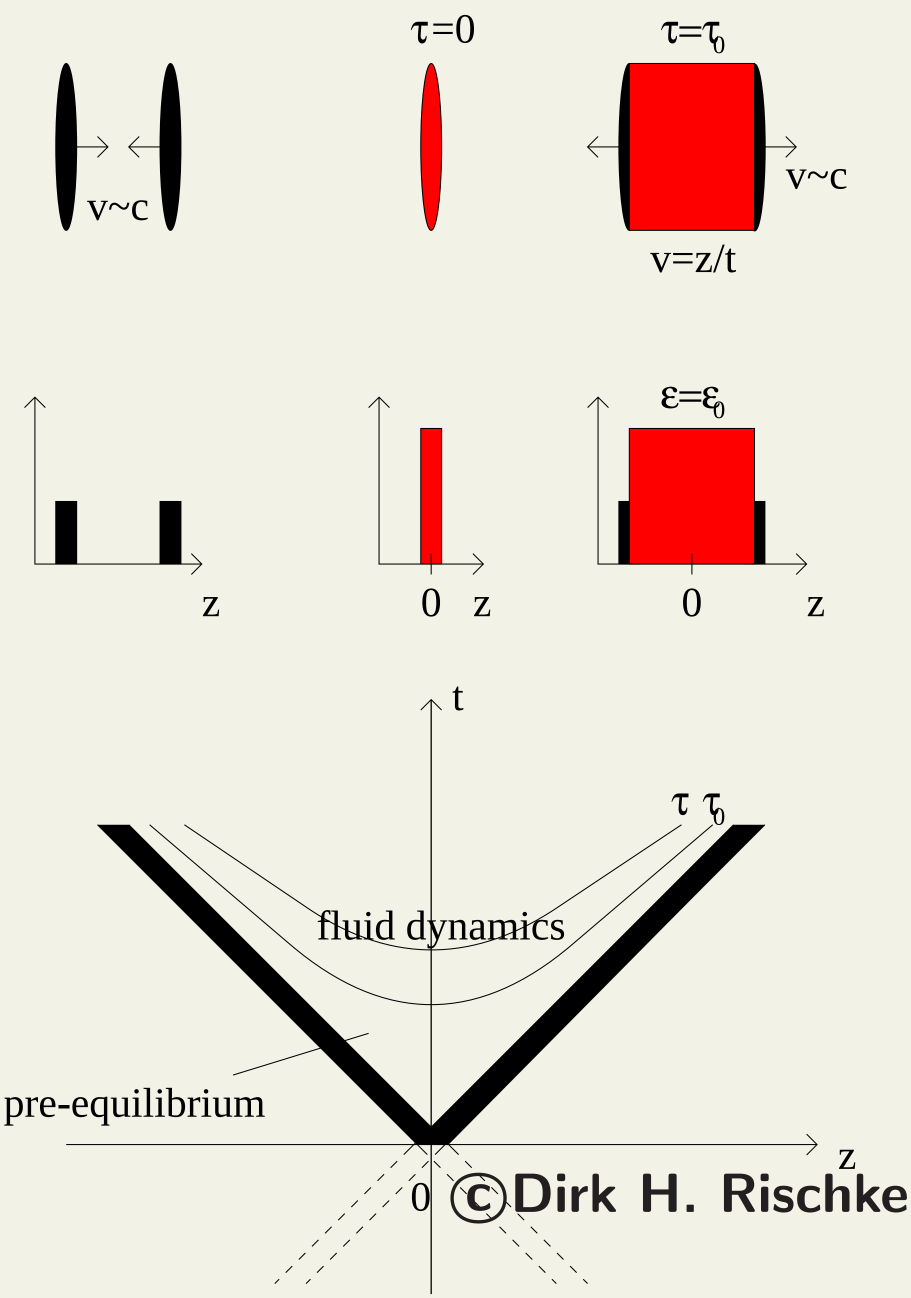
Conservation laws lead to the equations of motion for relativistic fluid:

$$\begin{aligned} Dn &= -n\partial_\mu u^\mu \\ D\epsilon &= -(\epsilon + P)\partial_\mu u^\nu \\ (\epsilon + P)Du^\mu &= \nabla^\mu P, \end{aligned}$$

where

$$D = u^\mu \partial_\mu \qquad \text{and} \qquad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu .$$

Bjorken hydrodynamics

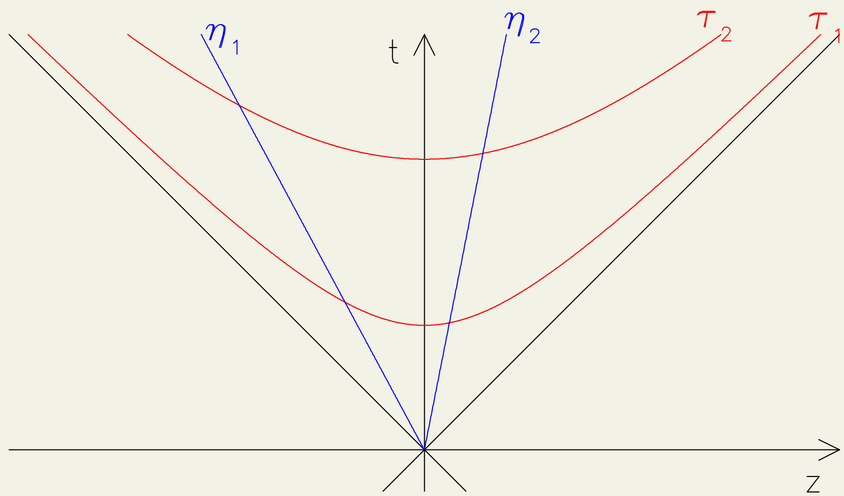


- **At very large energies, $\gamma \to \infty$ and "Landau thickness" $\to 0$**

- **Lack of longitudinal scale**
  $\Rightarrow$ **scaling flow**

$$v = \frac{z}{t}$$

- **Practical coordinates to describe scaling flow expansion are**



  - **Longitudinal proper time τ:**

$$\tau \equiv \sqrt{t^2 - z^2} \quad \Leftrightarrow \quad t = \tau \cosh \eta$$

  - **Space-time rapidity η:**

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z} \quad \Leftrightarrow \quad z = \tau \sinh \eta$$

- **Scaling flow $v = z/t \Rightarrow$ fluid flow rapidity $y = \eta$:**

$$y = \frac{1}{2}\ln\frac{1+v}{1-v} = \frac{1}{2}\ln\frac{1+z/t}{1-z/t} = \eta$$

- **Ignore transverse expansion:**
  Hydrodynamic equations turn out to be particularly simple:

$$\left.\frac{\partial\epsilon}{\partial\tau}\right|_\eta = -\frac{\epsilon+P}{\tau} \qquad (1)$$

$$\left.\frac{\partial P}{\partial\eta}\right|_\tau = 0 \qquad (2)$$

$$\left.\frac{\partial n}{\partial\tau}\right|_\eta = -\frac{n}{\tau} \qquad (3)$$

- **Eq. (2) $\Rightarrow$**
  - **No force between fluid elements with different η!**
  - **$P = P(\tau)$, no η-dependence!**

- **Eq. (2) + thermodynamics:**

$$0 = \frac{\partial P}{\partial \eta}\bigg|_\tau = s\,\frac{\partial T}{\partial \eta}\bigg|_\tau + n\,\frac{\partial \mu}{\partial \eta}\bigg|_\tau$$

If $n = 0$, $T = T(\tau) \Rightarrow T =$ const. on $\tau =$ const. surface.

- **In general T and ϵ not constant on $\tau =$ const. surface, but usually they are assumed to be**
  $\Rightarrow$ boost invariance: the system looks the same in all reference frames!

$$\epsilon = \epsilon(\tau), \qquad n = n(\tau)$$

- **Note that still**

$$\frac{\partial}{\partial \eta} T^{\mu\nu} \neq 0 \neq \frac{\partial}{\partial \eta} u^\mu$$

Vector and tensor quantities at finite η Lorentz boosted from values at $\eta = 0$

- **Thermodynamics:**

$$\begin{aligned} \mathrm{d}\epsilon &= T\,\mathrm{d}s + \mu\,\mathrm{d}n \\ \epsilon + P &= Ts + \mu n \end{aligned}$$

- **Eq. (1):**

$$\frac{\partial \epsilon}{\partial \tau} + \frac{\epsilon + P}{\tau} = 0$$

$$\Rightarrow \quad T\frac{\partial s}{\partial \tau} + \mu\frac{\partial n}{\partial \tau} + T\frac{s}{\tau} + \mu\frac{n}{\tau} = 0$$

(Eq. (3)) $\Rightarrow \quad \frac{\partial s}{\partial \tau} + \frac{s}{\tau} = 0$

$$\Rightarrow \quad s(\tau) = s_0\frac{\tau_0}{\tau}$$

$\Rightarrow \quad s\tau =$ **const.** $\Rightarrow \mathrm{d}S/\mathrm{d}\eta =$ **const**

independent of the equation of state!

- **Time evolution of baryon density:**

$$\textbf{Eq. (3)} \Rightarrow n(\tau) = n_0 \frac{\tau_0}{\tau} \Rightarrow \mathrm{d}N/\mathrm{d}\eta = \textbf{const}$$

also independent of the EoS.

- **Time evolution of energy density and temperature depend on the EoS.**

- **Assume ideal gas equation of state,** $P = \frac{1}{3}\epsilon$**,** $\epsilon \propto T^4$**:**

$$\begin{aligned} \textbf{Eq. (1)} \quad &\Rightarrow \quad \frac{\partial \epsilon}{\partial \tau} + \frac{4\epsilon}{3\tau} = 0 \\ &\Rightarrow \quad \epsilon(\tau) = \epsilon_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{4}{3}} \\ &\Rightarrow \quad T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}} \end{aligned}$$

- **Note:** τ_0**: initial time, thermalization time**

Application: Initial energy density estimate

1. **"Bjorken estimate"**

- **At $y = 0$, $E_T = E$**
- **Thus measuring**

$$\left.\frac{\mathrm{d}E_T}{\mathrm{d}y}\right|_{y=0}$$

gives total energy at $y = 0$.

- **Estimate the initial volume:**

$$V = A\,\Delta z = \pi R^2 \tau_0\,\Delta\eta$$

- **Thus**

$$\epsilon = \frac{1}{\pi R^2}\frac{E}{\tau_0 \Delta\eta} = \frac{1}{\pi R^2 \tau_0}\frac{\mathrm{d}E_T}{\mathrm{d}y}$$

- **Take $R = 6.3$ fm and $\tau_0 = 1$ fm/c:**

$$\textbf{@ SPS:}\quad \frac{\mathrm{d}E_T}{\mathrm{d}y} \approx 400\ \textbf{GeV} \quad \rightarrow \quad \epsilon \sim 3.2\ \textbf{GeV/fm}^3$$

$$\textbf{@ RHIC:}\quad \frac{\mathrm{d}E_T}{\mathrm{d}y} \approx 620\ \textbf{GeV} \quad \rightarrow \quad \epsilon \sim 5.0\ \textbf{GeV/fm}^3$$

- **Note that in this approach**

$$\epsilon(\tau) = \epsilon_0 \frac{\tau_0}{\tau}$$

No longitudinal work is done.

- **Pressure does work during expansion, $\mathrm{d}E = -P\mathrm{d}t$:**

$$\frac{\partial \epsilon}{\partial \tau} = \frac{\epsilon + P}{\tau} \Rightarrow \mathrm{d}(\epsilon\tau) = P\mathrm{d}\tau$$

Highly nontrivial

2. Entropy conservation

- **Assume ideal gas of massless particles:**

$$
\begin{aligned}
s &= 4n \Rightarrow \frac{\mathrm{d}S}{\mathrm{d}y} = 4\frac{\mathrm{d}N}{\mathrm{d}y} \\
s &= \frac{4g}{\pi^2} T^3 \\
\epsilon &= \frac{3g}{\pi^2} T^4
\end{aligned}
$$

- **With $s\tau =$ const. these give**

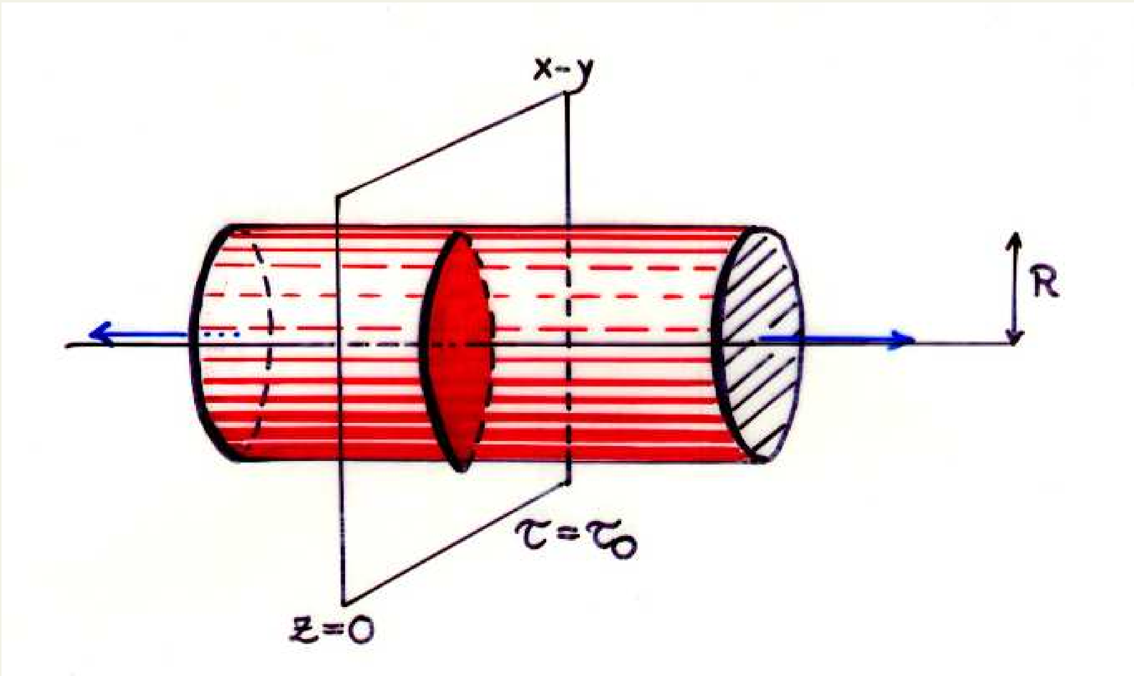
$$
\epsilon_0 = \frac{3}{\pi^{\frac{2}{3}} R^{\frac{8}{3}} \tau_0^{\frac{4}{3}} g^{\frac{1}{3}}} \left(\frac{\mathrm{d}N}{\mathrm{d}y}\right)^{\frac{4}{3}}
$$

@ RHIC: $\frac{\mathrm{d}N}{\mathrm{d}y} \approx 1000$

$g = 40$ **(2 flavours + gluons)**

$\Rightarrow \epsilon_0 \approx 6.0$ **GeV/fm^3**

Transverse expansion and flow



- Transverse expansion will set in latest at $\tau = R/c_s \approx 10$ fm
- Lifetimes in one dimensional expansion ~ 30 fm
- One dimensional expansion an oversimplification

- 2+1D: longitudinal Bjorken, transverse expansion solved numerically
- 3+1D: expansion in all directions solved numerically

- **Define speed of sound c_s:**

$$c_s^2 = \left.\frac{\partial P}{\partial \epsilon}\right|_{s/n_b}$$

- **large $c_s \Rightarrow$ "stiff EoS"**

- **small $c_s \Rightarrow$ "soft EoS"**

- **For baryon-free matter in rest frame**

$$(\epsilon + P)Du^\mu = \nabla^\mu P \qquad \Longleftrightarrow \qquad \frac{\partial}{\partial \tau} u_\mu = -\frac{c_s^2}{s}\partial_\mu s$$

$\Rightarrow$ **The stiffer the EoS, the larger the acceleration**

Initial conditions

- Initial time from early thermalization argument (+finetuning. . .)
- Total entropy to fit the multiplicity
- Density distribution?
- Multiplicity is proportional to the number of participants
- Glauber model: number of participants/binary collisions

$$N_{part}(b) = \int \mathrm{d}x\mathrm{d}y\, T_A(x + b/2, y)[\ldots$$

where

$$T_A(x, y) = \int_{-\infty}^{\infty} \mathrm{d}z\, \rho(x, y, z) \qquad \text{and} \qquad \rho(x, y, z) = \frac{\rho_0}{1 + e^{\frac{r-R_0}{a}}}$$

are nuclear thickness function and nuclear density distribution

- **"Differential Optical Glauber:"**
  Number of participants per unit area in transverse plane:

$$\begin{aligned} n_{\rm WN}(x,y;b) &= T_A(x+b/2,y)\left[1-\left(1-\frac{\sigma}{B}T_B(x-b/2,y)\right)^B\right] \\ &+ T_B(x-b/2,y)\left[1-\left(1-\frac{\sigma}{A}T_A(x-b/2,y)\right)^A\right] \end{aligned}$$

Number of binary collisions per unit area

$$n_{\rm BC}(x,y;b) = \sigma_{pp}T_A(x+b/2,y)T_B(x-b/2,y)$$

- **MC-Glauber:**
  - **sample $\rho(x,y,z)$ to get the positions of nucleons in 2 nuclei**
  - **count # of nucleons closer than $\sqrt{\sigma_{\rm pp}/\pi}$ in the collision**
  - **this gives $n_{\rm WN}$ and $n_{\rm BC}$**
  - **repeat to get enough statistics**

Various flavors of Glauber

1. **eWN: energy density** $\epsilon(x, y; b) \propto n_{\mathrm{WN}}$

2. **eBC: energy density** $\epsilon(x, y; b) \propto n_{\mathrm{BC}}$

3. **sWN: entropy density** $s(x, y; b) \propto n_{\mathrm{WN}}$

4. **sBC: entropy density** $s(x, y; b) \propto n_{\mathrm{BC}}$

5. **any combination of these!**

- **multiplicity as function of centrality**

$\Longrightarrow \epsilon(x, y; b) = \kappa \cdot \epsilon_{\mathrm{WN}} + (1 - \kappa) \cdot \epsilon_{\mathrm{BC}}$

or $s(x, y; b) = \lambda \cdot s_{\mathrm{WN}} + (1 - \lambda) \cdot s_{\mathrm{BC}}$

Equation of state

- **Final state includes π's, K's, nucleons. . .**
  $\Rightarrow$ EoS of interacting hadron gas
  $\Rightarrow$ well approximated by non-interacting gas of hadrons and resonances

$$P(T) = \sum_i \int \mathrm{d}^3p \, \frac{p^2}{3E} \, f(p,T)$$

- **Plasma EoS (=massless parton gas) with proper statistics and $\mu_B \neq 0$:**

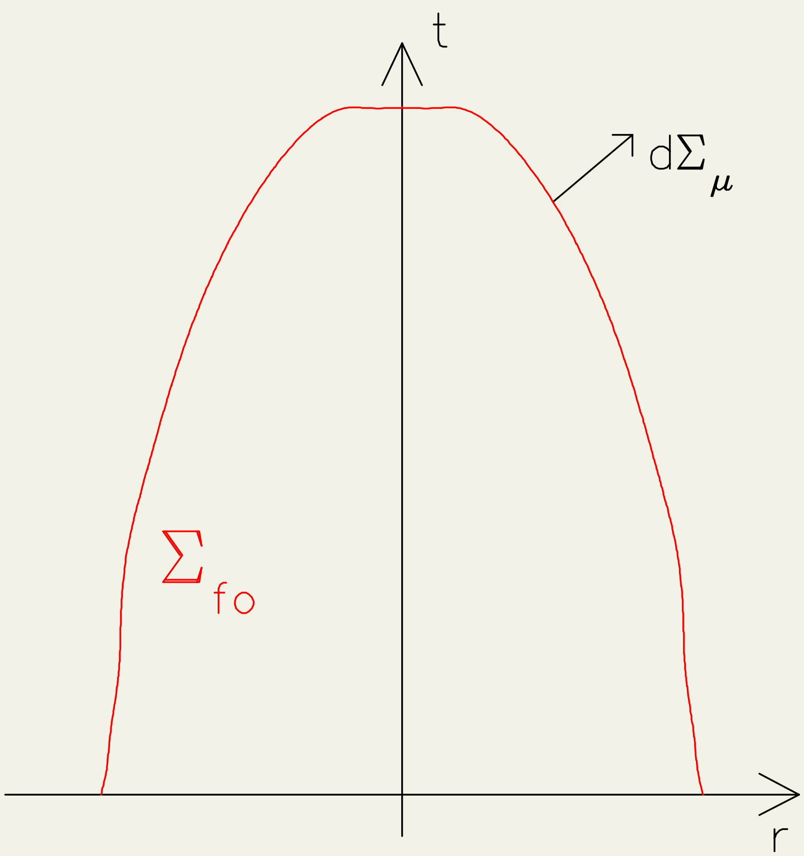
$$P(T,\mu) = \frac{(32+21N_f)\pi^2}{180}T^4 + \frac{1}{9}\mu_B^2 T^2 + \frac{1}{192\pi^2}\mu_B^4 - B$$

  $\Rightarrow$ First order phase transition by Maxwell construction

- **OR parametrized lattice result (only at $\mu_B = 0$):**
  $\Rightarrow$ match your favourite smoothly to HRG

When to end?

- Particles are observed, not fluid
- How and when to convert fluid to particles?
- i.e. how far is hydro valid?



- Kinetic equilibrium requires scattering rate $\gg$ expansion rate
- Scattering rate $\tau_{\rm sc}^{-1} \sim \sigma n \propto \sigma T^3$
- Expansion rate $\theta = \partial_\mu u^\mu$
- Fluid description breaks down when $\tau_{\rm sc}^{-1} \approx \theta$

$\rightarrow$ momentum distributions freeze-out

- $\tau_{\rm sc}^{-1} \propto T^3 \rightarrow$ rapid transition to free streaming
- Approximation: decoupling takes place on constant temperature hypersurface $\Sigma_{\rm fo}$, at $T = T_{\rm fo}$

Cooper-Frye

- Number of particles emitted = Number of particles crossing $\Sigma_{\rm fo}$

$$\Rightarrow \quad N = \int_{\Sigma_{\rm fo}} \mathrm{d}\Sigma_\mu \, N^\mu$$

- Frozen-out particles do not interact anymore: kinetic theory

$$\Rightarrow \quad N^\mu \quad = \quad \int \frac{\mathrm{d}^3\mathbf{p}}{E} p^\mu f(x, p \cdot u)$$

$$\Rightarrow \quad N \quad = \quad \int \frac{\mathrm{d}^3\mathbf{p}}{E} \int_{\Sigma_{\rm fo}} \mathrm{d}\Sigma_\mu \, p^\mu f(x, p \cdot u)$$

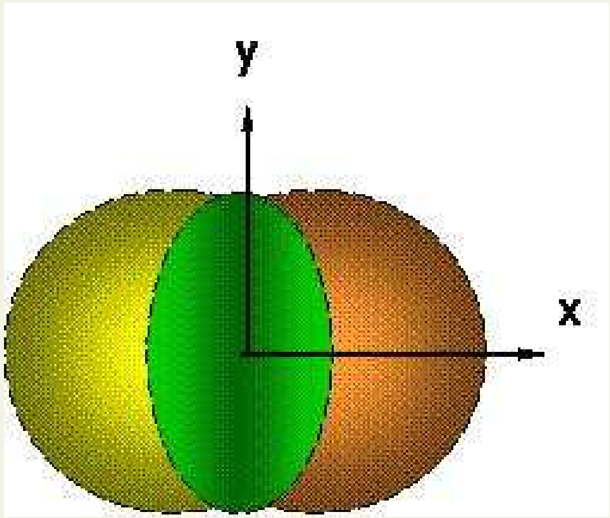
- Invariant single inclusive momentum spectrum: (Cooper-Frye formula)

$$\boxed{E\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}^3} = \int_{\Sigma_{\rm fo}} \mathrm{d}\Sigma_\mu \, p^\mu f(x, p \cdot u)}$$

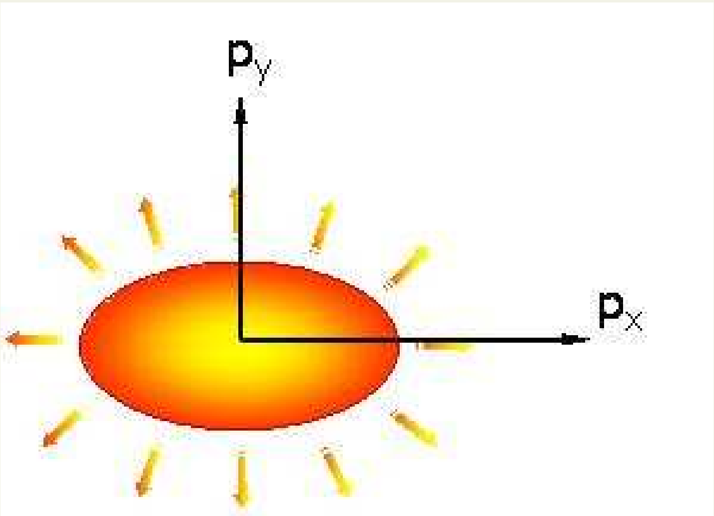
Cooper and Frye, PRD 10, 186 (1974)

Elliptic flow v_2

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy



$\Rightarrow$



- **Anisotropy in coordinate space + rescattering**

$\Rightarrow$ **Anisotropy in momentum space**

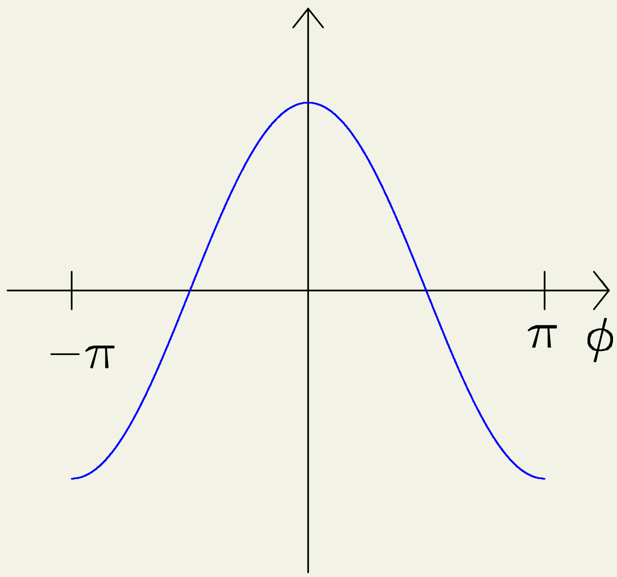
$$\frac{\partial}{\partial \tau} u_x = -\frac{c_s^2}{s}\frac{\partial}{\partial x} s \qquad \text{and} \qquad \frac{\partial}{\partial \tau} u_y = -\frac{c_s^2}{s}\frac{\partial}{\partial y} s$$

Elliptic flow v_2

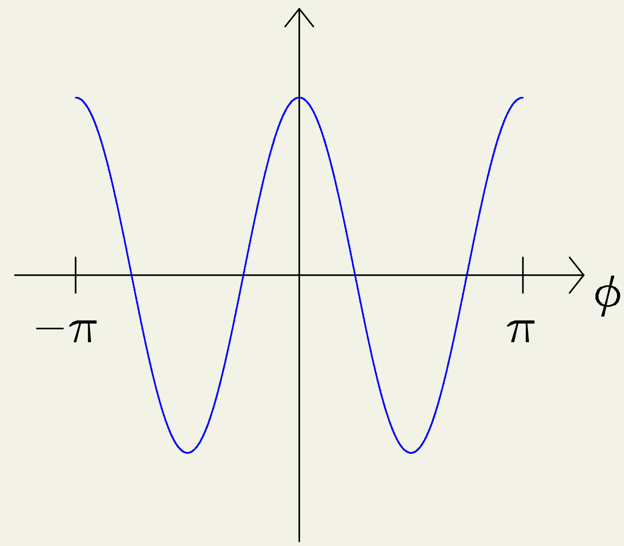
- **Fourier expansion of momentum distribution:**

$$\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T\, \mathrm{d}\phi} = \frac{1}{2\pi}\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T}(1 + 2v_1(y,p_T)\cos\phi + 2v_2(y,p_T)\cos 2\phi + \cdots)$$

$\mathbf{v_1}$: **Directed flow: preferred direction**

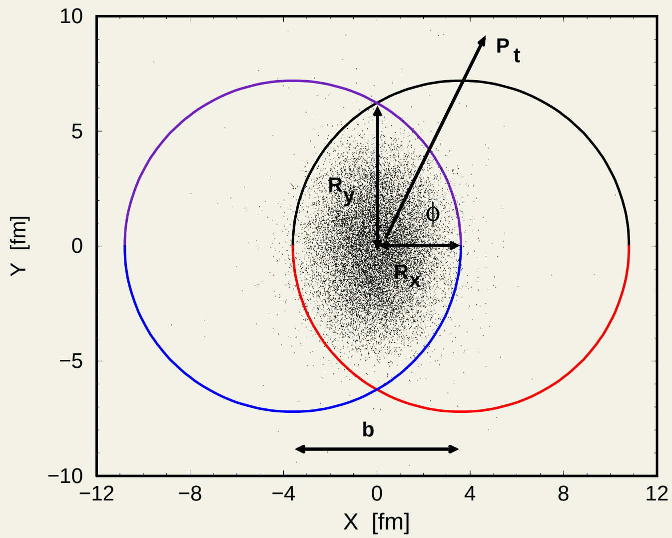


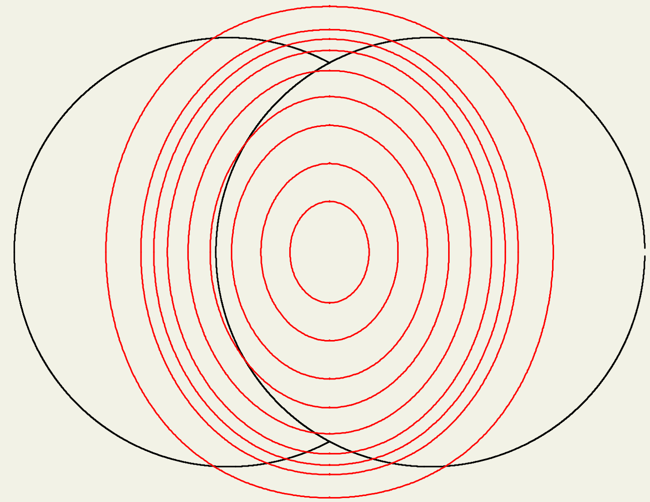
$\mathbf{v_2}$: **Elliptic flow: preferred plane**



sensitive to speed of sound $c_s^2 = \partial p/\partial e$ **and shear viscosity** η

Measures of anisotropy





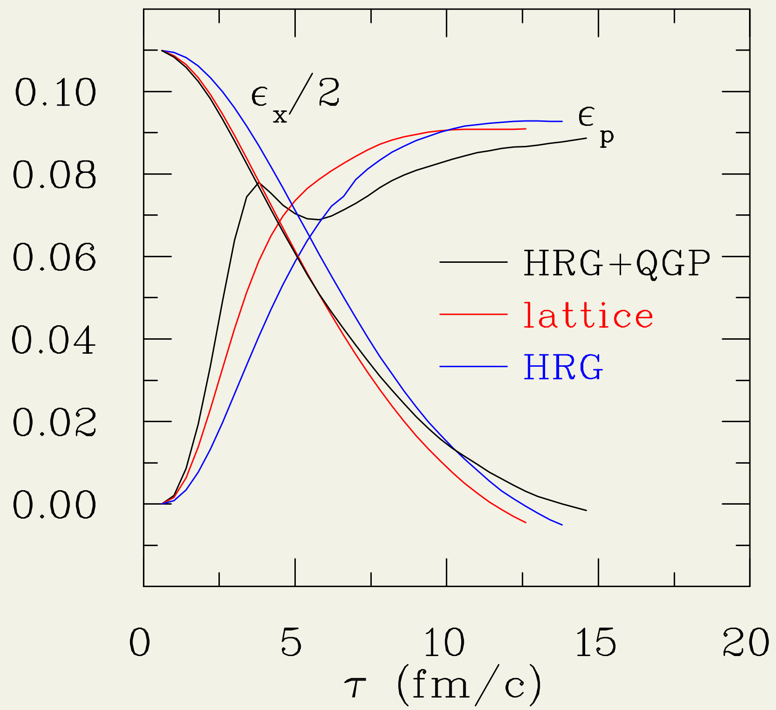
- **Spatial eccentricity**

$$\epsilon_x = \frac{\langle\langle y^2 - x^2 \rangle\rangle}{\langle\langle y^2 + x^2 \rangle\rangle} = \frac{\int \mathrm{d}x\, \mathrm{d}y\, \epsilon \cdot (y^2 - x^2)}{\int \mathrm{d}x\, \mathrm{d}y\, \epsilon \cdot (y^2 - x^2)}$$

- **Momentum anisotropy**

$$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle} = \frac{\int \mathrm{d}x\, \mathrm{d}y\, T^{xx} - T^{yy}}{\int \mathrm{d}x\, \mathrm{d}y\, T^{xx} - T^{yy}}$$

- **Au+Au @ RHIC, $b = 6$ fm:**



- **ϵ_x decreases during the evolution $\Rightarrow$ elliptic flow is self-quenching**

- **Most of ϵ_p is built up early in the evolution**

v_2

- **Not only collective but also thermal motion**
- **Elliptic flow v_2 a.k.a. p_T-averaged v_2:**

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} = \frac{\int \mathrm{d}\phi\ \cos(2\phi) \frac{\mathrm{d}N}{\mathrm{d}y\, \mathrm{d}\phi}}{\mathrm{d}N/\mathrm{d}y}$$

- **p_T-differential v_2**

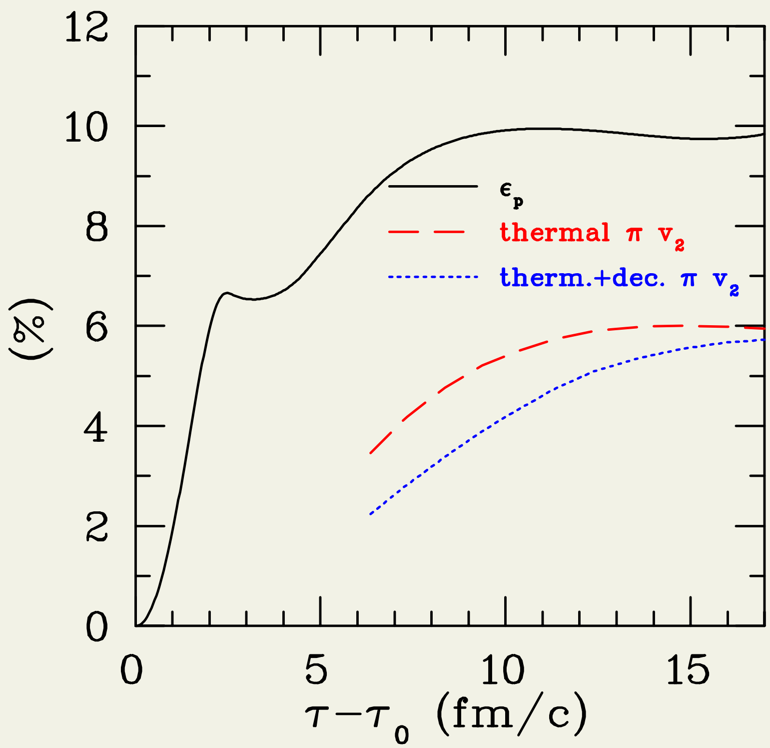
$$v_2(p_T) = \frac{\int \mathrm{d}\phi\ \cos(2\phi) \frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T\, \mathrm{d}\phi}}{\int \mathrm{d}\phi \frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T\, \mathrm{d}\phi}}$$

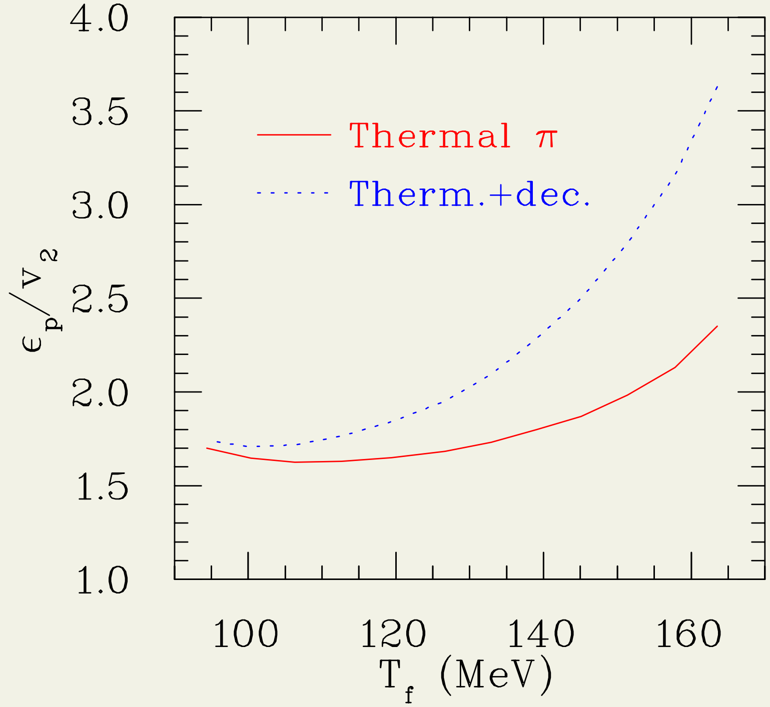
- **If $m_1 > m_2$, $v_2(m_1) > v_2(m_1)$, but $v_2(p_T, m_1) < v_2(p_T, m_2)$!**
- **No contradiction, since**

$$v_2 = \frac{\int \mathrm{d}p_T\, v_2(p_T) \frac{\mathrm{d}N}{\mathrm{d}p_T}}{\int \mathrm{d}p_T\, \frac{\mathrm{d}N}{\mathrm{d}p_T}}$$

ϵ_p vs. v_2

- **Au+Au @ RHIC, $b = 7$ fm:**

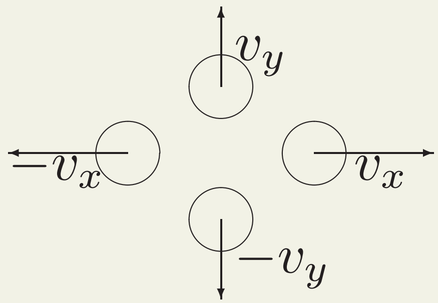




- **NO clear correspondence**
- **especially if one includes resonance decays**

Why $m_1 < m_2 \Rightarrow v_2(p_T, m_1) > v_2(p_T, m_2)$?

Simple source:



Each element has unit volume, decouples at the same time at the same temperature. Flow velocity in plane is larger than out of plane, $|v_x| > |v_y|$.

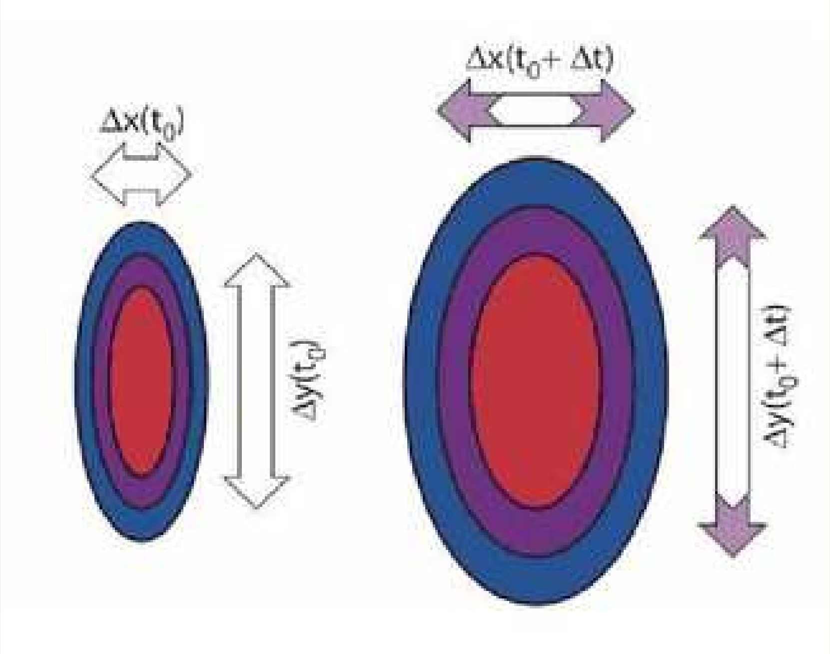
Boltzmann distribution and Cooper-Frye formula:

$$v_2(p_T) = \frac{\mathrm{I}_2\left(\frac{\gamma_x v_x p}{T}\right) - e^{\frac{E}{T}(\gamma_x - \gamma_y)}\mathrm{I}_2\left(\frac{\gamma_y v_y p}{T}\right)}{\mathrm{I}_0\left(\frac{\gamma_x v_x p}{T}\right) + e^{\frac{E}{T}(\gamma_x - \gamma_y)}\mathrm{I}_0\left(\frac{\gamma_y v_y p}{T}\right)} = \frac{C_1 - e^{\lambda\sqrt{m^2+p^2}}C_2}{C_3 + e^{\lambda\sqrt{m^2+p^2}}C_4}$$

mass increases, numerator decreases and denominator increases
$\rightarrow v_2$ decreases

Early thermalization?

- ϵ_p/ϵ_x almost independent of b, i.e. the initial value of ϵ_x
- Before thermalization, $\tau < \tau_0$ system expands to all directions, ϵ_x decreases

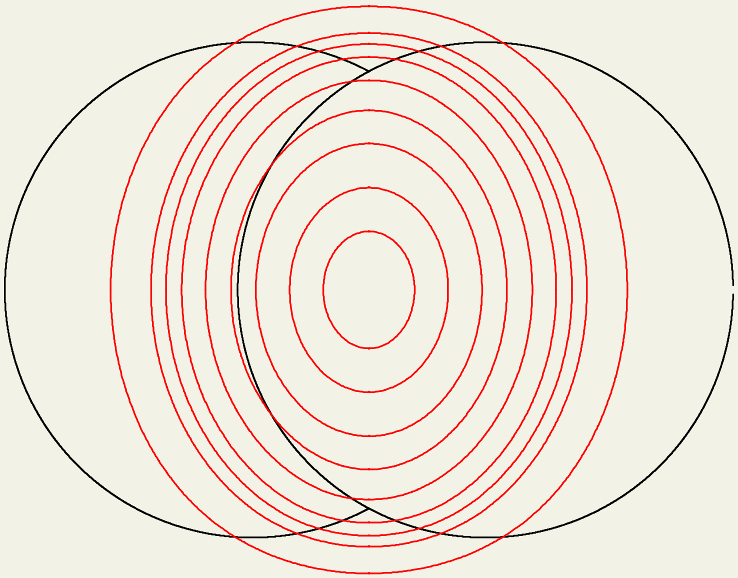


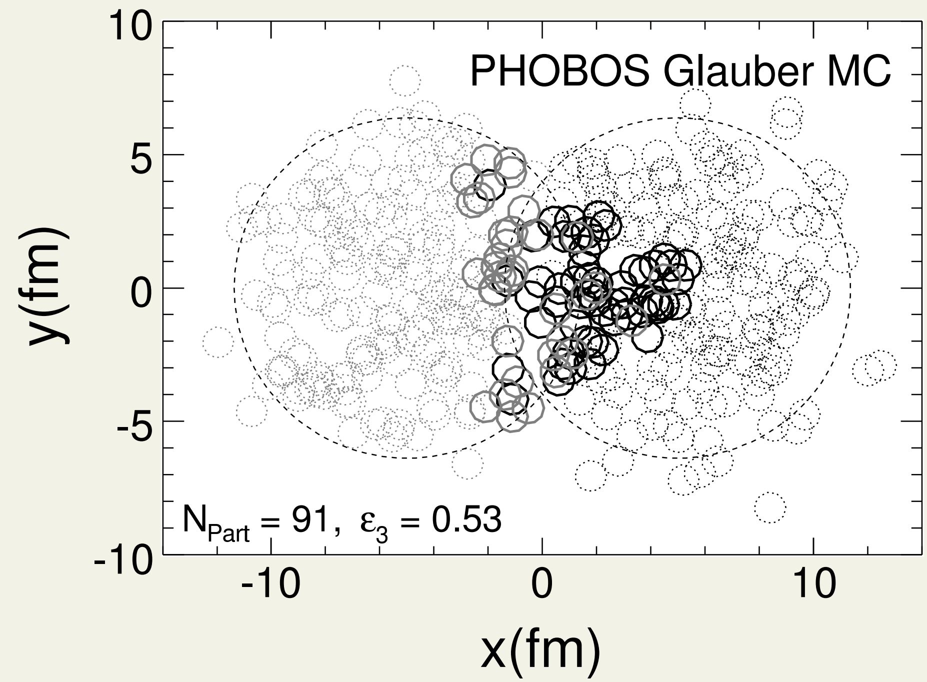
ⓒPeter F. Kolb

$\Longrightarrow$ Hydrodynamical evolution must start early or final v_2 is too small

- We do not know if v_2 could build up before thermalization...

event-by-event

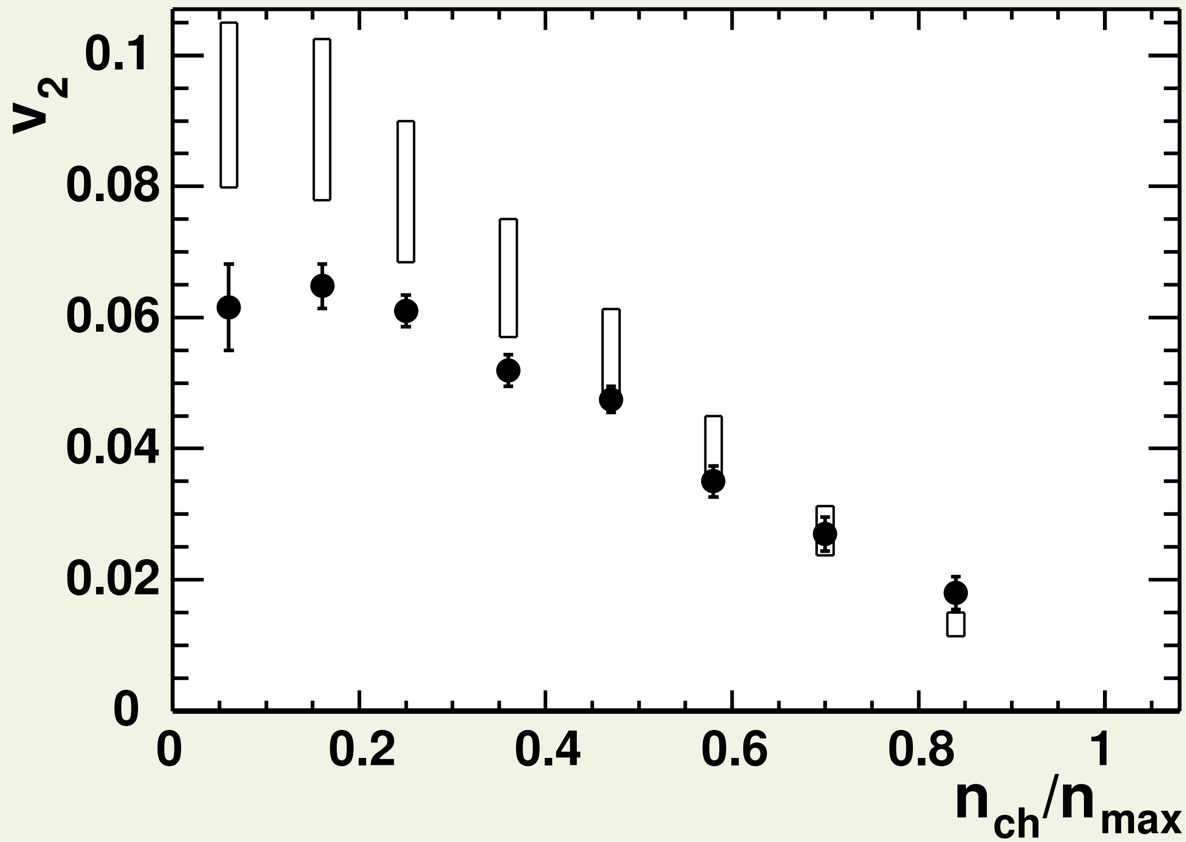




- **shape fluctuates event-by-event**
- **all coefficients v_n finite**

Success of ideal hydrodynamics

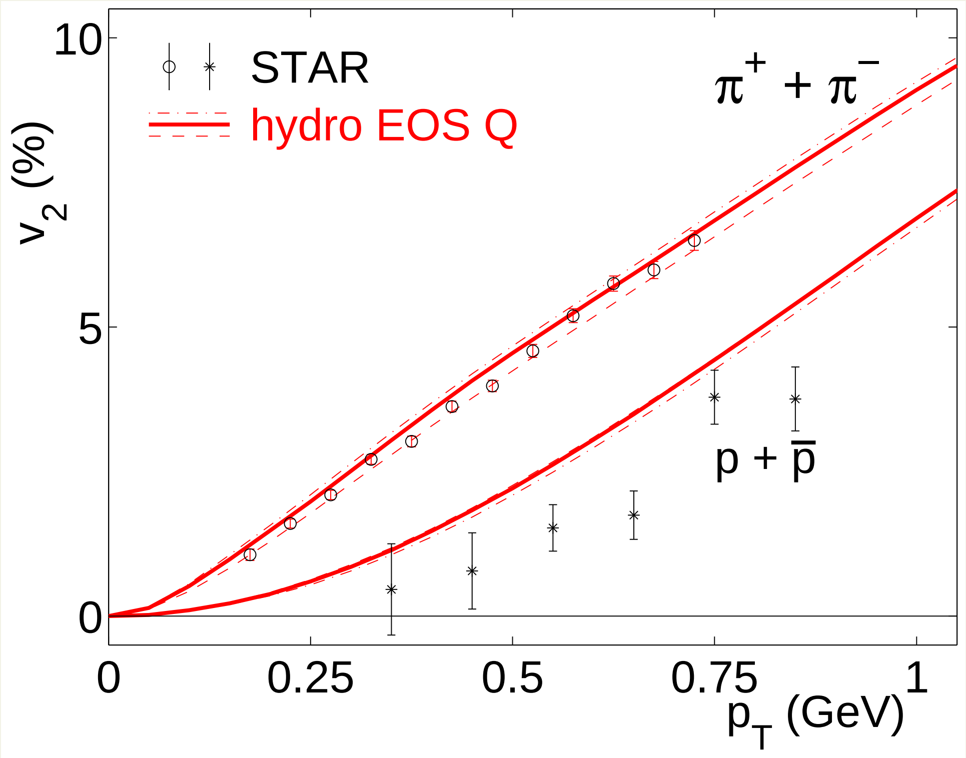
- p_T-**averaged** v_2 **of charged hadrons:**

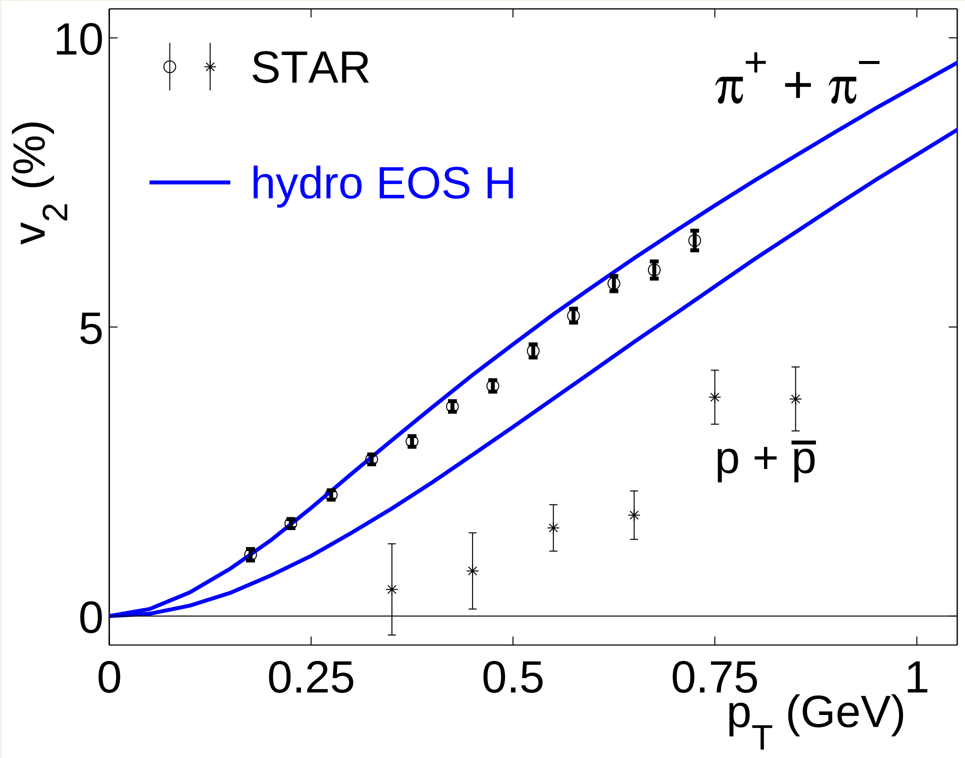


- **works beautifully in central and semi-central collisions**
- **but why is** $v_{2,\mathrm{obs}} > v_{2,hydro}$ **in most central collisions?**

Success of ideal hydrodynamics

Kolb, Heinz, Huovinen et al ('01) minbias Au+Au at RHIC

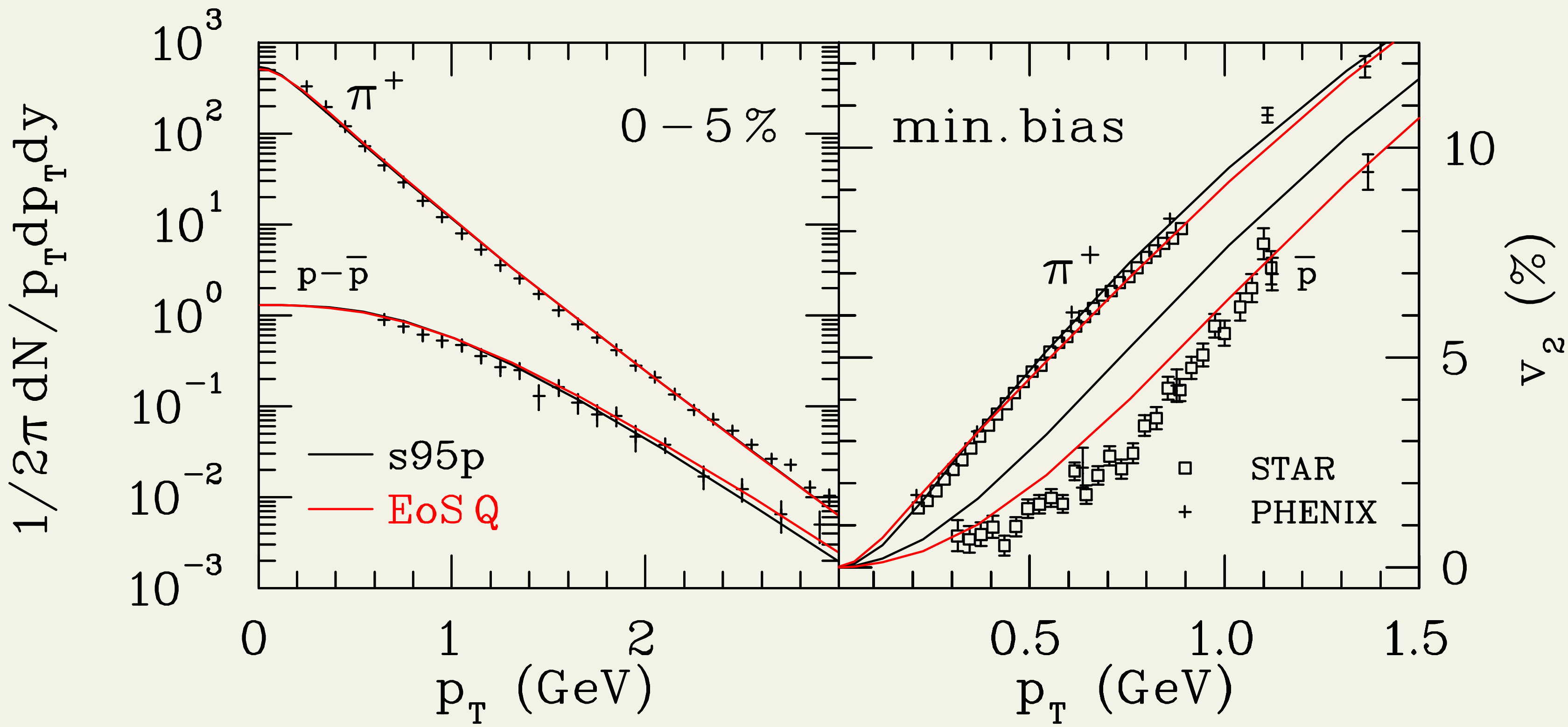




not perfect agreement but plasma EoS favored

Lattice EoS

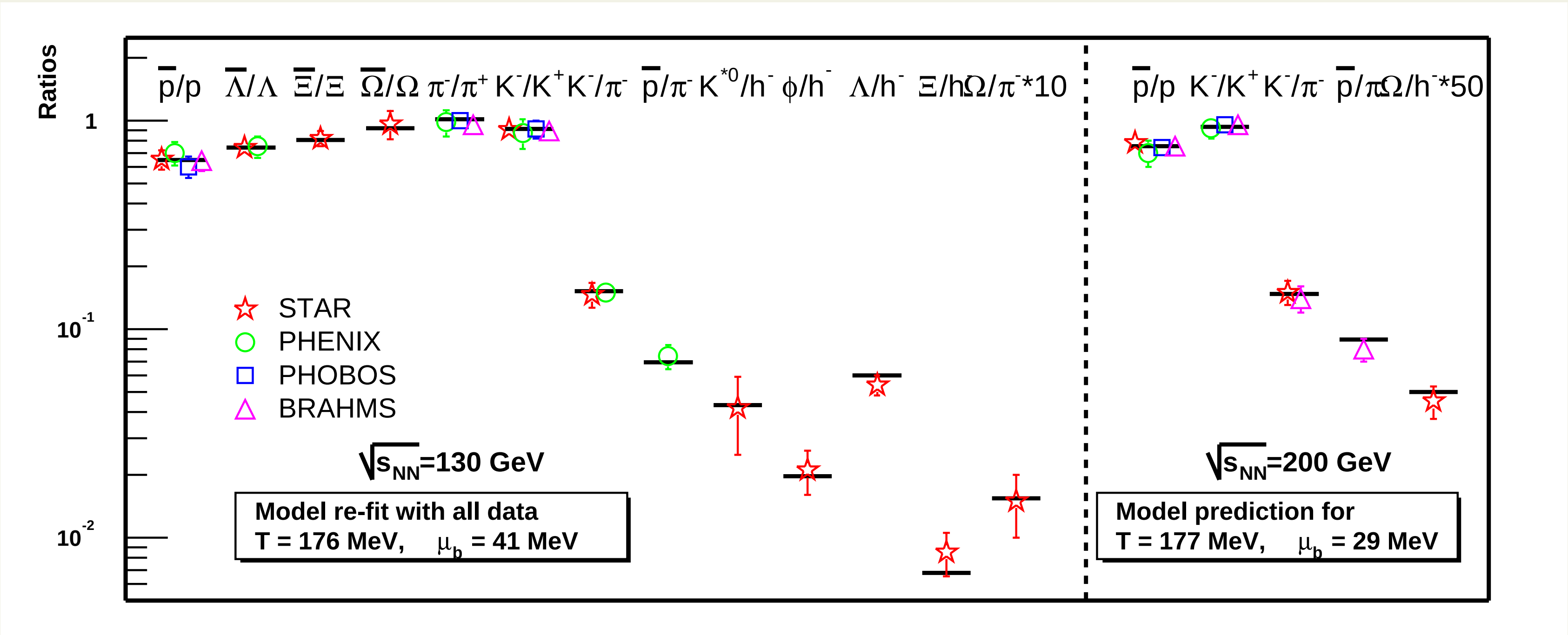
- **ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV**
- **chemical equilibrium**



- **s95p:** $T_{dec} = 140$ **MeV**
- **EoS Q: first order phase transition at $T_c = 170$ MeV, $T_{dec} = 125$ MeV**

Thermal models

- Hadronic phase: ideal gas of massive hadrons and resonances
- in chemical equilibrium

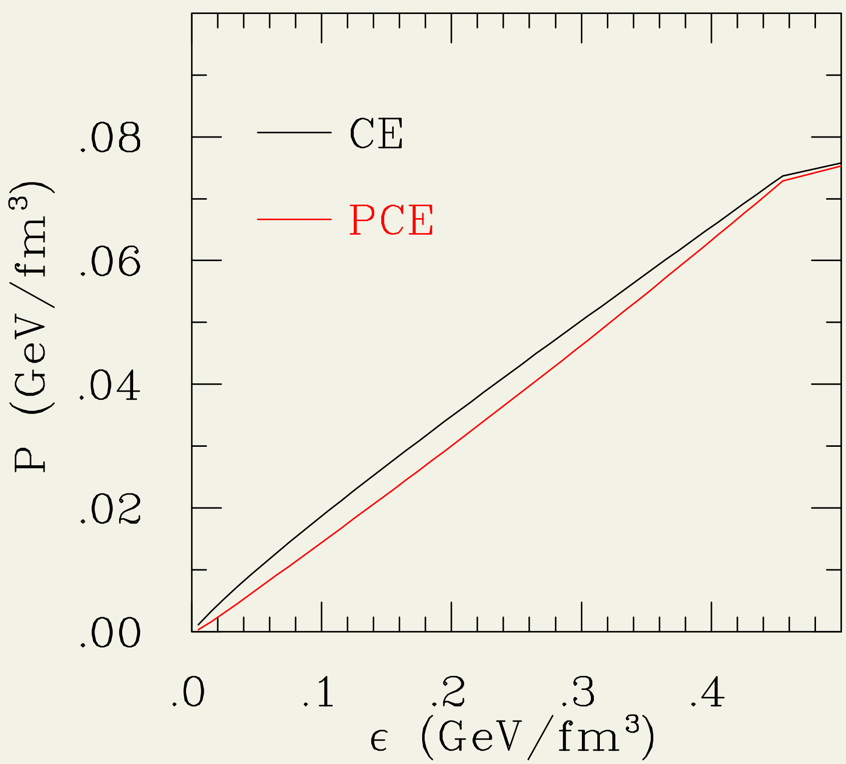


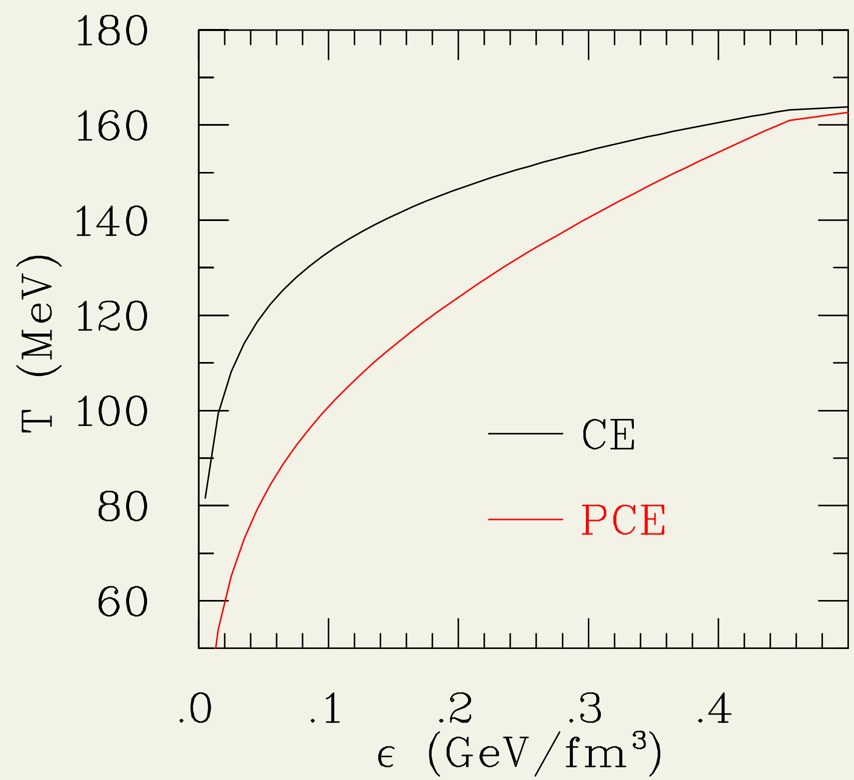
- **Particle ratios** $\Longleftrightarrow$ $T \approx 160$–170 **MeV temperature**
- **Evolution to** $T \approx 100$–120 **MeV temperature**

$\Rightarrow$ **In hydro particle ratios become wrong**

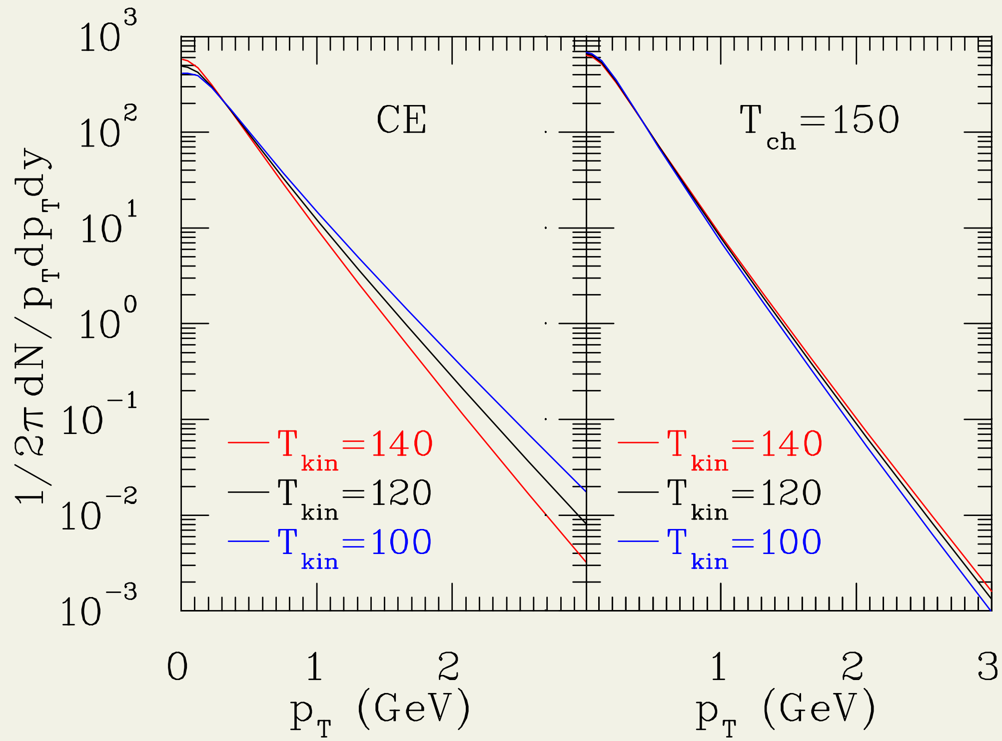
Chemical non-equilibrium

- **Treat number of pions, kaons etc. as conserved quantum numbers below T_{ch} (Bebie et al, Nucl.Phys.B378:95-130,1992)**
- $P = P(\epsilon, n_b)$ **changes very little, but** $T = T(\epsilon, n_b)$ **changes. . .**





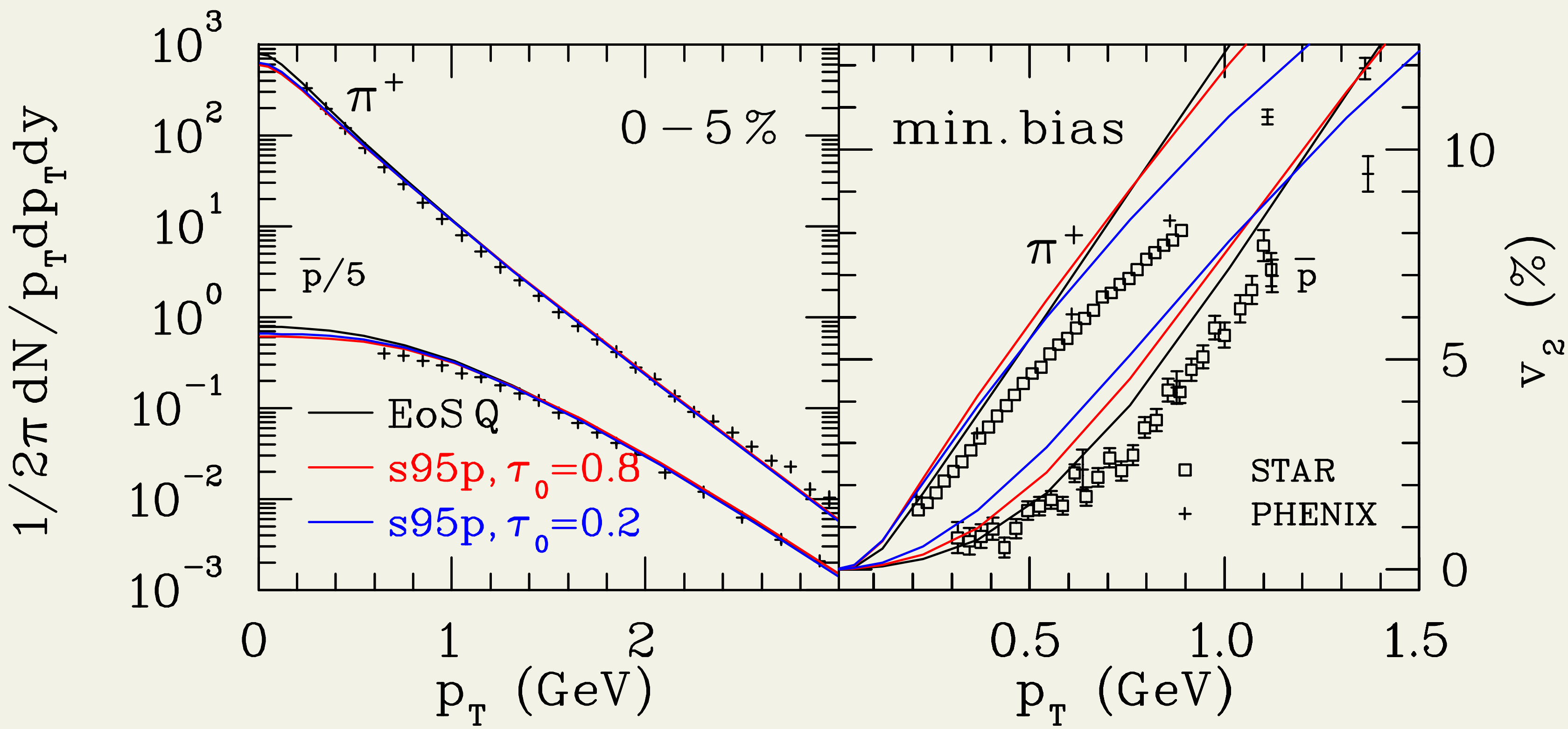
Effect of T_{kin} on pions



- **Longitudinal expansion does work ($p\,\mathrm{d}V$) $\Rightarrow$ $\frac{\mathrm{d}E_T}{\mathrm{d}y}$ decreases**
- **If particle # is conserved, $\langle p_T \rangle$ decreases**
- **In chemical equilibrium mass energy of baryon-antibaryon pairs is converted to kinetic energy $\Rightarrow$ $\langle p_T \rangle$ increases!**

More realistic EoS

- **ideal hydro, Au+Au at** $\sqrt{s_{NN}} = 200$ **GeV**
- $T_{\text{chem}} = 150$ **MeV**



- **EoS Q:** $T_{dec} = 120$ **MeV,** $s_{\text{ini}} \propto N_{bin}$ $\tau_0 = 0.2$ **fm/**c
- **s95p,** $\tau_0 = 0.8$**:** $T_{dec} = 120$ **MeV,** $s_{\text{ini}} \propto N_{bin}$**,** $\tau_0 = 0.8$ **fm/**c
- **s95p,** $\tau_0 = 0.2$**:** $T_{dec} = 120$ **MeV,** $s_{\text{ini}} \propto N_{bin} + N_{part}$**,** $\tau_0 = 0.2$ **fm/**c

Summary

- Hydrodynamics is a useful tool to model collision dynamics
  - approximation at its best
  - but it can reproduce (some of) the data
- we have observed hydrodynamical behaviour at RHIC and LHC