On the double-counting problem in Boltzmann equations and real-intermediate state subtraction

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Outline

Baryon asymmetry

$$\frac{n_B}{n_\gamma} = \begin{cases} 6.172 \pm 0.195 \times 10^{-10} & \text{(BBN)} \\ 6.108 \pm 0.060 \times 10^{-10} & \text{(CMB)} \end{cases}$$

Electroweak Baryogenesis

Electroweak phase transition creates bubbles. The wall front creates CP violating collisions at a semi-classical level: CP-asymmetry$\rightarrow$ Sphalerons $\rightarrow$ Baryon asymmetry. Requires a strong EW phase transition. Requires the sphalerons to not wash-out the asymmetry.

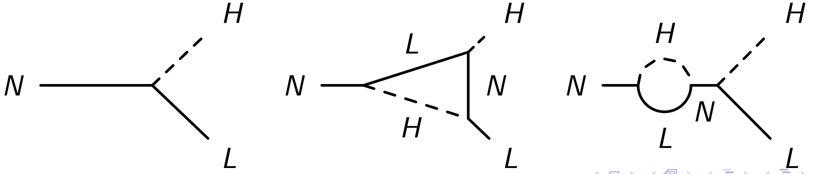
- Higgs not light enough to give a strong phase transition;
- Turn to beyond SM to tweak the phase transition.

Leptogenesis

Primordial heavy right-handed neutrinos decaying into SM in CP-violating processes (at loop order). Sphalerons convert CP-asymmetry to baryons followed by a $LH \to \bar{L}\bar{H}$ washout.

$$\mathcal{L} = \mathcal{L}_{SM} + \left(\frac{1}{2} m_{N_i}^2 N_i^2 + Y_{ij}^{\nu} L_i N_j H + h.c.\right) \qquad j = 1, 2, 3$$

Seesaw $m_\nu = -m_D m_N^{-1} m_D$, $m_D = Y\nu$. A wide range of mass is available for m_N.



First occurence

BARYON NUMBER GENERATION IN THE EARLY UNIVERSE*

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The generation of an excess of baryons over antibaryons in the very early universe due to *CP*- and *B*-violating interactions is described. The Boltzmann equation is used to perform detailed calculations of the time development of such an excess in several simple illustrative models. Complications encountered in applications of the results to specific models are discussed.

Baryogenesis semi-classical toy model: Let b be a nearly massless particle carrying $B = 1/2$ baryonic charge. Let X be a massive boson with baryon number violating interactions.

Boltzmann equation

$$\partial_t n_b + 3Hn_b = \Lambda^{X}_{12}[f_X|\mathcal{M}(X \to bb)|^2 + f_{\bar{X}}|\mathcal{M}(\bar{X} \to bb)|^2$$
$$- f_b f_b|\mathcal{M}(bb \to X)|^2 - f_b f_b|\mathcal{M}(bb \to \bar{X})|^2]$$
$$+ \Lambda^{34}_{12}\left[f_{\bar{b}} f_{\bar{b}}|\mathcal{M}'(\bar{b}\bar{b} \to bb)|^2 - f_b f_b|\mathcal{M}'(bb \to \bar{b}\bar{b})|^2\right]$$

With

$$|\mathcal{M}'|^2 := |\mathcal{M}|^2 - |\mathcal{M}_{RIS}|^2$$

Where RIS stands for Real-Intermediate State; where the intermediate particle is on-shell.

$$|\mathcal{M}_{RIS}|^2 = \frac{\pi}{m_X\Gamma_X}\delta(s - m_X^2)$$
$$\times \left(|\mathcal{M}(bb \to X)|^2|\mathcal{M}(X \to \bar{b}\bar{b})|^2 + |\mathcal{M}(bb \to \bar{X})|^2|\mathcal{M}(\bar{X} \to \bar{b}\bar{b})|^2\right)$$

RIS subtraction

$$\mathcal{M} \propto \frac{1}{s - m^2 + im\Gamma} = \frac{(s - m^2) - im\Gamma}{(s - m^2)^2 + m^2\Gamma^2}$$

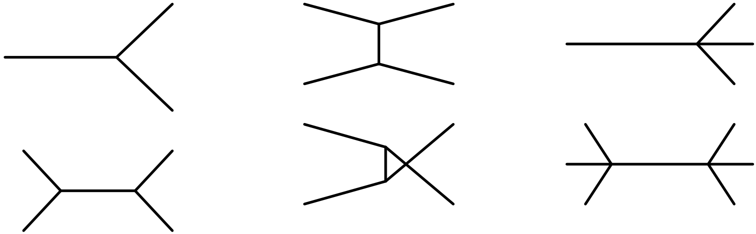
Notice that 1st term$\rightarrow$ 0 when on-shell: the on-shell contribution is the second part. When on-shell, the second term gives:

$$\left(\frac{m\Gamma}{(s - m^2)^2 + m^2\Gamma^2}\right)^2 \propto \frac{\epsilon^2}{(x^2 + \epsilon^2)^2} \rightarrow \frac{\pi}{\epsilon}\delta(x)$$

Hence this is the quantity which needs to be removed.

Examples of double-counting

Double-counting not only applies to s-channel resonances:



Any higher level graph which can go on-shell can be cut and separated into lower level graphs. Theoretically, each possible cut implies one RIS subtraction to be made in the Boltzmann network.

What's the problem?

- Effects are of the same order of the graph, but subleading compared to other graphs;
- There seems to be intuitive ways to proceed;

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- Effects are of the same order of the graph, but subleading compared to other graphs;
- There seems to be intuitive ways to proceed;
- But this is completely ad.hoc. and denotes a gap in the theory;
- $\Gamma \neq 0$ in practical cases, $O(\Gamma^2)$ effects of unknown importance;
- Baryogenesis models have moved from qualitative to being numerically precise;
- There is no consensus;

Goal

- Find simple and consistent subtraction schemes which give the most reliable Boltzmann equation;

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- Find simple and consistent subtraction schemes which give the most reliable Boltzmann equation;

OR

- Find a stronger equation from more fundamental physics to derive the semi-classical Boltzmann equation.

History

- *Evolution of cosmological baryon asymmetries*;
  Fry, Oliver and Turner (1980);
- *Protecting from the primordial baryon asymmetry from erasure by sphalerons*;
  J. Cline, K. Kainulainen and K. Olive (1993);
- *Towards a complete theory of thermal leptogenesis*;
  G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia (2003);
- *ARS Leptogenesis*;
  Bjorn Garbrech (2017).

Method 1

Take a boson propagator with total width Γ (decay+thermal):

$$|D|^2 = \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2} \rightarrow \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2} - \frac{\pi}{m\Gamma}\delta(p^2 - m^2)$$

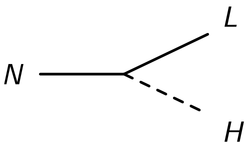
The goal of the method is to remove the pole at $p^2 = m^2$ in the limit $\Gamma \rightarrow 0$.
Pros: Easy to implement and to compute for the s-channel.
Cons: Has only a numerical meaning when $\Gamma = 0$, as the presence of Γ moves the position of the pole in the complex plane.
Sometimes gives negative cross-sections.

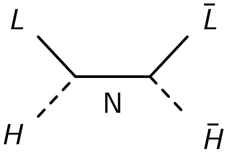
Another approach to method 1

From *Towards a complete lepto...*, this leptogenesis model has another intuitive classical approach:

$$\gamma^{RIS} = \gamma_{Decay} BR(N \to \bar{H}\bar{L}) = \gamma_D/4$$
$$\Rightarrow \gamma^{sub}_{N_s} = \gamma_{N_s} - \gamma_D/4$$

Removal of an on-shell part is not explicit, here we only remove a rate of events.



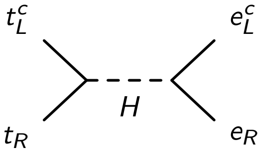


Method 1: Negative cross-sections

From *Protecting from sphalerons...*, compute the thermal integral:

$$\gamma \propto \frac{T}{32\pi^4}\int_{2m_t^2}^{\infty}\frac{ds}{\sqrt{s}}\lambda(s,m_t^2,m_e^2)K_1(\sqrt{s}/T)\sigma^{sub}(t_R t_L^c \to e_R e_L^c)$$

But this turns out to be negative for $T < m_H/2$! Negative cross-section make sense in terms of rates?



Method 2

To have a positive result, subtract before squaring the propagator. For example, try to take the principal part of the propagator:

$$D(p) \to \mathcal{P}(D) = \frac{p^2 - m^2}{(p^2 - m^2)^2 + m^2\Gamma^2} \qquad \text{(bosons)}$$

$$S(p) \to \mathcal{P}(S) = \frac{(p^2 - m^2)(\not{p} + m)}{(p^2 - m^2)^2 + m^2\Gamma^2} \qquad \text{(fermions)}$$

Method 2:

- Clear procedure for t/u-channel resonances and for sums of channels (cross-term hard to deal with method 1):

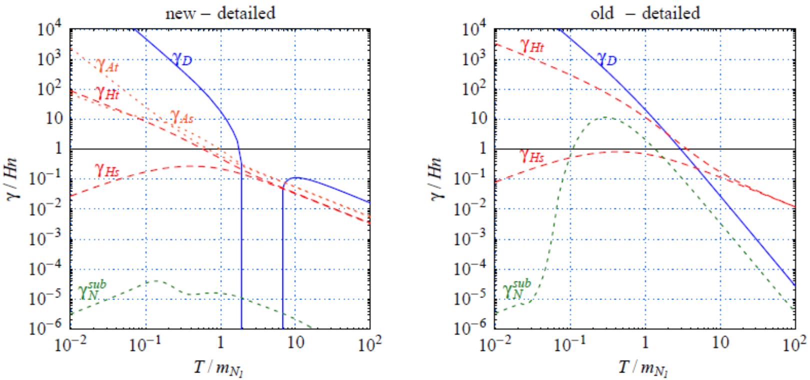
$$|\mathcal{M}|^2 \propto |D(s) + D(t)|^2 \sim D(s)^2 + 2D(s)D(t) + D(t)^2 \qquad (1)$$

- No negative cross-sections!

But it is equivalent, up to a factor of 2 to the first method:

$$|\mathcal{P}(D)| \simeq |D_{N_1}|^2 - \frac{\pi}{2m\Gamma}\delta(s - p^2) \qquad (\Gamma \ll m)$$

Comparison



"As shown [...], the properly subtracted rate has no spurious peaks around the resonance region, in contrast with the result of refs. [6,10]" (new = method 1, old = method 2) Ref: Towards a complete lepto...

Take-home message

When an intermediate particle is allowed to go on-shell, make sure you are not double-counting. Use one of the methods mentioned.

Our first goal seems well explored, and no "breakthrough" seems to have been found.

What about our second goal? Can we find the correct subtraction scheme from first principles?

Non-Equilibrium Quantum Field Theory

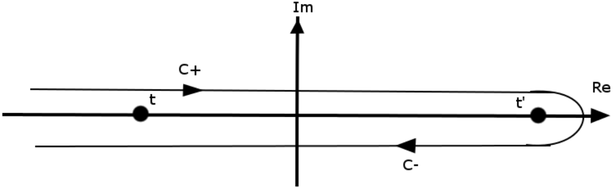
An Introduction to Non-equilibrium Many-Body Theory,
Joseph Maciejko;
Quantum Transport in EWBG,
Thomas Konstandin.

- CTP formalism
- Schwinger-Dyson Equation
- Kadanoff-Baym equation
- Quantum Boltzmann Equation

CTP formalism

Assume knowledge of the initial state, but not the final state.

Closed-Time Path:



$$i\Delta(x,t;x',t') = \langle\Omega|\, T_c\left[\hat{\phi}(x,t)\hat{\phi}(x',t')^\dagger\right]|\Omega\rangle$$

NEQFT is about expressing the final state in terms of the initial state, in exchange for integrating the unordered square of the physics.

CTP formalism

Branch off the propagator on each of these segments to try and split up the contour effects:

$$\Delta(x,x') := \begin{cases} \Delta^{T} = -i\langle T[\phi(x)\phi^{\dagger}(x')]\rangle & \text{On } C_{+} \\ \Delta^{<} = i\langle \phi^{\dagger}(x')\phi(x)\rangle & t \in C_{+}, t' \in C_{-} \\ \Delta^{>} = -i\langle \phi(x)\phi^{\dagger}(x')\rangle & t \in C_{-}, t' \in C_{+} \\ \Delta^{\bar{T}} = -i\langle \bar{T}[\phi(x)\phi^{\dagger}(x')]\rangle & \text{On } C_{-} \end{cases} \tag{2}$$

This is reminescent of the real-time formalism of thermal QFT. Where a ghost field lives on the bottom path.

Schwinger-Dyson Equation

Redefine the propagator:

$$\Delta(x,x') := \begin{bmatrix} \Delta^{T} & \Delta^{<} \\ \Delta^{>} & \Delta^{\bar{T}} \end{bmatrix}$$

Using the 2PI effective action formalism and functional derivatives, one can derive the form of the full propagator, as usual.

Schwinger-Dyson Equation:

$$\Delta(x,x') = \Delta_0(x,x') + \int_c \int_c d^4y d^4z \; \Delta_0(x,y)\Sigma(y,z)\Delta(z,x')$$

Kadanoff-Baym equation

Act with $\hat{\Delta}_0^{-1}$ on $<, >$ equations, use $(X, r) = (x + y/2, x - y)$. Change r to its fourier coordinate, p, to get:

$$(p^2 - m^2 - \Sigma^h) \star \Delta^{>,<} - \Sigma^{>,<} \star \Delta^h = Coll.$$

$$Coll. = (\Sigma^> \star \Delta^< - \Sigma^< \star \Delta^>)/2$$

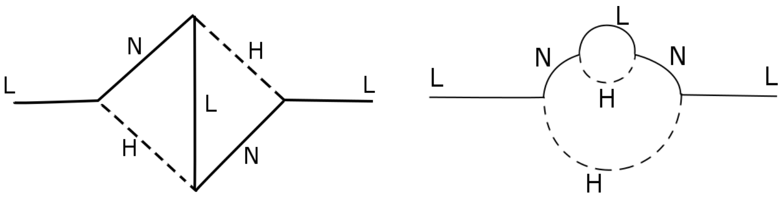
$$\Delta^h = \Delta^T - (\Delta^< + \Delta^>)$$

$$A \star B = A \; e^{-i(\overleftarrow{\partial}_p \vec{\partial}_X - \overleftarrow{\partial}_X \vec{\partial}_p)/\hbar} B$$

The $>$ and $<$ part the generalisations of the statistical part and spectral part of the propagator. The remaining 2 equations give the *pole-mass equation*.

Solution to double-counting

The KB equation contains only self-energies; no cross-sections or decays. These are contained within the on-shell cuts of the self-energies. *ARS Leptogenesis*, Bjorn Garbrecht(2017,ppt); *Resonant Leptogenesis*, Daniele Teresi(2017,ppt).



QBE

To get the quantum analog of the Boltzmann equation, use:

- Gradient expansion: slow background $\Rightarrow \star$ becomes a 2nd order differentials;
- Fluid approximation: plasma close to equilibrium $\Rightarrow n(\beta) \sim$ Bose-Einstein/Fermi-Dirac;
- Weak couplings: far away from a non-equilibrium source, chemical equilibrium is reached.

Take-home message

Manipulating the QBE allows you to construct the corrections to the Boltzmann/diffusion equations, which compute the baryon asymmetry.

This is currently work in progress. Derivations of the Boltzmann equation are known; derivations of its corrections are more subtle and unexplored.

Summary

Double-counting is the problem you get when solving classical time-dependent equations with quantum scattering rates.

- Change to NEQFT schemes: communications with Quark-Gluon Plasma people;
- Advanced study of the moment double-counting appears in the calculation when deriving from NEQFT;
- Advanced study of the numerics of the KB equation and lowering the difficulty of their integrations.

For Further Reading I

Edward W. Kolb and Stephen Wolfram, *Baryon number generation in the early universe*.

J. N. Fry, Keith A. Olive, and Michael S. Turner, *Evolution of cosmological baryon asymmetries. I.The role of gauge bosons*.

J. Cline, K. Kainulainen and K. Olive *Protecting the Primordial Baryon Asymmetry from Erasure by Sphalerons*. Addison-Wesley, Reading, Massachusetts, 1993.

G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM*

Joseph Maciejko, *An Introduction to Non-equilibrium Many-Body Theory*

Thomas Konstandin, *Quantum Transport and Electroweak Baryogenesis*