Chameleons in the early Universe

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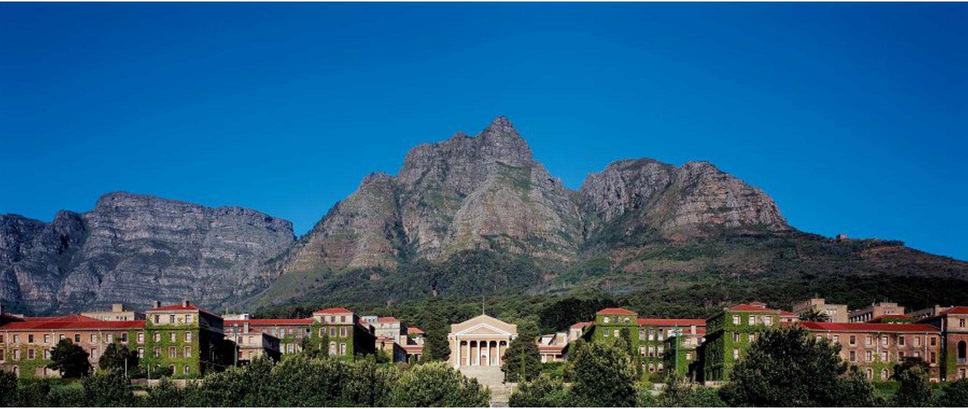

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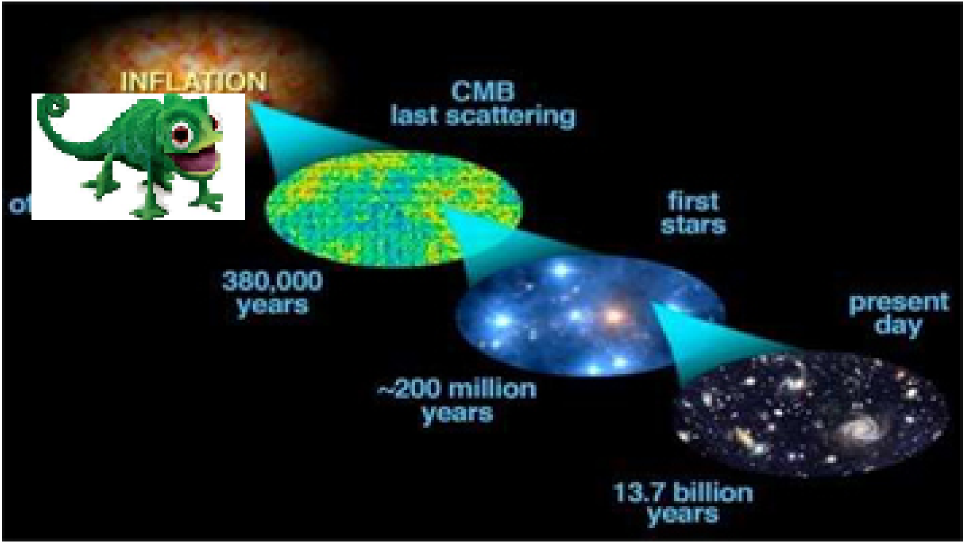
General idea


INFLATION
CMB
last scattering
first
stars
380,000
years
~200 million
years
present
day
13.7 billion
years

General idea

Hyphotesis:

- Chameleon field as the inflaton
- There is some non-relativistic matter at the time of inflation

Question:

- What happens with n_s and r?

Chameleon field: Short review

Chameleon field: Short review

Chameleon fields are scalar fields whose coupling to matter is such that its effective mass depends on the local matter density[1].

$$S = S_{inf}[g_{\mu\nu}, \varphi] + \int d^4 \sqrt{-g} \mathcal{L}_m(\tilde{g}_{\mu\nu}, \psi) \tag{1}$$

with[2]

$$\tilde{g}_{\mu\nu} = F^2(\varphi) g_{\mu\nu}, \qquad F(\varphi) = e^{c\varphi/M_{Pl}}. \tag{2}$$

If we vary it w.r.t. φ, we get

$$\Box\varphi = -\frac{dV}{d\varphi} - \frac{dF}{d\varphi}\rho_m = -\frac{dV_{\text{eff}}}{d\varphi}. \tag{3}$$

where

$$V_{\text{eff}}(\phi) = V(\phi) + F(\phi)\rho. \tag{4}$$

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[1] Khoury and Weltman 2004a,b.
[2] Hinterbichler et al. 2013.

Conformal inflation

Conformal inflation: short review

Consider[3]

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[\partial_\mu\chi\partial^\mu\chi - \partial_\mu\phi\partial^\mu\phi + \frac{\chi^2-\phi^2}{6}R - \frac{\lambda}{18}(\phi^2-\chi^2)^2\right]. \tag{5}$$

It is invariant under[4]

$$g_{\mu\nu} \to e^{-2\sigma(x)}g_{\mu\nu}; \quad \chi \to e^{\sigma(x)}\chi, \qquad \phi \to e^{\sigma(x)}\phi. \tag{6}$$

Fixing the gauge

$$\chi = \sqrt{6}M_{\mathrm{Pl}}\cosh\frac{\varphi}{\sqrt{6}M_{\mathrm{Pl}}}; \quad \phi = \sqrt{6}M_{\mathrm{Pl}}\sinh\frac{\varphi}{\sqrt{6}M_{\mathrm{Pl}}}. \tag{7}$$

We get

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \lambda M_{\mathrm{Pl}}^4\right]. \tag{8}$$

[3] Bars, Steinhardt, and Turok 2013; Kallosh and Linde 2013.

[4] It is also invariant under global $SO(1,1)$.

Conformal inflation: propaganda

To have inflation we deform the $SO(1,1)$ symmetry keeping the local conformal invariance[5]

$$V = \lambda f(\phi/\chi)\left[\phi^2 - g(\phi/\chi)\chi^2\right]^2 . \tag{9}$$

Choosing $f(x) = 1$ and

$$g(x) = \omega^2 + (1-\omega^2)x^n \,, \tag{10}$$

we can have the Higgs potential at low φ values and inflation when $\varphi \to \infty$.

[5] RC and H. Nastase 2014

Conformal inflation: general feature

At $\varphi \to \infty$,

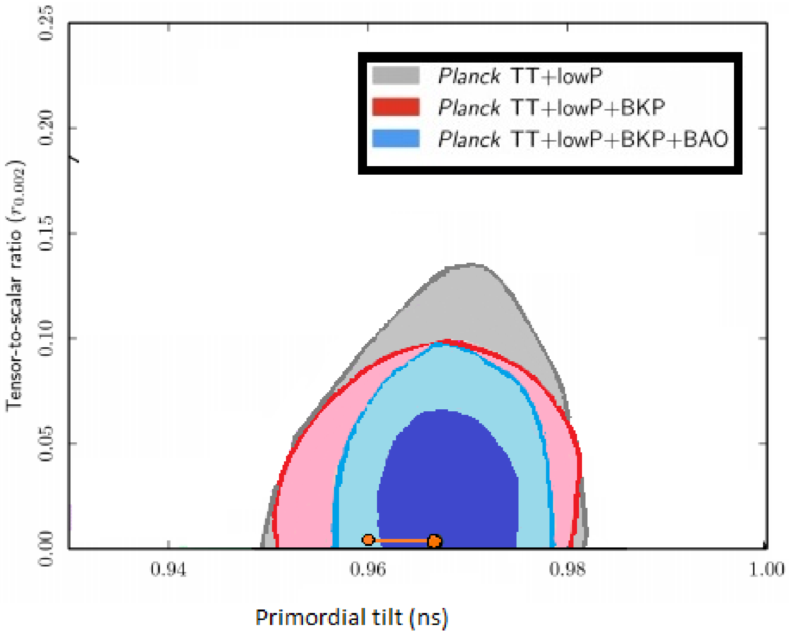
$$V(\varphi) \simeq V_0 \left[1 - B e^{-\sqrt{\frac{2}{3}} \frac{\varphi}{M_{\rm Pl}}} \right] . \tag{11}$$

which implies

$$\begin{aligned} 1 - n_s &\simeq \frac{2}{N_e} \\ r &\simeq 3(n_s - 1)^2 \simeq \frac{12}{N_e^2} . \end{aligned} \tag{12}$$

For 50 e-folds, $n_s = 0.9600$, while for 60 e-folds $n_s = 0.9667$.

Sweet spot



Planck TT+lowP
Planck TT+lowP+BKP
Planck TT+lowP+BKP+BAO
Tensor-to-scalar ratio ($r_{0.002}$)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Primordial tilt (ns)

Attractors

Modified KG equation

At the time of inflation we have

$$\rho_X = F(\varphi)\rho_m = \frac{F(\varphi)\rho_{m0}a_0^3}{a^3} \propto e^{-3N}F(\varphi). \tag{13}$$

and radiation scales as

$$\rho_{\rm rad} = \frac{\rho_{\rm rad,0}\,a_0^4}{a^4} \propto e^{-4N}. \tag{14}$$

In terms of $N = \ln a$, the KG becomes

$$H^2\frac{d^2\varphi}{dN^2} + \frac{1}{3M_{\rm Pl}^2}\left(\frac{3}{2}\rho_X + \rho_{\rm rad} + 3V\right)\frac{d\varphi}{dN} = -\frac{\rho_X}{F}\frac{dF}{d\varphi} - \frac{dV}{d\varphi} = -\frac{dV_{\rm eff}}{d\varphi}\,, \tag{15}$$

Attractors

Since

$$F(\varphi) = e^{c\varphi} \tag{16}$$

and

$$\rho_X = F(\varphi)\rho_m = \frac{F(\varphi)\rho_{m0}a_0^3}{a^3} \propto e^{-3N}e^{c\varphi}, \tag{17}$$

If we find $\varphi = \varphi_i + kN$, it implies

$$\rho_{\mathrm{X}} \sim const. \tag{18}$$

if $k = 3/c$.

Friedmann and Modified KG equations

Defining

$$\Omega_X \equiv \frac{\rho_X}{3H^2 M_{\rm Pl}^2}; \quad \Omega_{\rm rad} \equiv \frac{\rho_{\rm rad}}{3H^2 M_{\rm Pl}^2}; \quad \Omega_{{\rm kin},\varphi} \equiv \frac{\varphi'^2}{6M_{\rm Pl}^2} , \quad \Omega_V \equiv \frac{V}{3H^2 M_{\rm Pl}^2} , \tag{19}$$

The Friedmann equation,

$$\Omega_X + \Omega_{\rm rad} + \Omega_{{\rm kin},\varphi} + \Omega_V = 1 , \tag{20}$$

which implies

$$\frac{d\varphi}{dN} = M_{\rm Pl}\sqrt{6(1 - \Omega_X - \Omega_{\rm rad} - \Omega_V)}. \tag{21}$$

The nontrivial equation is the KG equation, which becomes

$$\frac{d^2\varphi}{dN^2} + \left(\frac{3}{2}\Omega_X + \Omega_{\rm rad} + 3\Omega_V\right)\frac{d\varphi}{dN} = 3cM_{\rm Pl}\Omega_X - \frac{1}{H^2}\frac{dV}{d\varphi} , \tag{22}$$

Analytical results

Assuming that $\varphi = \varphi_i + kN$, and ignoring Ω_{rad}, implies

$$3\left(\frac{1}{2}\Omega_X + \Omega_V\right)k = 3c\Omega_X - \frac{1}{H^2}V', \quad (23)$$

$$\frac{k^2}{6} = 1 - \Omega_X - \Omega_V. \quad (24)$$

Assuming now $V' \sim \tilde{\alpha} V_0$ and $H^2 \sim const$, implies

$$\Omega_V \simeq \frac{1}{1 + \frac{\tilde{\alpha}/c}{1 - \frac{3}{2c^2}}}\left(1 + \frac{3}{2c^2}\right). \quad (25)$$

Analytical results

$$\Omega_V \simeq \frac{1}{1+\frac{\tilde{\alpha}/c}{1-\frac{3}{2c^2}}}\left(1+\frac{3}{2c^2}\right). \tag{26}$$

From Friedmann equation

$$\Omega_X = 1-\Omega_V-\frac{3}{2c^2} \geq 0 \Rightarrow \Omega_V \leq 1-\frac{3}{2c^2}. \tag{27}$$

The results above implies

$$\tilde{\alpha} \geq \frac{3}{c}. \tag{28}$$

Typically

$$\Omega_X \sim 10^{-4}, \quad \Omega_{\rm kin} \sim 10^{-5}, \quad \Omega_V \sim 1-\Omega_X-\Omega_{\rm kin}. \tag{29}$$

Numerical results

The numerical solution of

$$\frac{d^2\varphi}{dN^2} + \left(\frac{3}{2}\Omega_X + \Omega_{\rm rad} + 3\Omega_V\right)\frac{d\varphi}{dN} = 3cM_{\rm Pl}\Omega_X - \frac{1}{H^2}\frac{dV}{d\varphi}\ , \qquad (30)$$

with

$$V(\varphi) \simeq V_0\left[1 - Be^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm Pl}}}\right]. \qquad (31)$$

gives, in fact, an attractor.

Numerical results

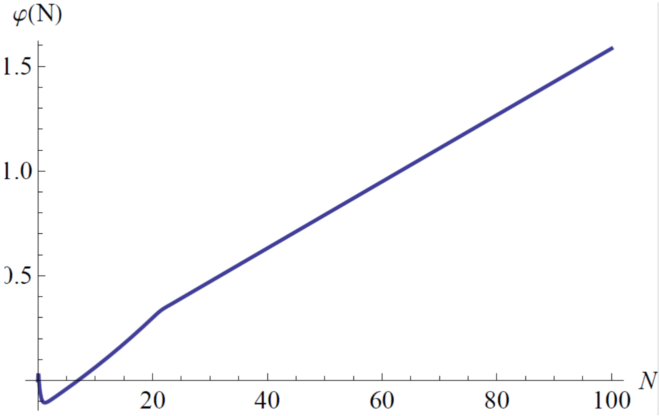


Figure 1: Attractor is achieved after 20 e-folds.

Caution

Some comments

- In usual inflation, the perturbations in the canonical scalar φ get mixed with the perturbations in the (scalar part of the) gravitational perturbations, leading to the Mukhanov-Sasaki equation for the combined variable $\varphi_k(\eta)$ (η is conformal time).
- We have, on the top of that, fluctuations on the original nonrelativistic particle density at the begining of the phase, $\rho_{m,0}$, as well as in principle fluctuations in the number of e-folds, $N(\eta)$, since $dN(\eta) = da(\eta)/a(\eta)$.
- This is not only more complicated, but also in principle model-dependent, as we need to explain the mechanism of production of $\rho_{m,0}$, that will be tied in with the mechanism for generating perturbations for them.

Cosmological observables

A coarse description

$$H^2\varphi'' + \left(\tfrac{1}{2}\rho_X + V\right)\varphi' = -\frac{d}{d\varphi}V_{\rm eff}(\varphi, N)\ ,$$

$$\downarrow$$

$$H^2\varphi'' + V_{\rm infl}\varphi' = -\frac{d}{d\varphi}V_{\rm infl}(\varphi)\ ,$$

- How do we choose $V_{\rm infl}(\varphi)$?

A coarse description

- Solve

$$H^2\varphi'' + \left(\frac{1}{2}\rho x + V\right)\varphi' = -\frac{d}{d\varphi}V_{\text{eff}}(\varphi, N)\ , \tag{32}$$

to get $V_{\text{eff}}(N)\Big|_{\text{att}} \equiv f(N)$.

- Guess a potential $V_{\text{infl}}(\varphi)$ that after solving

$$H^2\varphi'' + V_{\text{infl}}\varphi' = -\frac{d}{d\varphi}V_{\text{infl}}(\varphi) \tag{33}$$

we get $V_{\text{infl}}(N) \equiv g(N) = f(N)$.

A coarse description

For

$$V_{\mathrm{infl}}(\varphi) = c_1(1 - c_2\varphi)^q, \tag{34}$$

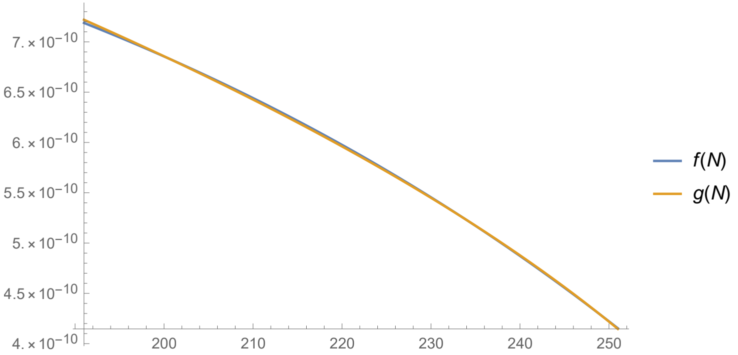


Figure 2: $c_1 = 1.256 \times 10^{-9}$, $c_2 = 0.04357$ and $q = 0.9375$. The relative is less than 1 percent.

A coarse description

Finally, using

$$\epsilon = \frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{V'_{\mathrm{infl}}(\varphi)}{V_{\mathrm{infl}}(\varphi)}\right)^2, \qquad \eta = M_{\mathrm{Pl}}^2 \frac{V''_{\mathrm{infl}}(\varphi)}{V_{\mathrm{infl}}(\varphi)}, \tag{35}$$

and then

$$n_s = -6\epsilon + 2\eta + 1, \qquad r = 16\epsilon. \tag{36}$$

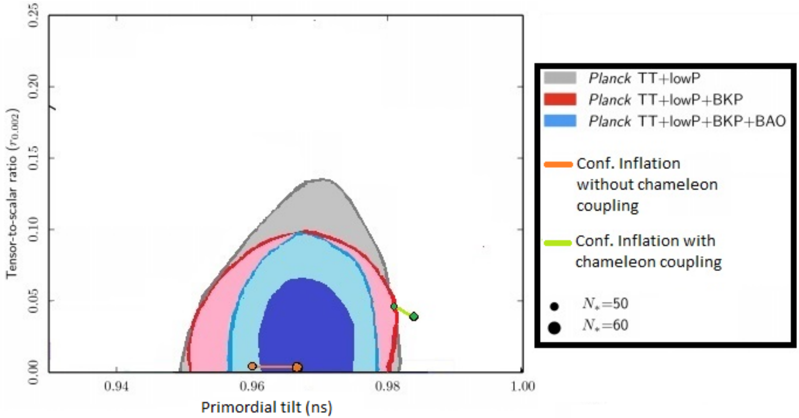
For 50 e-folds we get

$$n_s = 0.9807, \qquad r = 0.049, \tag{37}$$

while for 60 e-folds

$$n_s = 0.9829, \qquad r = 0.043. \tag{38}$$

Final result



Conclusion

Summary

- We have investigated the possibility that the inflaton in conformal inflation models is also a chameleon, coupling with some heavy non-relativistic particles present during inflation.
- The chameleon coupling leads to attractor solutions for the energy densities.
- A coarse treatment suggests that the values of the cosmological observables goes from the sweet spot of Planck data to almost out of the allowed region.

Questions?

Ghosts and Tachyons

Consider

$$S(\phi) = c_1 \int d^4x(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - c_2 m^2\phi^2). \tag{39}$$

In term of the modes,

$$S(\phi) = c_1 \int dt(\dot{\phi}_k^2 - m_k^2\phi_k^2), \tag{40}$$

where $m_k^2 = m^2 + c_2\vec{k}^2$.

Ghosts and Tachyons

Remember $m_k^2 = m^2 + c_2 \vec{k}^2$.

(i) Normal healthy field has $c_1 = c_2 = 1$. Oscillatory type with usual boundary conditions

$$\ddot{\phi}_k + m_k^2 \phi_k = 0. \tag{41}$$

(ii)Tachyon has $c_1 = 1$ and $c_2 = -1$. For small momenta $m_k^2 < 0$,

$$\ddot{\phi}_k - \omega_k^2 \phi_k = 0, \qquad \omega_k^2 = |m_k^2|. \tag{42}$$

Exponential solutions. If the particle moves faster than the speed of light we recover normal oscillatory solutions.

Ghosts and Tachyons

Remember $m_k^2 = m^2 + c_2 \vec{k}^2$.

(iii) Massive ghost has $c_1 = -1$ and $c_2 = 1$. It implies negative kinetic energy. Oscillatory equations. Problem arises only when coupled to other healthy fields.

(iv)Tachyonic ghosts has $c_1 = -1$ and $c_2 = -1$. All the problems associated to ghosts and tachyons.