Dissipative and Stochastic Effects During Inflation $¹$ </sup>

Rudnei O. Ramos

Rio de Janeiro State University Department of Theoretical Physics

McGill University

September 8th, 2017

[Inflation](#page-2-0)

- [Warm Inflation](#page-10-0)
- [WI Model Building](#page-13-0)
- [Dissipation and Noise](#page-21-0)
- [Primordial Perturbations](#page-32-0)
- [Observational Predictions](#page-34-0)

[Summary](#page-39-0)

Inflation: Early phase of accelerated expansion dominated by vacuum energy $V(\phi)$ (ϕ = scalar field – the inflaton) $\ddot{a} > 0$, $p < -\rho/3$

Inflation: Early phase of accelerated expansion dominated by vacuum energy $V(\phi)$ (ϕ = scalar field – the inflaton) $\ddot{a} > 0$, $p < -\rho/3$

CMB anisotropies require inflation to occur at high energies:

$$
V^{1/4} \sim 10^{16} \left(\frac{r}{0.1}\right)^{1/4} \, \mathrm{GeV}
$$

 $r =$ tensor-to-scalar curvature perturbation ratio. Planck TT+BKP 2015: $r \leq 0.08$.

Inflation: Early phase of accelerated expansion dominated by vacuum energy $V(\phi)$ (ϕ = scalar field – the inflaton) $\ddot{a} > 0$, $p < -\rho/3$

CMB anisotropies require inflation to occur at high energies:

$$
V^{1/4} \sim 10^{16} \left(\frac{r}{0.1}\right)^{1/4} \, \mathrm{GeV}
$$

 $r =$ tensor-to-scalar curvature perturbation ratio. Planck TT+BKP 2015: $r \leq 0.08$.

 \Rightarrow Can the inflaton be embedded into a fundamental theory ?

Inflation: Early phase of accelerated expansion dominated by vacuum energy $V(\phi)$ (ϕ = scalar field – the inflaton) $\ddot{a} > 0$, $p < -\rho/3$

CMB anisotropies require inflation to occur at high energies:

$$
V^{1/4} \sim 10^{16} \left(\frac{r}{0.1}\right)^{1/4} \, \mathrm{GeV}
$$

 $r =$ tensor-to-scalar curvature perturbation ratio. Planck TT+BKP 2015: $r \leq 0.08$.

 \Rightarrow Can the inflaton be embedded into a fundamental theory ?

 \Rightarrow We need to know how it interacts with other fields !

Cold Inflation

Cold Inflation^a: Inflaton \implies Reheating \implies Radiation \implies Matter Inflaton interactions with other d.o.f. only important during reheating $\dot{\rho}_r + 4H\rho_r = 0$, $(H = \dot{a}/a \sim cte)$ \Rightarrow The radiation density during inflation redshifts away: $\rho_{\mathsf{r}} \sim 1/a^4$ \Rightarrow density perturbations sourced by inflaton's quantum fluctuations

[Inflation](#page-2-0) [Warm Inflation](#page-10-0) [WI Model Building](#page-13-0) [Dissipation and Noise](#page-21-0) [Primordial Perturbations](#page-32-0) [Observational Predictions](#page-34-0) Summar

Cold Inflation

Inflaton \implies Radiation (p)reheating \implies Matter $\phi \longrightarrow \chi, \psi_{\chi} \longrightarrow SM \text{ dof}$ for example : $\mathcal{L}_I = -V(\phi) - \frac{1}{2}$ $\frac{1}{2}g_{\chi}^2\phi^2\chi^2 - g_{\psi}\phi\bar{\psi}_{\chi}\psi_{\chi} + \mathcal{L}_I[\chi,\psi_{\chi},SM]$

If the universe did not supercool, then...

- \bullet can reduce the need for very small couplings
- more opportunities for particle phenomenology
- make structure from thermal fluctuations
- \bullet can produce observable amounts of non-gaussianity
- \bullet exit nicely

Warm Inflation^a:

^alan G. Moss PLB154 1985, Yokoyama and Maeda PLB207 1988, A. Berera

Inflaton \implies Decay \implies Radiation \implies Matter

Inflaton interactions with other d.o.f. is important during inflation generate dissipation/viscosity terms \Rightarrow small fraction of vacuum energy density can be converted to radiation

$$
\dot{\rho}_r + 4H\rho_r = \Upsilon \dot{\phi}^2
$$

 \Rightarrow The radiation density during inflation stabilises: $\Rightarrow \rho_r \sim \text{T} \dot{\phi}^2/(4H)$

Warm Inflation

The production of radiation is associated with a friction term in the inflaton equation,

$$
\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \Upsilon \dot{\phi} = \xi
$$

- \cdot T_{ϕ} (friction term) describes how inflaton's interactions with other fields backreact on the inflaton dynamics.
- ϵ describes quantum and thermal stochastic fluctuations. Dynamics for the inflaton is similar to a Langevin equation with quantum and thermal noise terms (stochastic process w/ Gaussian noises $)^2$
- Effectiveness of warm inflation measured by $Q = \frac{\gamma}{3}$ 3H
- $\overline{}$ Cold or warm inflation: $T{\leq}H$ or $T{\geq}H$
- amplitude $\delta \phi^2 \sim H^2+HT+\Upsilon$ T, (WI: $\tau > H$), curvature perturbations $\zeta = H \delta \phi / \dot{\phi}$.

Warm Inflation Model Building

Challenges:

• Coupling the inflaton to light particles is hard

$$
\mathcal{L}_{\text{int}} = -g\phi\bar{\psi}\psi \quad \Rightarrow \quad m_{\psi} = g\phi \gtrsim 7
$$

Warm Inflation Model Building

Challenges:

• Coupling the inflaton to light particles is hard

$$
\mathcal{L}_{\text{int}} = -g\phi\bar{\psi}\psi \quad \Rightarrow \quad m_{\psi} = g\phi \gtrsim 7
$$

Light particles induce large thermal mass corrections:

$$
\Delta m_\phi^2 \sim g^2 T^2 \gg H^2
$$

Warm Inflation Model Building

Challenges:

• Coupling the inflaton to light particles is hard

$$
\mathcal{L}_{\text{int}} = -g\phi\bar{\psi}\psi \quad \Rightarrow \quad m_{\psi} = g\phi \gtrsim 7
$$

Light particles induce large thermal mass corrections:

$$
\Delta m_{\phi}^2 \sim g^2 T^2 \gg H^2
$$

 \bullet Need very small couplings \Rightarrow results in very little dissipation ...

[Inflation](#page-2-0) [Warm Inflation](#page-10-0) [WI Model Building](#page-13-0) [Dissipation and Noise](#page-21-0) [Primordial Perturbations](#page-32-0) [Observational Predictions](#page-34-0) Summar

PHYSICAL REVIEW D. VOLUME 60, 083509

Is warm inflation possible?

Jun'ichi Yokovama* Department of Physics, Stanford University, Stanford, California 94305-4060 and Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Andrei Linde

Department of Physics, Stanford University, Stanford, California 94305-4060 (Received 25 August 1998; published 24 September 1999)

We show that it is extremely difficult and perhaps even impossible to have inflation supported by thermal effects. [S0556-2821(99)00916-9]

[Inflation](#page-2-0) [Warm Inflation](#page-10-0) [WI Model Building](#page-13-0) [Dissipation and Noise](#page-21-0) [Primordial Perturbations](#page-32-0) [Observational Predictions](#page-34-0) Summar

PHYSICAL REVIEW D. VOLUME 60, 083509

Is warm inflation possible?

Jun'ichi Yokovama* Department of Physics, Stanford University, Stanford, California 94305-4060 and Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Andrei Linde

Department of Physics, Stanford University, Stanford, California 94305-4060 (Received 25 August 1998: published 24 September 1999)

We show that it is extremely difficult and perhaps even impossible to have inflation supported by thermal effects. [S0556-2821(99)00916-9]

VOLUME 83. NUMBER 2

PHYSICAL REVIEW LETTERS

12 JULY 1999

A First Principles Warm Inflation Model that Solves the Cosmological Horizon and Flatness Problems

Arjun Berera,¹ Marcelo Gleiser,² and Rudnei O. Ramos³ ¹Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235 ²Department of Physics and Astronomy, Dartmouth College, Hanover, New Hampshire 03755 ³Universidade do Estado do Rio de Janeiro, Instituto de Física, Departamento de Física Teórica, 20550-013 Rio de Janeiro, RJ, Brazil (Received 30 September 1998; revised manuscript received 2 November 1998)

A quantum field theory warm inflation model is presented that solves the horizon and flatness problems. The model obtains, from the elementary dynamics of particle physics, cosmological scale factor trajectories that begin in a radiation dominated regime, enter an inflationary regime, and then smoothly exit back into a radiation dominated regime, with non-negligible radiation throughout the evolution.

Is warm inflation possible ? YES !

The inflaton does not need to be coupled directly to the radiation fields: It can couple indirectly through heavy mediator fields

Working model: the two-stage decay model: $\phi \qquad \longrightarrow \qquad \chi, \psi_{\chi} \qquad \longrightarrow \qquad \qquad \psi_{\sigma}, \sigma$ inflaton heavy field light fermions or scalars $(m_X \gg T)$ ($m_Y \ll T$) for example : $\mathcal{L}_I = -\frac{\lambda}{4}$ $\frac{\lambda}{4}\phi^4 - \frac{1}{2}$ $\frac{1}{2}g_{\chi}^2\phi^2\chi^2-g_{\psi}\phi\bar{\psi}_{\chi}\psi_{\chi}-h_{\sigma}M\chi\sigma^2-h_{\psi}\chi\bar{\psi}_{\sigma}\psi_{\sigma}$

Working model: the two-stage decay model:

$$
\begin{array}{rcl}\n\phi & \longrightarrow & \chi, \psi_{\chi} & \longrightarrow & \psi_{\sigma}, \sigma \\
\text{inflaton} & \text{heavy field} & \text{light fermions or scalars} \\
(m_{\chi} \gg T) & (m_{\gamma} \ll T)\n\end{array}
$$

for example : $\mathcal{L}_I = -\frac{\lambda}{4}$ $\frac{\lambda}{4} \phi^4 - \frac{1}{2}$ $\frac{1}{2}g_{\chi}^2\phi^2\chi^2-g_{\psi}\phi\bar{\psi}_{\chi}\psi_{\chi}-h_{\sigma}M\chi\sigma^2-h_{\psi}\chi\bar{\psi}_{\sigma}\psi_{\sigma}$

- decouple the radiation from the inflaton $(m_\chi, m_{\psi_\gamma} \gg T, m_{\sigma, \psi_\sigma} \ll T)$
- couplings between fields of order 0.1
- SUSY to reduce vacuum corrections, e.g., $W = g\Phi X^2 + hXY^2$ where Φ , X, Y are superfields with scalar and fermion components given by (ϕ, ψ_{ϕ}) , (χ, ψ_{χ}) and (σ, ψ_{σ}) respectively.

Let $\phi \equiv \phi(x, t)$ and average out (integrate over) the other fields. This gives a stochastic (Langevin-like) system $^3_\cdot$

$$
\ddot{\phi}(\mathbf{x},t)+3H\dot{\phi}(\mathbf{x},t)+\int d^4x'\Sigma_R(x,x')\phi(x')+V_{,\phi}-\frac{1}{a^2}\nabla^2\phi(\mathbf{x},t)=\xi(\mathbf{x},t).
$$

The kernel:

$$
\Sigma_R(x') = -i\left(\frac{g_1^2}{2}\right)^2 \varphi^2 \sum_{i=1}^2 \theta(t-t') \langle [\chi_i^2(x'), \chi_i^2(0)] \rangle
$$

$$
-i\frac{g_2^2}{2} \theta(t-t') \text{tr} \left\{ \langle [\bar{\psi}_X(x')\psi_X(x'), \bar{\psi}_X(0)\psi_X(0)] \rangle \right\} ,
$$

is connected to the two-point correlation function of the noise term $\xi(\mathbf{x},t)$: generalized fluctuation-dissipation relation

The self-energy contribution is a dissipative term 4 (in the adiabatic approximation, $\dot{\phi}/\phi, H, \, \dot{T}/\, \tau < \Gamma_\chi \approx h^2 m_\chi/(8 \pi)$), $\int d^4x' \Sigma_R(x,x')\phi(x') \approx \Upsilon \dot{\phi}(x,t)$

$$
\begin{array}{rcl}\n\Upsilon & = & \frac{2}{T} \left(\frac{g_1^2}{2} \right)^2 \varphi^2 \int \frac{d^4 p}{(2\pi)^4} \, \rho_\chi(p_0, \mathbf{p})^2 \, n_B(p_0) \left[1 + n_B(p_0) \right] \\
& + & \frac{g_2^2}{2T} \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left[\rho_{\psi_\chi}(p_0, \mathbf{p})^2 \right] n_F(p_0) \left[1 - n_F(p_0) \right] \,,\n\end{array}
$$

where ρ_χ and ρ_{ψ_χ} are the spectral functions for the intermediate fields χ and $\psi_{\rm v}$

The dissipation coefficient Υ can be explicitly computed in QFT once the interactions are given.

Table 1. A summary of all the dissipation coefficients in the low-temperature regime coming from each of the interaction cases involving a heavy intermediate field (all expressions evaluated with $m_{\chi_1} = m_{\chi_2}$).

 \Rightarrow leading friction coefficient for $T \ll m_{\chi}$ is:

$$
\Upsilon \sim g^2 h^4 (T^3/m_\chi^2),
$$

Going back to the old WI model

New Motivation: Little Higgs model⁵. The Higgs is a PNGB of some spontaneously broken global symmetry: $G \rightarrow H$, with a property of collective symmetry breaking. Global symmetry is explicitly broken by two sets of interactions, with each preserving a subset of the symmetry:

$\mathcal{L} = \mathcal{L}_0 + \lambda_1 \mathcal{L}_1 + \lambda_2 \mathcal{L}_2$

 \mathcal{L}_0 symmetric part, $\mathcal{L}_{1,2}$ are explicit symm. breaking terms.

Fig. 3. A global symmetry G is spontaneously broken down to a subgroup H . A subgroup F of G is gauged and it is broken down to the intersection of F and H, $I = F \cap H$, which is identified as the SM electroweak gauge symmetry. The number of uneaten PNGBs is given by the number of generators of $(N(G) - N(H)) - (N(F) - N(I))$. They are identified as the Higgs.

⁵Arkani-Hamed, Cohen & Georgi (2001)

Consider a $U(1)$ gauge theory spontaneously broken by two complex scalar fields:

$$
\langle \phi_1 \rangle = \langle \phi_1 \rangle \equiv M/\sqrt{2}
$$

Only one Nambu-Goldstone boson is "eaten" by the gauge field, while the other remains as a physical singlet scalar (inflaton):

$$
\phi_1 = \frac{M}{\sqrt{2}} e^{i\phi/M}, \quad \phi_2 = \frac{M}{\sqrt{2}} e^{-i\phi/M}
$$

Couple the inflaton to charged and singlet Weyl fermions:

$$
-\mathcal{L}_{\phi\psi} = \frac{g}{\sqrt{2}}(\phi_1 + \phi_2)\bar{\psi}_{1L}\psi_{1R} - i\frac{g}{\sqrt{2}}(\phi_1 - \phi_2)\bar{\psi}_{2L}\psi_{2R} + \text{h.c.}= gM\cos(\phi/M)\bar{\psi}_1\psi_1 + gM\sin(\phi/M)\bar{\psi}_2\psi_2.
$$

with interchange symmetry:

$$
\phi_1 \leftrightarrow i\phi_2, \qquad \psi_{1L,R} \leftrightarrow \psi_{2L,R}
$$

Fermion masses are bounded and can be light!

$$
gM \lesssim T \lesssim M
$$

Effective potential at high temperature:

$$
V_T \simeq \sum_{i=1,2} \left[-\frac{7\pi^2}{180} T^4 + \frac{m_i^2 T^2}{12} + \frac{m_i^4}{16\pi^2} \left(\log \left(\frac{\mu^2}{T^2} \right) - c_f \right) \right]
$$

No thermal inflaton masses!

Alternatively, expand Lagrangian to quadratic order:

$$
\mathcal{L}_{\phi\psi} = -\sum_{i} \left[m_i + g_i \delta\phi + \frac{f_i}{2} \delta\phi^2 + \ldots \right] \bar{\psi}_i \psi_i
$$

$$
\Sigma_{\phi}(0) = [(g_1^2 + m_1 f_1) + (g_2^2 + m_2 f_2)] I_T
$$

= $g^2 [-\cos(2\phi/M) + \cos(2\phi/M)] I_T = 0$,

where
$$
I_T \simeq -(\Lambda^2/2\pi^2) + (T^2/6)
$$
.

Cancellation of quadratic divergences and thermal masses!

Dissipation comes from non-local terms in the effective action, which come only from diagram (a):

No cancellation of dissipative terms!

$$
\begin{array}{rcl}\n\Upsilon & = & \int d^4x' \Sigma_R(x, x') \left(t' - t \right) \\
& = & \sum_i 4 \frac{g_i^2}{T} \int \frac{d^3 p}{(2\pi)^3} \frac{m_i^2}{\Gamma_{\psi_i} \omega_p^2} n_F(\omega_p) \left[1 - n_F(\omega_p) \right] \\
& = & \sqrt{2\pi \omega_p^2} \end{array}
$$

where
$$
\omega_p = \sqrt{|\mathbf{p}|^2 + m_i^2}
$$
.

[Bastero-Gil, Berera & Ramos (2001)]

Fermion decay from additional Yukawa interactions:

$$
\mathcal{L}_{\psi\sigma} = -h\sigma \sum_{i=1,2} (\bar{\psi}_{iL}\psi_{\sigma R} + \bar{\psi}_{\sigma L}\psi_{iR})
$$

Dissipation coefficient proportional to the temperature:

$$
\Upsilon \simeq \alpha(h) \frac{g^2}{h^2} T
$$
, $\alpha(h) \simeq \frac{3}{1 - 0.34 \log(h)}$

with $m_i^2 \simeq \Delta m_T^2 \simeq h^2 T^2/8$. [c.f. Yokoyama & Linde (1998)]

Example of the dynamical evolution in warm inflation with a quartic potential. The dashed lines in the bottom plot correspond to M and gM (in Planck units).

Fluctuations & primordial spectrum: coupled system $\delta\ddot{\phi}_{\mathsf{k}}^{\mathsf{GI}} + (3\mathsf{H} + \mathsf{Y})\delta\dot{\phi}_{\mathsf{k}}^{\mathsf{GI}} + \dot{\phi}\delta\mathsf{Y}^{\mathsf{GI}} + \left(\frac{\mathsf{k}^2}{a^2} + \mathsf{V}_{\phi\phi}\right)\delta\phi_{\mathsf{k}}^{\mathsf{GI}} \approx (2\,\mathsf{Y}\,\mathsf{T})^{1/2}\hat{\boldsymbol{\xi}}_{\mathsf{k}}$ Field EOM: fluctuation force & $\frac{\delta Y^{GI}}{Y} = \frac{1}{4} \frac{\delta \rho_r^{GI}}{\rho_r} = \frac{\delta T}{T}$ Coupled system $($ light d. of f .) inflaton-radiation 10 $Q = Y/(3H) = 0.01$ $Q = 10$ $\frac{2}{n}$ $\frac{8}{5}$ = 10 **Growing mode!** 10^{-5} $\frac{5}{N_a}$ R is constant after horizon crossing (freeze-out) $\frac{5}{N}$ $P_R \simeq \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 (1+2N+\frac{T}{H}\frac{4\pi Q}{\sqrt{1+4\pi Q/3}}) \times G[Q], \quad Q = Y/(3H)$

Dissipative processes may maintain a non-trivial distribution of inflaton particles:

$$
N\!\simeq\!n_{\text{BE}}\!\!=\!\!(e^{k/aT}\!-\!1)^{-1}
$$

Spectral tilt $n_s \times$ Tensor-to-Scalar ratio r PRD95, 023517 (2017)

Breaking Degeneracies

WARM INFLATION DISSIPATIVE EFFECTS: ...

PHYSICAL REVIEW D 95, 023517 (2017)

TABLE I. The values of dissipation ratio Q_* and T_*/H_* , along also the values of r, α_* and β_* , and the $\Delta \chi^2$ with respect to the minimal Λ CDM model when n_s is fixed at the value $n_s \approx 0.9655$, for each model considered in this work.

$V(\phi)$	Υ	Q_*	T_*/H_*	r	α_s	β_s	$\Delta\chi^2_{\rm min}$
Quartic	$\propto \frac{T^3}{\phi^2}$	1.697×10^{-3}	7.246	36×10^{-3}	-9.840×10^{-4}	-2.557×10^{-5}	-0.2
Sextic		0.187	41.945	5.225×10^{-3}	1.540×10^{-3}	1.972×10^{-4}	$+0.3$
Hilltop		0.186	41.656	1.741×10^{-4}	-9.997×10^{-5}	-3.101×10^{-6}	$+0.1$
Higgs		1.417	214.829	2.317×10^{-6}	-1.857×10^{-4}	-4.333×10^{-6}	-0.1
Plateau sextic		5.645×10^{-3}	10.766	1.085×10^{-7}	-4.692×10^{-4}	3.369×10^{-5}	Ω
Quartic	$\propto T$	1.256	273.472	1.276×10^{-4}	-3.019×10^{-4}	4.156×10^{-6}	Ω
Sextic		4.966	769.074	1.064×10^{-5}	3.731×10^{-3}	8.381×10^{-4}	$+0.7$
Hilltop		0.028	50.303	1.092×10^{-4}	-1.588×10^{-4}	-1.028×10^{-5}	Ω
Higgs		0.020	44.492	2.947×10^{-4}	-2.505×10^{-4}	-1.424×10^{-5}	-0.1
Plateau sextic		0.810	210.187	4.448×10^{-9}	-3.862×10^{-4}	-2.708×10^{-6}	Ω

Non-Gaussianities

 $\langle \Phi(\mathbf{k}_1) \Phi(\mathbf{k}_2) \Phi(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{\Phi}(k_1, k_2, k_3)$

 $\Phi \equiv (3/5)\zeta$, $\zeta =$ comoving curvature perturbation

Nonlinearity parameter f_{NI} :

 $B_{\Phi}(k_1, k_2, k_3) = f_{NL} F(k_1, k_2, k_3)$

function $F(k_1, k_2, k_3)$ depends on the shape (type of triangle)

Warm Inflation and Non-Gaussianity⁶

 f_{NL} relative to the equilateral shape: $f_{NL} = \frac{18}{5}$ 5 $B(k,k,k)$ $P(k)^2$

Planck constraints: $f_{NL}^{\rm local} = 2.7 \pm 5.8$, $f_{NL}^{\rm equi} = -42 \pm 75$, $f_{NL}^{\rm warm} = 4 \pm 33$ 6M. Basteiro-Gil, A. Berera, I. G. Moss, R.O.R., JCA[P12](#page-36-0), [0](#page-38-0)[08](#page-36-0) [\(](#page-37-0)[20](#page-38-0)[1](#page-33-0)[4](#page-34-0)[\)](#page-38-0)

Summary

- **Dissipative and stochastic effects due to interactions can be** relevant and modify the inflationary predictions $(r, n_s, f_{NL},$ etc).
- Warm inflation has been previously successfully constructed in the low- T regime for dissipation (inflaton+mediator fields+radiation fields), but requires large number of heavy mediators ($\mathsf{N}_\chi \sim 10^6).$
- Warm inflation in the high-T regime for dissipation has been considered "impossible".
- \bullet Little Higgs like model for the inflaton (PNGB of a broken $U(1)$) symmetry $+$ exchange symmetry) demonstrates for the first time that this is not true.
- Cancellation of thermal masses and quadratic divergences but not of dissipative effects: a warm inflation model with minimal matter content (only two light fermions !).
- ■ Both low-T and high-T particle physics models for WI lead to consistent dynamics with observables well within the Planck values.