Glasma to Plasma: instabilities, quantum decoherence and thermalization

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Lecture iII, JET school, June, 2012

Outline of lectures

- **Lecture I: QCD in Regge-Gribov asymptotics: Gluon Saturation and the Color Glass Condensate**

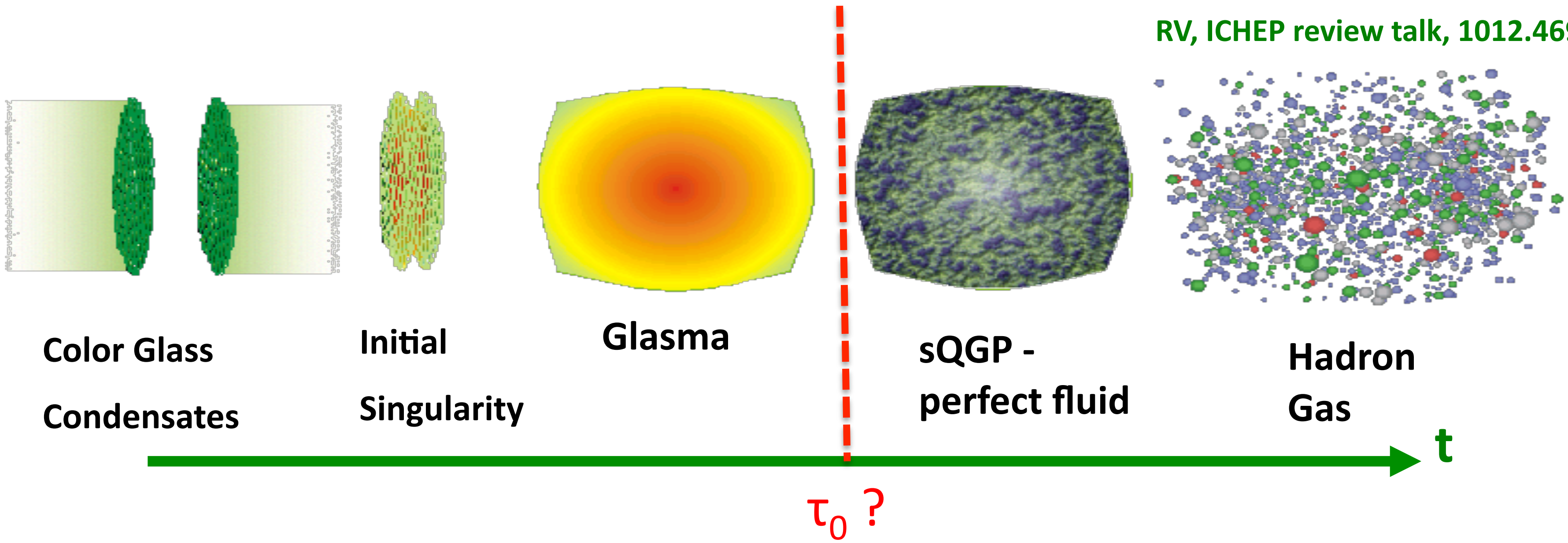
- **Lecture II: Quantum field theory in strong fields. Factorization. the Glasma, long range correlations, multi-particle production**

- **Lecture III: Quantum field theory in strong fields.**
  Instabilities, the spectrum of initial quantum fluctuations, decoherence, hydrodynamics, Bose-Einstein condensation and thermalization

HI theory draws concretely on concepts in perturbative and non-perturbative QCD, string holography, reaction-diffusion systems, topological effects, plasma physics, thermodynamics and stat. mech, quantum chaos, Bose-Einstein condensates, pre-heating in inflationary cosmology

- **Motivation: the unreasonable effectiveness of hydrodynamics in heavy ion collisions**

Ab initio approach to heavy ion collisions



- Compute properties of relevant degrees of freedom of wave fns. in a systematic framework (as opposed to a "model")?
- How is matter formed ? What are its non-equilibrium properties & lifetime? Can one "prove" thermalization or is the system "partially" thermal ?
- When is hydrodynamics applicable? How much jet quenching occurs in the Glasma? Are there novel topological effects (sphaleron transitions?)

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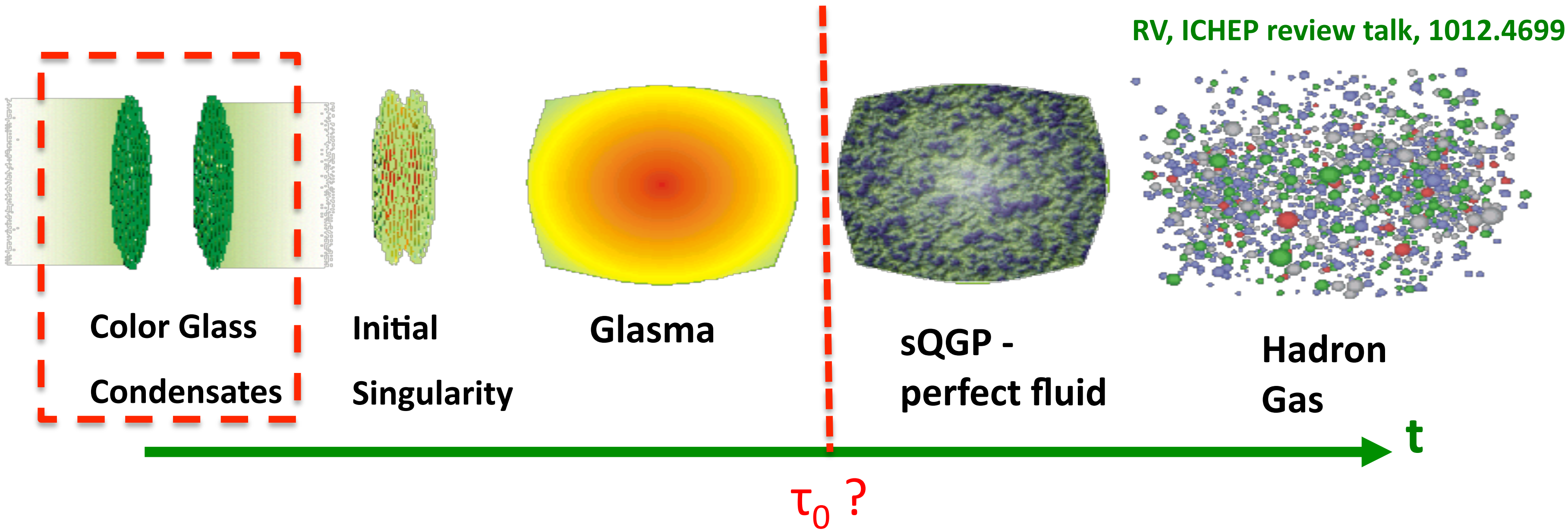

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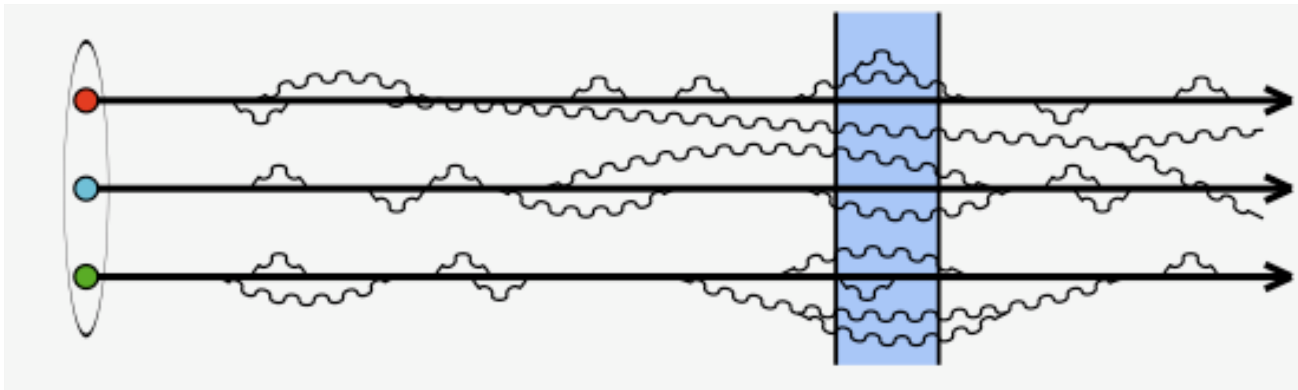
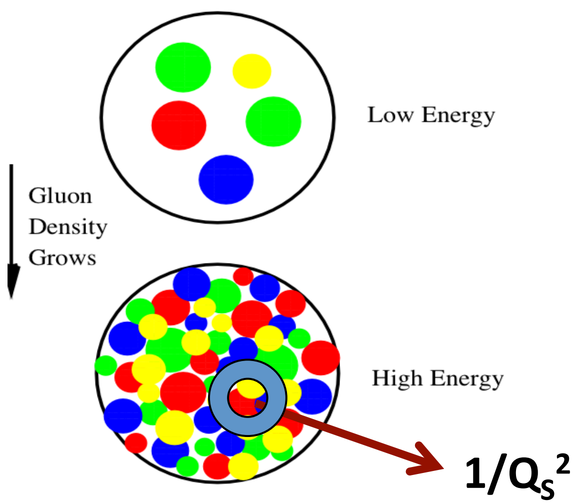
Gluon Saturation in large nuclei: classical coherence from quantum fluctuations



Wee parton fluctuations time dilated on strong interaction time scales

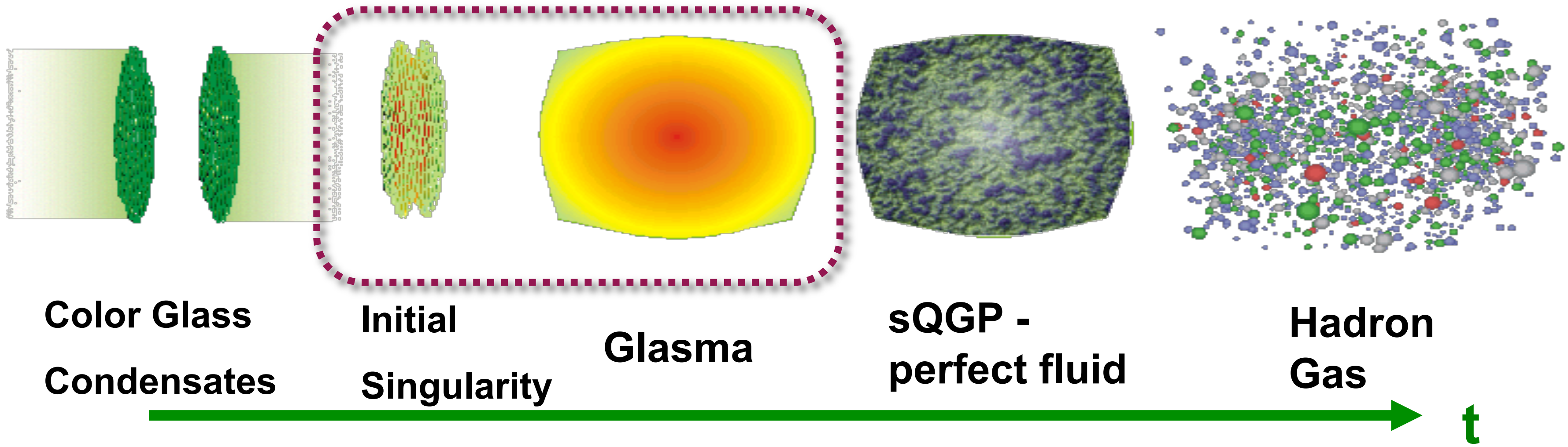


The gluon density saturates at a maximal value of ~ $1/\alpha_S$ ➔ gluon saturation

Large occupation # => classical color fields

|P>_pert ➔ |P>_classical

Quantum decoherence from classical coherence



Glasma (\Glahs-maa\): *Noun:* non-equilibrium matter between CGC and QGP

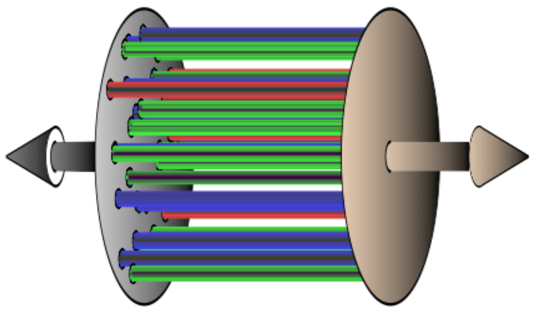
<u>Computational framework</u>

Gelis,RV NPA (2006)

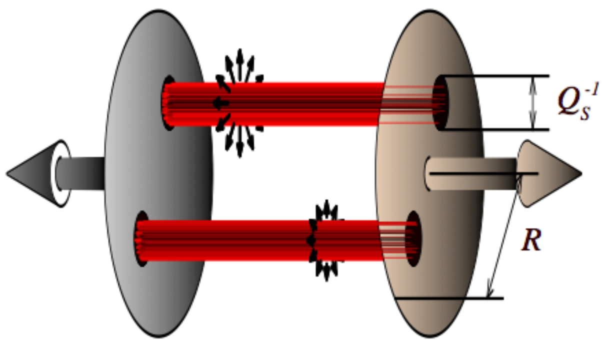
Schwinger-Keldysh: for strong time dependent sources (ρ ~ 1/g) ,
initial value problem for inclusive quantities

For eg., Schwinger mechanism for pair production, Hawking radiation, …

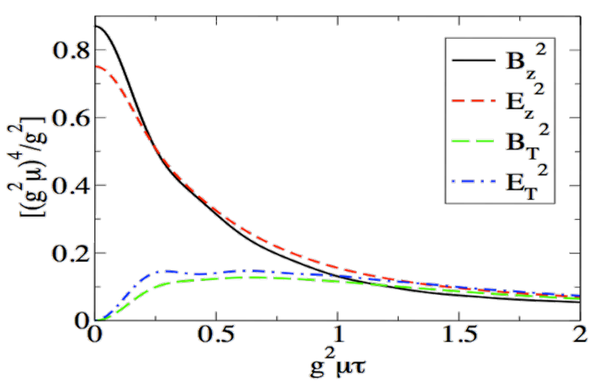
Lumpy classical configurations



Solutions of Yang-Mills equations produce (nearly) boost invariant gluon field configurations: "Glasma flux tubes"

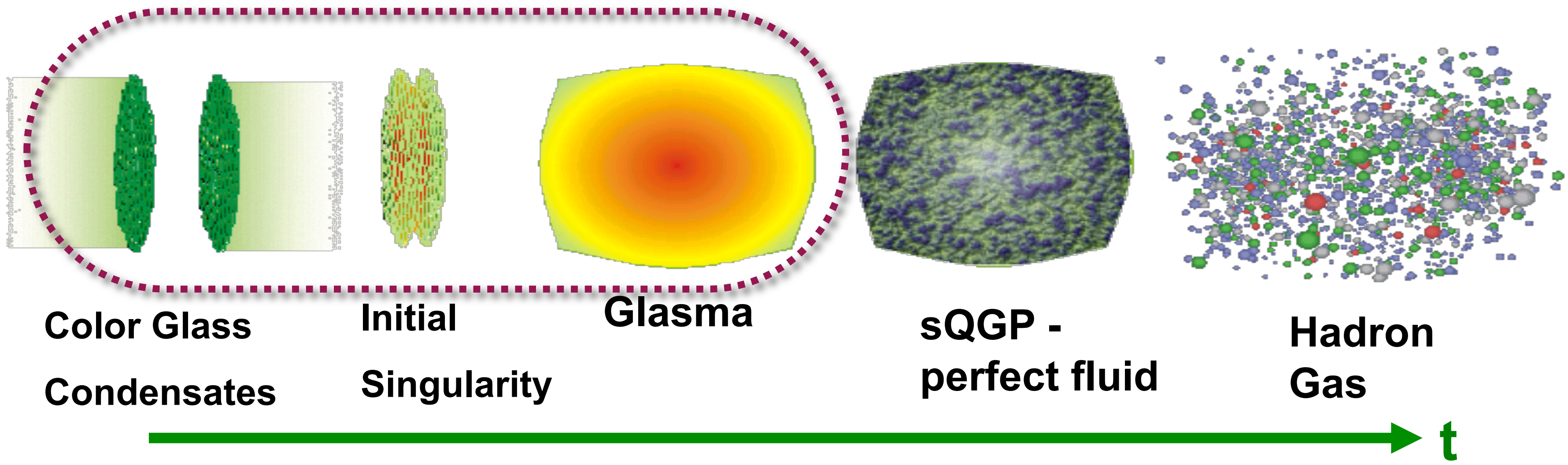


Lumpy gluon fields are color screened in transverse plane over distances ~ $1/Q_S$
- Negative Binomial multiplicity distribution.



"Glasma flux tubes" have non-trivial longitudinal color E & B fields at early times
--generate Chern-Simons topological charge

Their quantum descendents



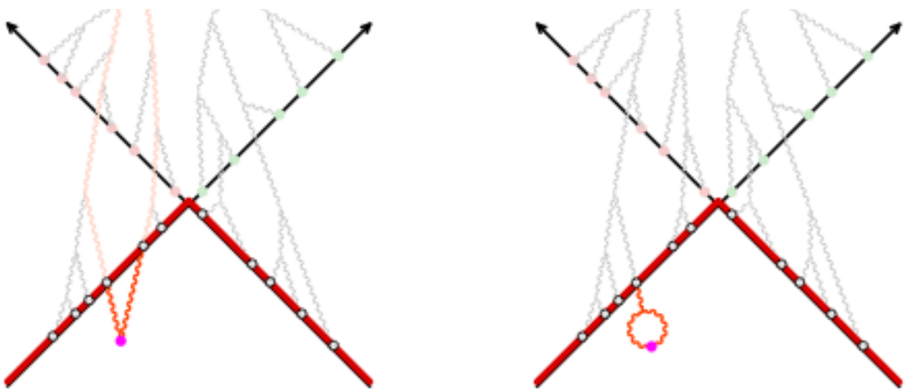
Two kinds of important quantum fluctuations:

a) Before the collision: p_η=0 modes factorized into the wavefunctions
- responsible for energy/rapidity evolution of wavefunctions

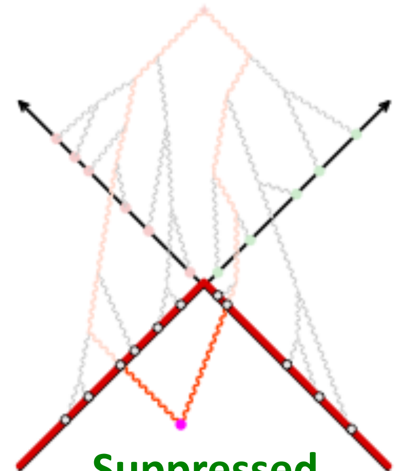
a) After the collision $p_\eta \neq 0$; hold the key to early time dynamics
- responsible for decoherence, isotropization, thermalization

Quantum fluctuations in classical backgrounds: I

Gelis,Lappi,RV: 0804.2630, 0807.1306,0810.4829



Factorized into energy evolution of wavefunctions



Suppressed

JIMWLK factorization: $p^\eta=0$ (small x !) modes that are coherent with the nuclei can be factorized for inclusive observables

$$\langle T^{\mu\nu}(\tau,\underline{\eta},x_\perp)\rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau,x_\perp)$$

$$Y_1 = Y_{\rm beam} - \eta\,;\; Y_2 = Y_{\rm beam} + \eta$$

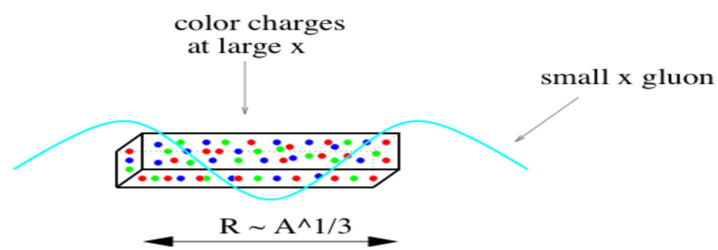
W's are universal "functional density matrices" describing distribution of large x color sources ρ_1 and ρ_2 of incoming nuclei; can be extracted from DIS or hadronic collisions

Initial conditions for quantum evolution

For large nuclei, general considerations about the color structure of higher dimensional representations of color charge density ρ^a probed give as an initial condition for evolution (MV model)

$$W_{x_0}[\rho] = \exp\left[-\int d^2x_\perp \frac{\rho^a(x_\perp)\rho^a(x_\perp)}{2\;\mu^2}\right]$$

μ^2 = Color charge squared per unit area ~ $A^{1/3}$

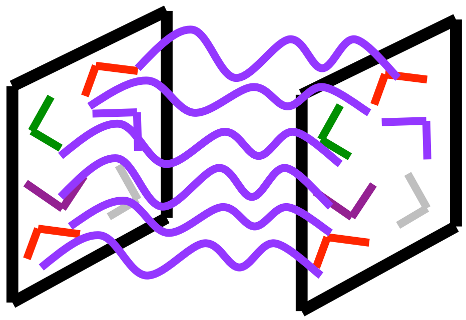


Other (sub-leading in A) contributions to these initial conditions

Jeon, RV
Dumitru, Jalilian-Marian, Petreska

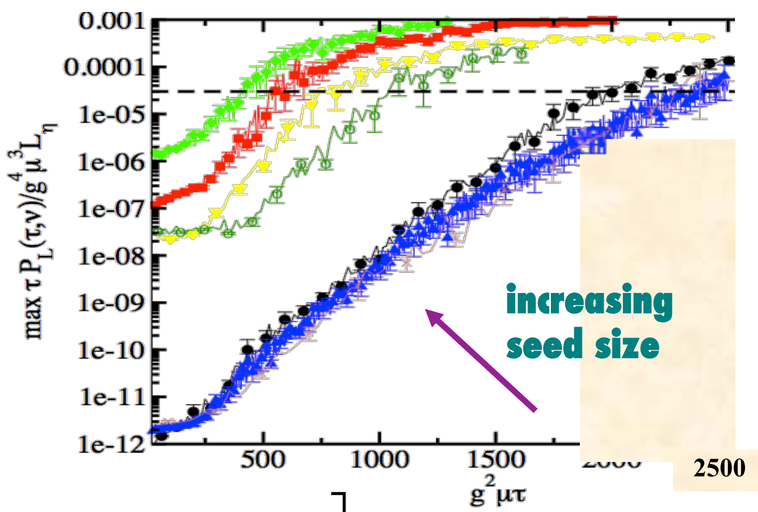
From Glasma to Plasma

Romatschke,RV
Fukushima,Gelis,McLerran



Quant. fluct. grow exponentially after collision

As large as classical field at 1/Qs !



$$T^{\mu\nu}_{\mathrm{N^{n-1}LO}} = \left[\int_\Sigma d^3u_1 \cdots d^3u_n \, \Gamma_n(u_1 \cdots u_n) \cdot \mathbf{T}_{u_1} \cdots \mathbf{T}_{u_n}\right] T^{\mu\nu}_{\mathrm{LO}}$$

Gelis,Lappi,RV

For $p_\eta \neq 0$ modes: $$\mathbf{T}_u \mathcal{A}(x) \sim \frac{\delta \mathcal{A}(x)}{\delta \mathcal{A}(0,y)} \sim \exp\left(\sqrt{Q_s \tau}\right)$$

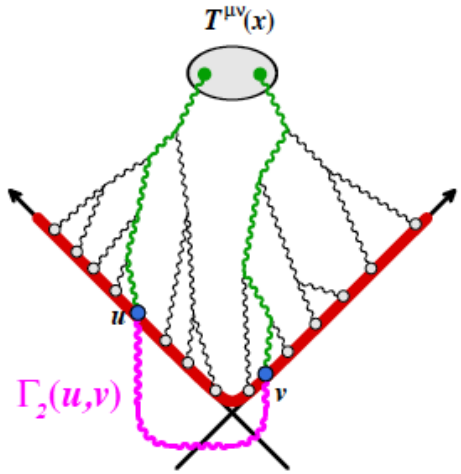
- **Requires resummation of ``secular" divergences to all orders in pert. theory**

$$\left[g \exp\left(\sqrt{Q_S \tau}\right)\right]^n$$

Quantum fluctuations: power counting

Dusling, Gelis, RV, arXiv1106.3297 (2011)

NLO:

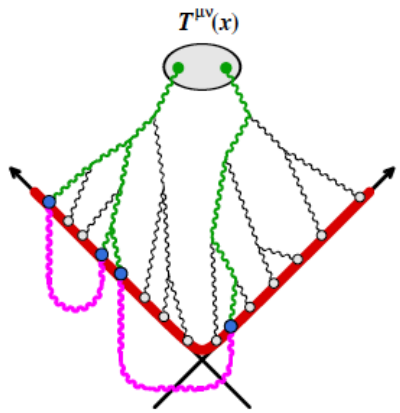


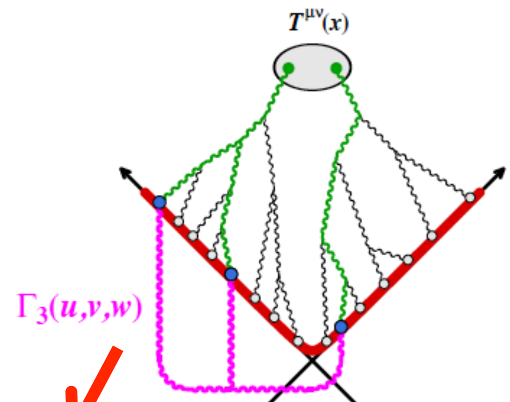
$$\Gamma_2^{\mu\nu}(u_1,u_2) = \int \frac{d^3k}{(2\pi)^3 2E_k}\, a^\mu_{-k}(u_1)\, a^\nu_{+k}(u_2)$$

$$\left[\frac{\delta^2 S_{\rm YM}}{\delta A^\mu A^\nu}\right]_{\rm A=A_{cl}} a^\nu_{\pm k} = 0$$

$$\lim_{x^0\to-\infty} a^\mu_{\pm k,\lambda a}(x) = \epsilon^\mu(k)\, T^a\, e^{\pm i k\cdot x}$$

Higher orders:





Suppressed by g relative to term on left in the power counting

Resum leading contributions: $T^{\mu\nu}_{\rm resummed}(x) = \exp\left[\frac{1}{2}\int_\Sigma d^3u\, d^3v\, \Gamma_2(u,v)\cdot \mathbf{T}_u \mathbf{T}_v\right] T^{\mu\nu}_{\rm LO}$

Spectrum of initial fluctuations

Dusling,Gelis,RV, arXiv1106.3297 (2011)

$$T^{\mu\nu}_{\text{resummed}}(x) = \int \mathcal{D}\alpha \, F_0[\alpha] \, T^{\mu\nu}_{\text{LO}}[A_{\text{cl.}} + \alpha](x)$$

$$F_0[\alpha] \propto \exp\left[-\frac{1}{2}\int_\Sigma d^3u \, d^3v \, \alpha(u) \, {\Gamma_2}^{-1}\alpha(v)\right]$$

Initial spectrum of fluctuations

$$\langle\langle T^{\mu\nu}\rangle\rangle_{\text{LLx+Linst.}} = \int [D\rho_1][D\rho_2] \, W_{\text{Y}_{\text{beam}}-\text{Y}}[\rho_1] \, W_{\text{Y}_{\text{beam}}+\text{Y}}[\rho_2]$$
$$\times \int [da(u)] \, F_{\text{init}}[a] \, T^{\mu\nu}_{\text{LO}}[A_{\text{cl}}(\rho_1, \rho_2) + a]$$

Computing small fluctuations in the Glasma

1) Construct τ-independent inner product on initial Cauchy surface at $\tau=0^+$

2) Solve small fluctuation equations in Glasma background at $\tau=0^+$

3) Determine physical solutions

$$\langle c_{\nu k} c_{\mu l}\rangle = 0$$

Gaussian random variable

$$\langle c_{\nu k} c^*_{\mu l}\rangle = 2\pi\delta(\nu-\mu)\delta_{kl}$$

$$A(\tau,\eta,x_\perp) = A_{\rm cl.}(\tau,x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi} d\mu_K\, c_{\nu K}\, e^{i\nu\eta}\, \chi_K(x_\perp)\, H^{(2)}_{i\nu}(\lambda_K\tau) + c.c$$

$$[D^2 + V''(A_{\rm cl.})]\chi_K(x_\perp) = \lambda_K^2\, \chi_K(x_\perp)$$

4) Well defined algorithm – numerical computations feasible

The first fermi: a master formula

Also correlators of $T^{\mu\nu}$

✓ From solutions of B-JIMWLK

$$\langle\langle T^{\mu\nu}\rangle\rangle_{\rm LLx+Linst.} = \int [D\rho_1][D\rho_2]\, W_{\rm Y_{beam}-Y}[\rho_1]\, W_{\rm Y_{beam}+Y}[\rho_2]$$
$$\times \int [da(u)]\, F_{\rm init}[a]\, T^{\mu\nu}_{\rm LO}[A_{\rm cl}(\rho_1,\rho_2)+a]$$

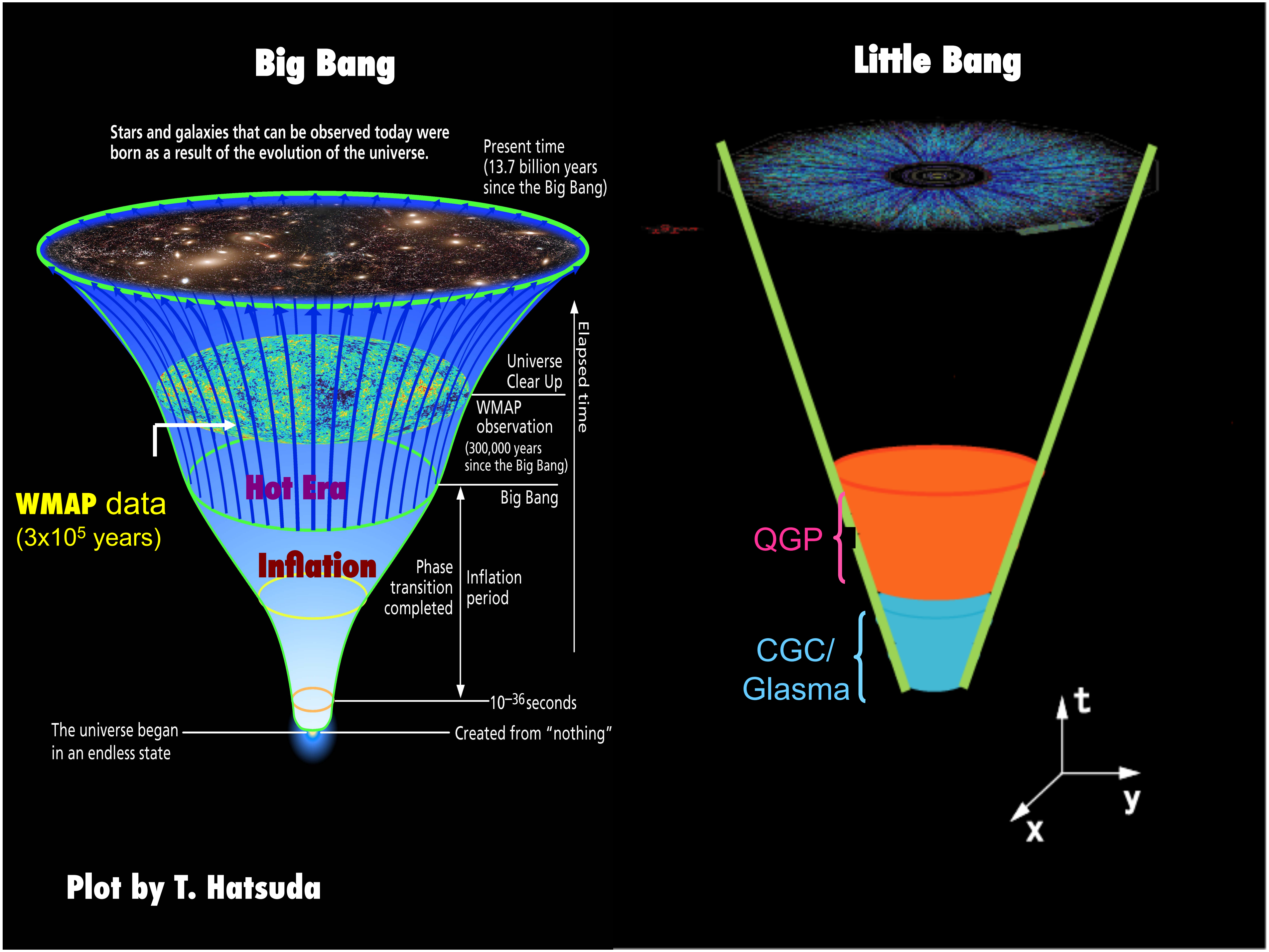
✧ Gauge invariant Gaussian spectrum of quantum fluctuations

✓ 3+1-D solutions of Yang-Mills equations

✧ Expression computed recently-numerical evaluation in progress

Dusling,Gelis,RV

- This is what needs to be matched to viscous hydrodynamics, event-by-event
- All modeling of initial conditions for heavy ion collisions includes various degrees of over simplification relative to this “master” formula



Big Bang
Stars and galaxies that can be observed today were born as a result of the evolution of the universe.
Present time (13.7 billion years since the Big Bang)
Elapsed time
Universe Clear Up
WMAP observation (300,000 years since the Big Bang)
Big Bang
Hot Era
WMAP data (3x10^5 years)
Inflation
Phase transition completed
Inflation period
10^-36 seconds
The universe began in an endless state
Created from "nothing"
Little Bang
QGP
CGC/ Glasma
t
y
x
Plot by T. Hatsuda

Big Bang vs. Little Bang

Decaying Inflaton
with occupation # $1/g^2$ ⟷ Decaying Glasma
with occupation # $1/g^2$

Explosive amplification
of low mom. small
fluctuations (preheating) ⟷ Explosive amplification
of low mom. small fluct.
(Weibel instabilities)

Int. of fluctutations/inflaton
-> thermalization ? ⟷ Int. of fluctutations/Glasma
-> thermalization ?

Other common features: topological defects, turbulence ?

Glasma spectrum of initial quantum fluctuations

Path integral over small fluctuations equivalent to

$$A(x_\perp,\tau,\eta) = A_{\rm cl.}(x_\perp,\tau) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\, c_{\nu k}\, e^{i\nu\eta}\, \chi_k(x_\perp)\, H_{i\nu}(\lambda_k\tau) + c.c$$

Gaussian random variables

Berry conjecture: High lying quantum eigenstates of classically chaotic systems, linear superpositions of Gaussian random variables

Yang-Mills is a classically chaotic theory

B. Muller et al.

Srednicki: Systems that satisfy Berry's conjecture exhibit "eigenstate thermalization"

Also, Jarzynski, Rigol, …

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

scalar Φ^4 theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{g^2}{4}\phi^4 + J\phi$$

$$J = \theta(-x^0)\frac{Q^3}{g}$$

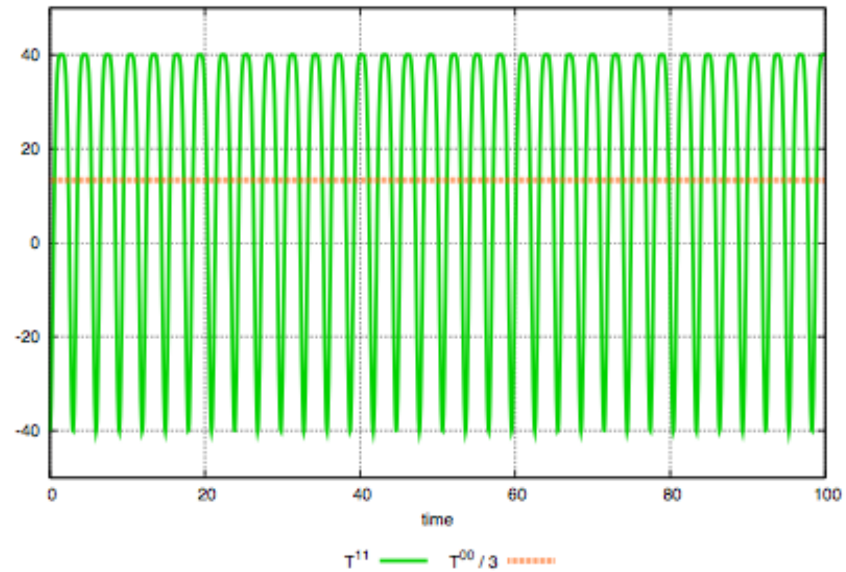
Components of Stress-Energy tensor:

$$\begin{aligned}
\varepsilon &= \frac{1}{2}\Big(\dot{\phi}^2 + (\nabla_\perp \phi)^2 + \tau^{-2}(\partial_\eta \phi)^2\Big) + V(\phi) \\
T^{xx} &= \frac{1}{2}\Big(\dot{\phi}^2 + (\partial_x \phi)^2 - (\partial_y \phi)^2 - \tau^{-2}(\partial_\eta \phi)^2\Big) - V(\phi) \\
T^{yy} &= \frac{1}{2}\Big(\dot{\phi}^2 - (\partial_x \phi)^2 + (\partial_y \phi)^2 - \tau^{-2}(\partial_\eta \phi)^2\Big) - V(\phi) \\
\tau^2 T^{\eta\eta} &= \frac{1}{2}\Big(\dot{\phi}^2 - (\partial_x \phi)^2 - (\partial_y \phi)^2 + \tau^{-2}(\partial_\eta \phi)^2\Big) - V(\phi)\,.
\end{aligned}$$

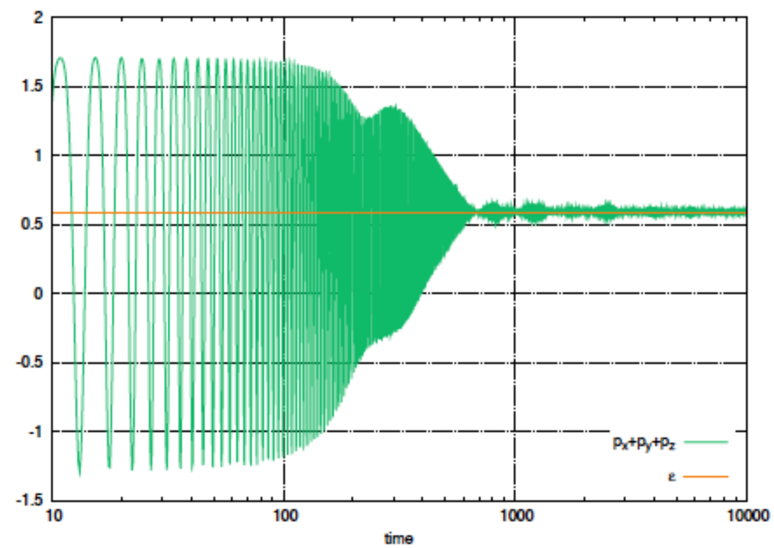
Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

scalar Φ^4 theory in fixed volume:



Energy density and pressure without averaging over fluctuations



Energy density and pressure after averaging over fluctuations

→ **Converges to single valued relation "EOS"**

Hydrodynamics from quantum fluctuations

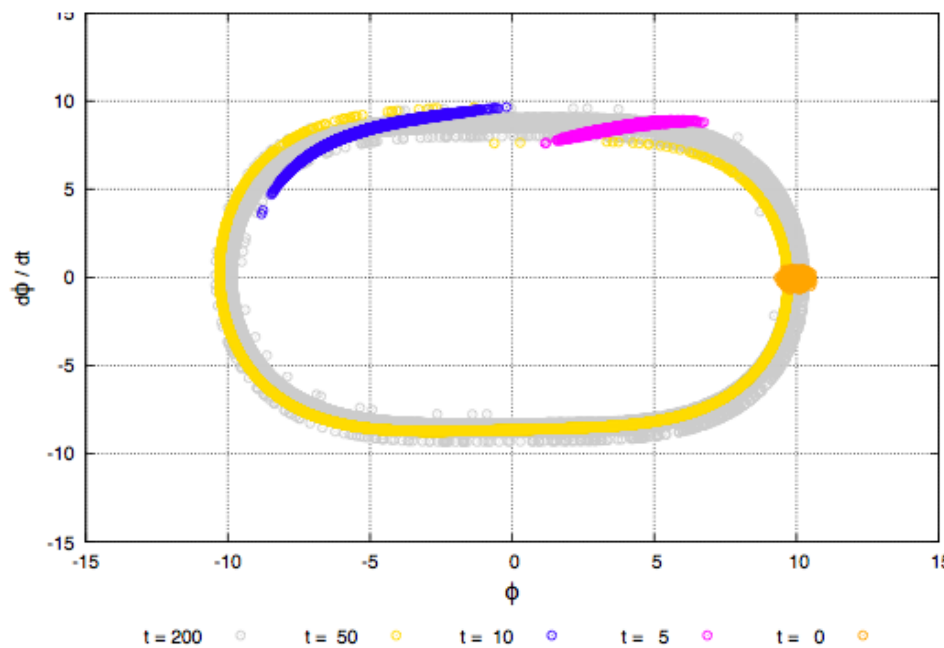
Dusling,Epelbaum,Gelis,RV (2011)

Anatomy of phase decoherence:

$\Delta\Theta = \Delta\omega\ t$

$T_{period} = 2\pi\ /\ \Delta\omega$

➔ $T_{period} \cong 18.2\ /\ g\ \Delta\Phi_{max}$



Different field amplitudes from different initializations of the classical field

$$\langle T^\mu_\mu \rangle = \int d\phi\, d\dot\phi\ \rho_t(\phi,\dot\phi)\ T^\mu_\mu(\phi,\dot\phi) \equiv \int dE\, d\theta\ \tilde\rho_t(E,\theta)\ T^\mu_\mu(E,\theta)$$

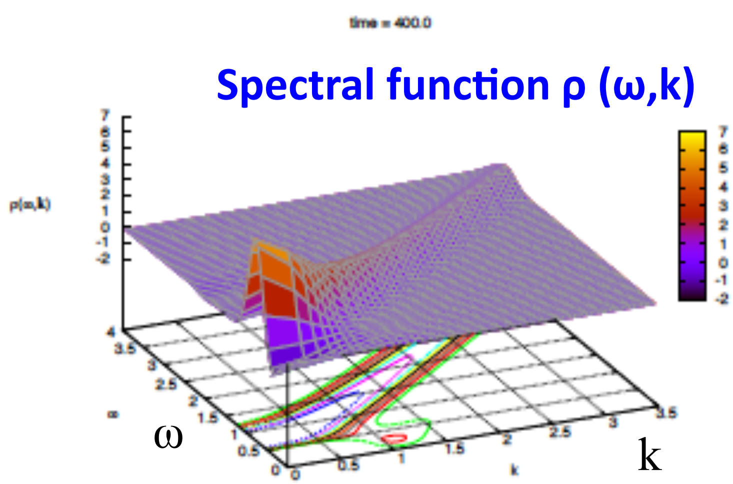
$$t \overset{\approx}{\to} \infty \quad \int dE\ \tilde\rho_t(E) \int d\theta\ T^\mu_\mu(E,\theta) = 0$$

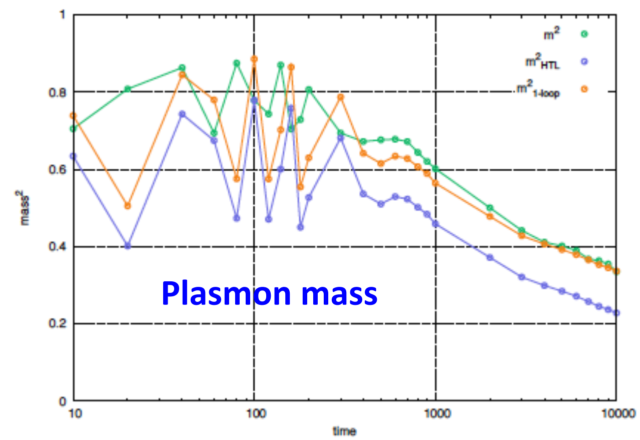
$$\int d\theta\ T^\mu_\mu(E,\theta) = \frac{2\pi}{T} \int_t^{t+T} d\tau\ T^\mu_\mu(\phi(\tau),\dot\phi(\tau)) = 0$$

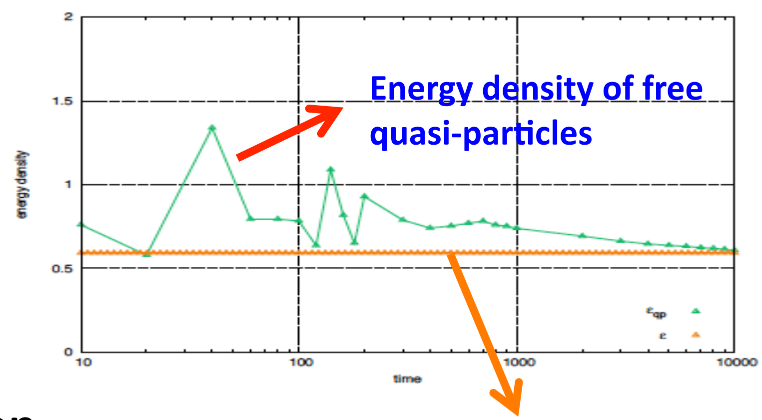
Because $T_\mu^{\ \mu}$ for scalar theory is a total derivative and ϕ is periodic

Quasi-particle description?

Epelbaum,Gelis (2011)





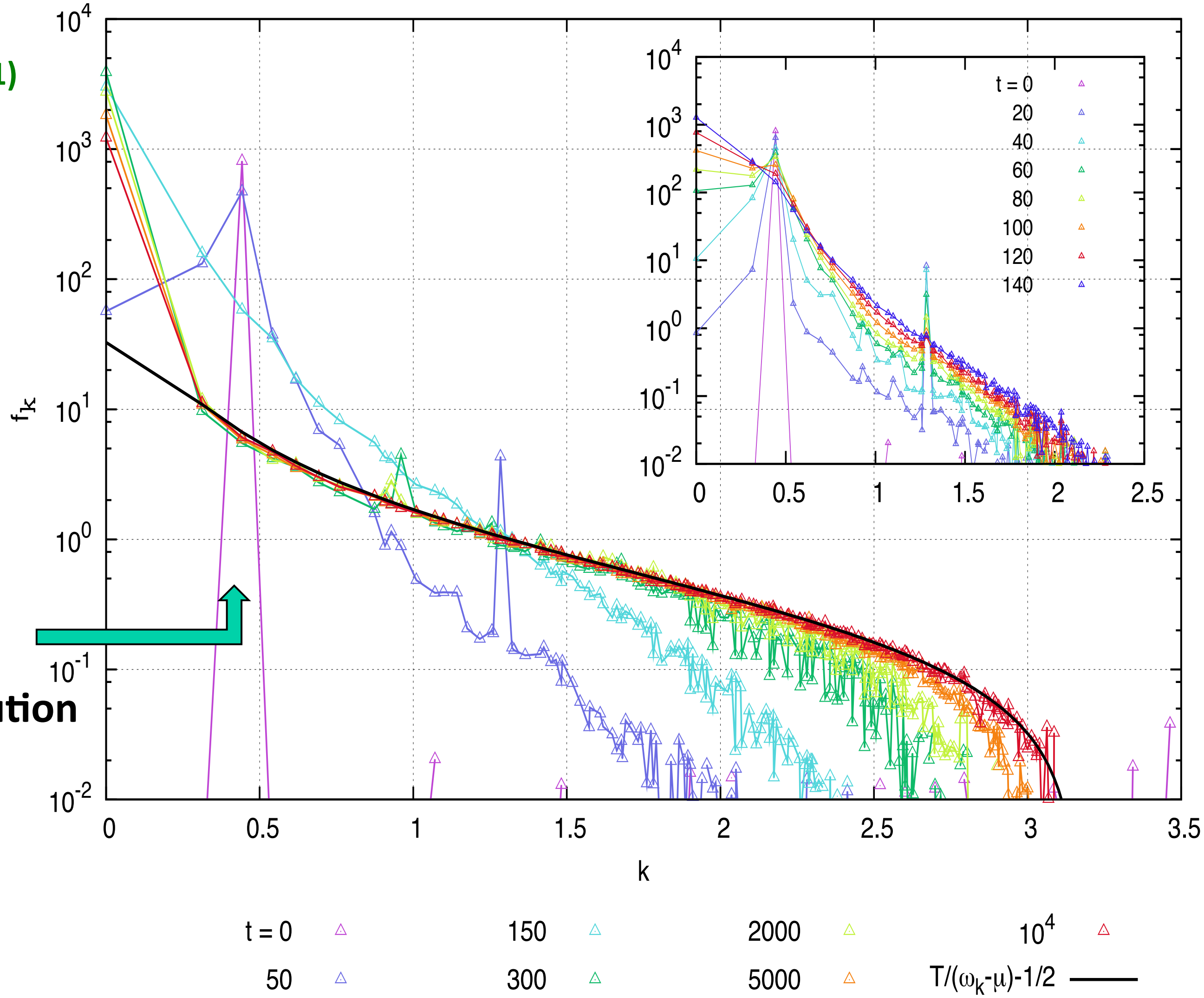


- At early times, no quasi-particle description

- May have quasi-particle description at late times. Effective kinetic "Boltzmann" description in terms of interacting quasi-particles at late times ?

Quasi-particle occupation number

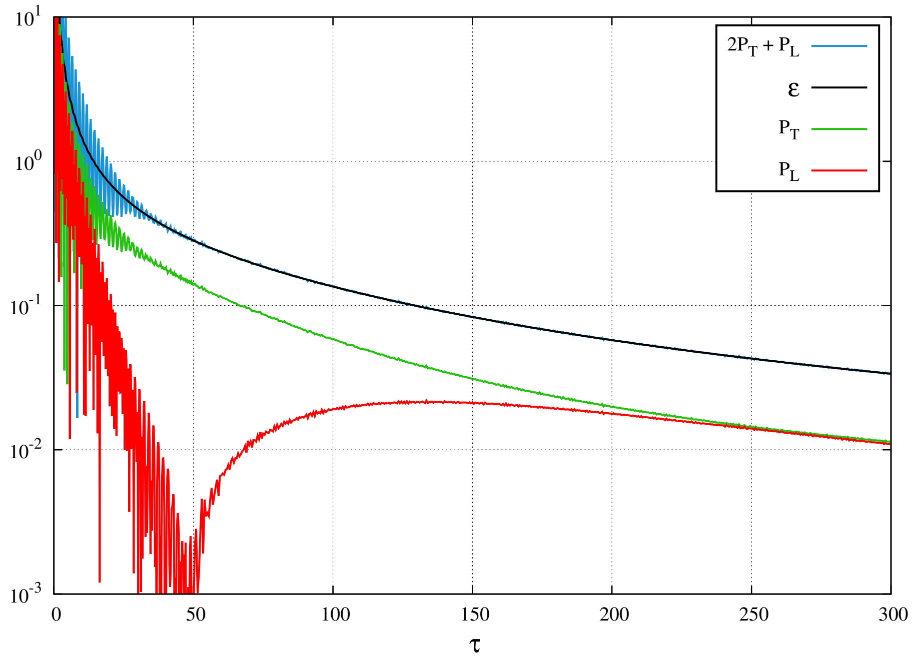
Gelis,Epelbaum (2011)



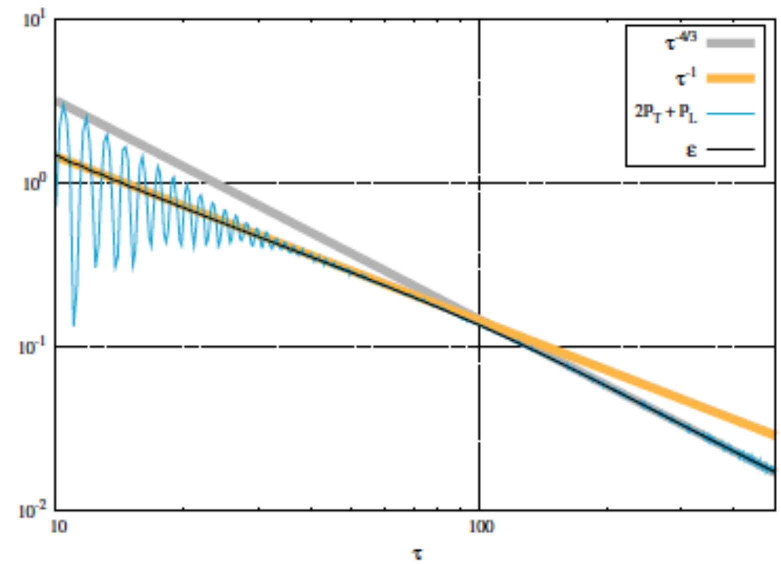
Initial
mode distribution

System becomes over occupied relative to a thermal distribution…

Proof of concept: isotropization of longitudinally expanding fields in scalar Φ^4



Dusling,Epelbaum,Gelis,RV: arXiv:1206.3336



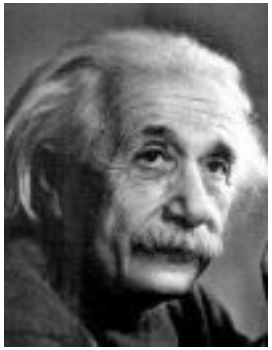
QCD – similar framework – more challenging computationally and conceptually

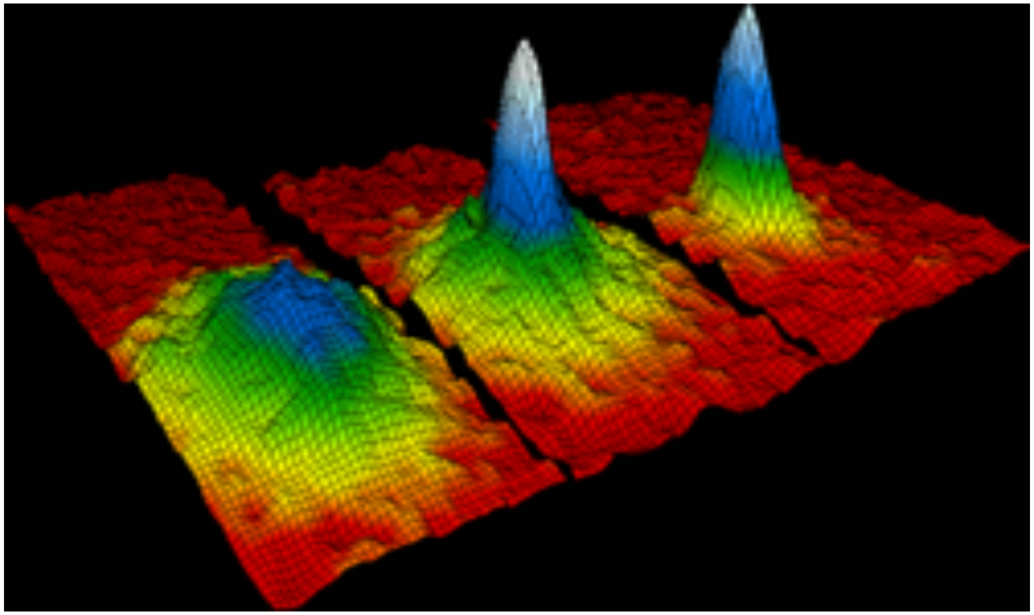
Dusling, Gelis, RV: arXiv 1106.3927

Numerical development underway – results hopefully very soon…

Bose-Einstein Condensation in HI Collisions ?

Blaizot,Gelis,Liao,McLerran,RV: arXiv:1107.5295v2





Cold rubidium atoms in a magnetic trap

Gell-Mann's Totalitarian Principle of Quantum Mechanics:
Everything that is not forbidden is Compulsory

✧ *Possible phenomenological consequences…*

Mickey Chiu et al., 1202.3679

Bose-Einstein Condensation and Thermalization

Blaizot,Gelis,Liao,McLerran,RV: arXiv:1107.5295v2

Assumption: Evolution of "classical" fields in the Glasma can be matched to a quasi-particle transport description

See also, Mueller, Son (200)2
Jeon (2005)

All estimates are "parametric": $\alpha_S << 1$

System is over-occupied: $n \approx Q_S^3/\alpha_s$; $\varepsilon = Q_S^4/\alpha_S$

$\Rightarrow n \bullet \varepsilon^{-3/4} \approx 1/\alpha_S^{1/4} >> 1$

In a thermal system, $n \bullet \varepsilon^{-3/4} = 1$

If a system is over-occupied near equilibrium and elastic scattering dominates, it can generate a Bose-Einstein condensate

Known in context of inflation:
Khlebnikov, Tkachev (1996)
Berges et al. (2011)

Bose-Einstein Condensation and Thermalization

$$n_{\rm eq} = \int_{\mathbf{p}} f_{\rm eq}(\mathbf{p}) \; ; \; \varepsilon_{\rm eq} = \int_{\mathbf{p}} \omega_{\mathbf{p}} \, f_{\rm eq}(\mathbf{p})$$

$$f_{\rm eq}(\mathbf{p}) = \frac{1}{e^{\beta(\omega_p - \mu)-1}}$$

In a many-body system, gluons develop a mass
$\omega_{p=0} = m \approx \alpha_S^{1/2}\, T$

If over-occupation persists for μ = m, system develops a condensate

$$f_{\rm eq}(\mathbf{p}) = n_c \delta^3(\mathbf{p}) + \frac{1}{e^{\beta(\omega_p - m)-1}}$$

$$n_c = \frac{Q_s^3}{\alpha_S}\left(1 - \alpha_S^{1/4}\right)$$

As α_S ➔ 0, most particles go into the condensate

$$\varepsilon_c = m\, n_c \approx \alpha_S^{1/4}\, T^4 << T^4$$

It however carries a small fraction of the energy density…

Transport in the Glasma

$$\frac{df}{dt} \equiv \partial_\tau f - \frac{p_z}{\tau} \partial_{p_z} f = C[f]$$

"Landau" equation for small angle 2 → 2 scattering:

$$\frac{df}{dt}|_{\text{coll}} \sim \frac{\Lambda_S^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[\frac{df}{dp} + \frac{\alpha_S}{\Lambda_S} f(p)(1 + f(p)) \right] \right\}$$

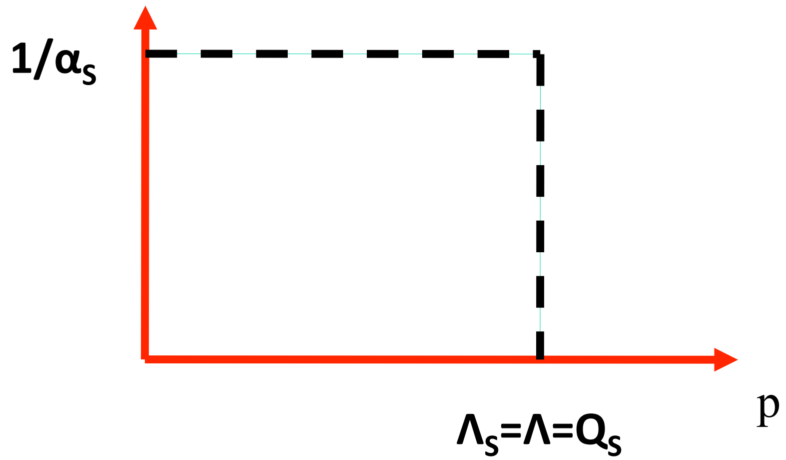
This is satisfied by a distribution where

$$f \sim \frac{1}{\alpha_S} \; ; p < \Lambda_S \qquad \sim \frac{1}{\alpha_S} \frac{\Lambda_S}{p} \; ; \Lambda_S < p < \Lambda \qquad \sim 0 \; ; \Lambda < p$$

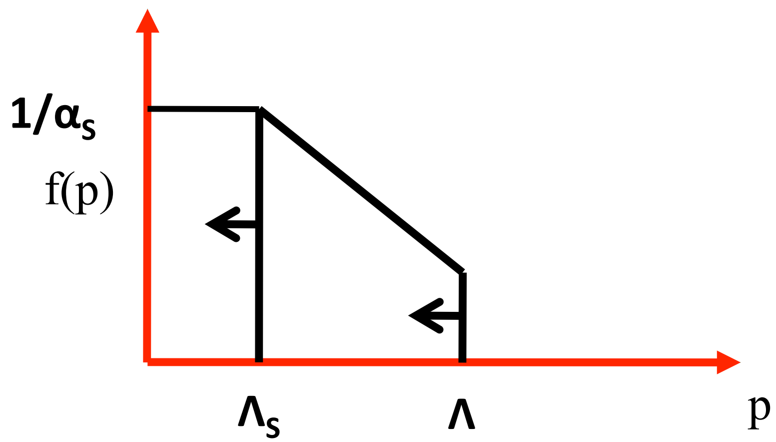
Λ_S and Λ are dynamical scales determined by the transport equation

Transport in the Glasma

At $\tau \sim 1/Q_S$



At $\tau > 1/Q_S$



When $\Lambda_S = \alpha_S \Lambda$, the system thermalizes;
one gets the ordering of scales: $\Lambda = T$, $m = \Lambda \Lambda_S = \alpha^{1/2} T$, $\Lambda_S = \alpha_S T$

Thermalization: from Glasma to Plasma

<u>Fixed box</u>: Energy conservation gives $\Lambda^3 \Lambda_S$ = constant

From moments of transport eqn., , $\tau_{coll} = \Lambda / \Lambda_S^2 \sim t$

From these two conditions, $\Lambda_S \sim Q_s \left(\frac{t_0}{t}\right)^{3/7}$ $\Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{1/7}$

Thermalization time: $t_{\mathrm{therm.}} \sim \frac{1}{Q_S} \left(\frac{1}{\alpha_S}\right)^{7/4}$

Also, Kurkela, Moore (2011))

Entropy density s = Λ^3 increases and saturates at t_{therm} as T^3

$N_{quark} \sim \Lambda^3 = N_{gluon} (\Lambda^2 \Lambda_S/\alpha_S)$ at t_{therm} when $\Lambda_S = \alpha_S \Lambda$

Thermalization: from Glasma to Plasma

Expanding box : **matter is strongly self interacting for fixed anisotropy**

$$\varepsilon_g(t) \sim \varepsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta}$$

$0 < \delta \leq 1/3$

$$\Lambda_S \sim Q_S \left(\frac{t_0}{t}\right)^{(4+\delta)/7} \qquad \Lambda \sim Q_S \left(\frac{t_0}{t}\right)^{(1+2\delta)/7}$$

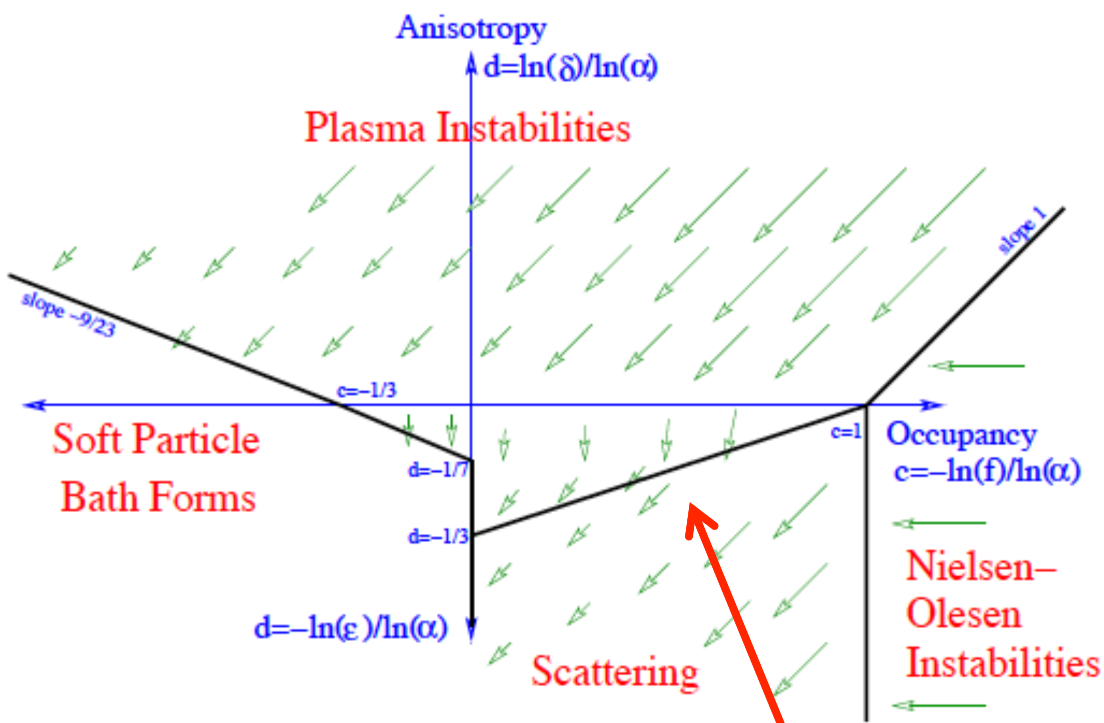
Thermalization time t_{therm} $= \frac{1}{Q_S}\left(\frac{\tau_0}{\tau}\right)^{7/(3-\delta)}$

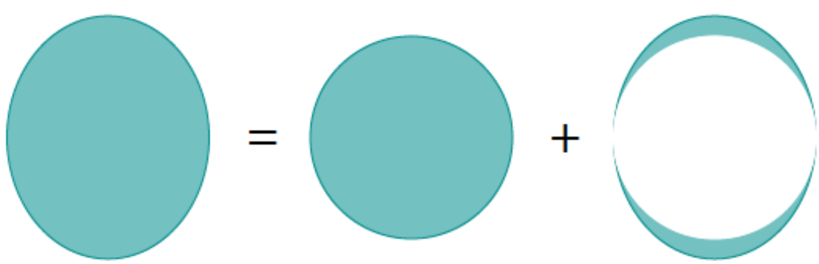
For $\delta = -1$, recover fixed box results…

A condensate can still form in the expanding case for $\delta > 1/5$

What about plasma instabilities ?

Kurkela, Moore(2011)
Schlichtling,Mclerran, RV, in preparation





Relevant case for us: in general need careful numerical study to gauge impact on scaling solutions

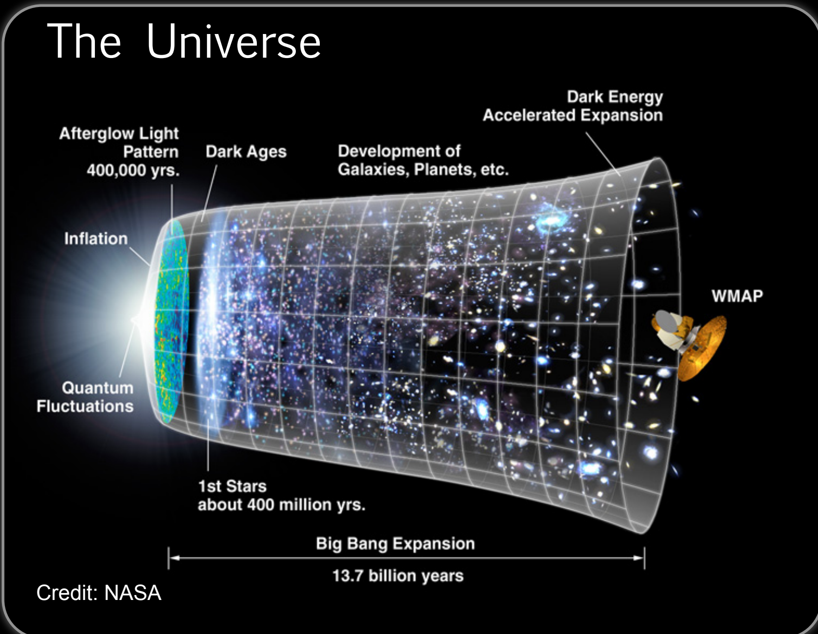
Summary

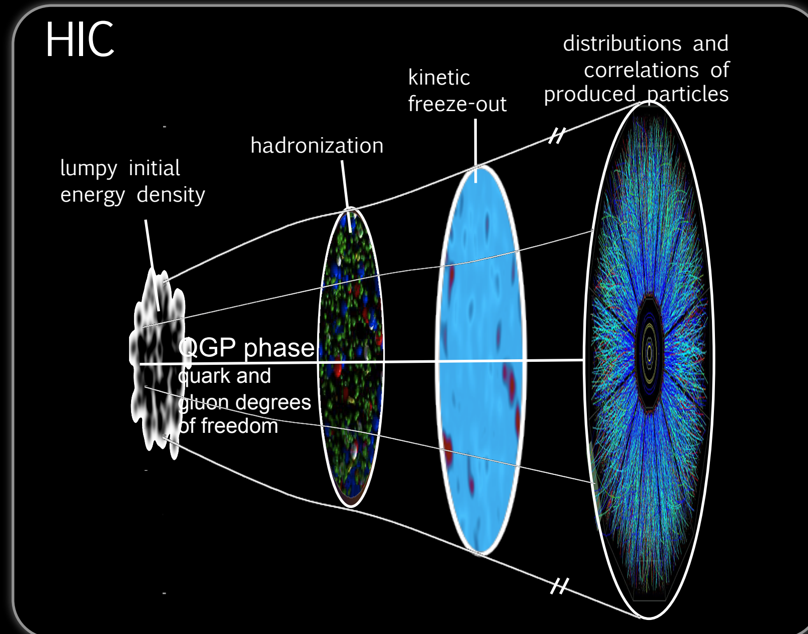
- **Presented *ab initio* picture of multi-particle production and thermalization in heavy ion collisions**
- **Thermalization is a subtle business even in weak coupling**
- **Hydrodynamics may be unreasonably effective because it requires rapid decoherence of classical fields and strong self-interactions, not thermalization**
- **Exciting possibility of a transient Bose-Einstein Condensate**

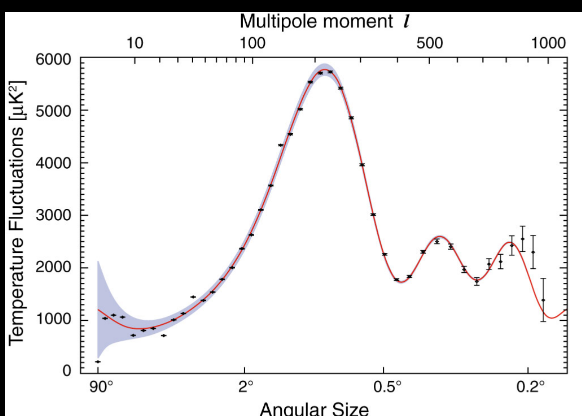
THE END

An Analogy with the Early Universe

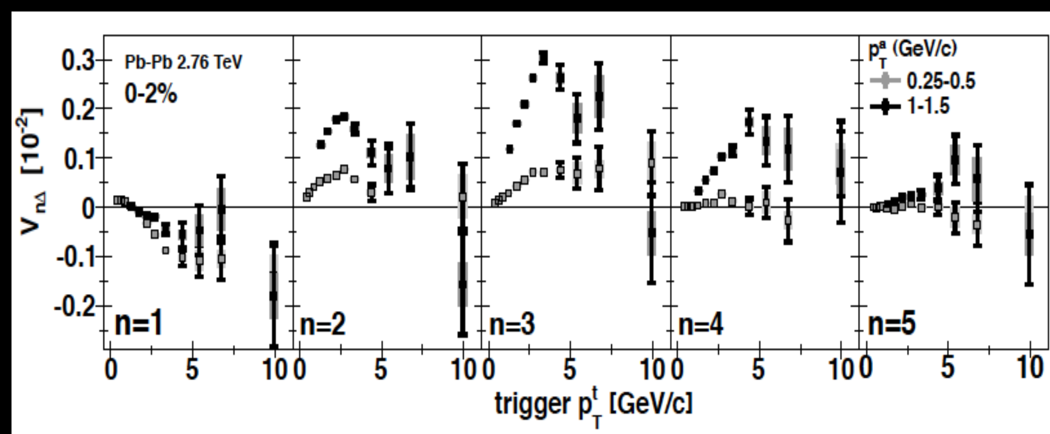
Mishra et al; Mocsy- Sorensen





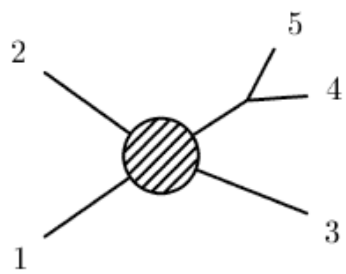


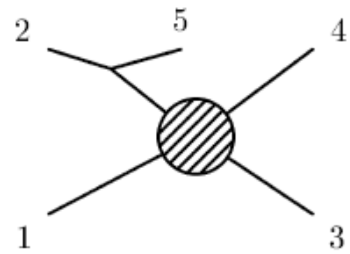
WMAP



HIC-ALICE

Role of inelastic processes ?





Wong (2004)
Mueller,Shoshi,Wong (2006)

Power counting for n ➔ m processes contributions to the collision integral

Vertices contribute α_S^{n+m-2}

Factor of $(\Lambda_S/\alpha_S)^{n+m-2}$ from distribution functions

Screened infrared singularity: $(1/\Lambda\ \Lambda_S)^{n+m-4}$

Remaining phase space integrals Λ^{n+m-5}

Net result is $\tau_{inelas} \sim \Lambda / \Lambda_S^2 = \tau_{elas}$

At most parametrically of the same order as elastic scattering.
So a transient Bose-Einstein condensate can form.

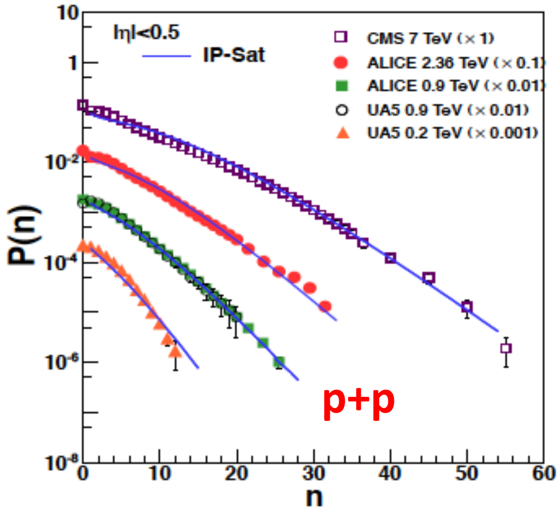
Numerical simulations will be decisive

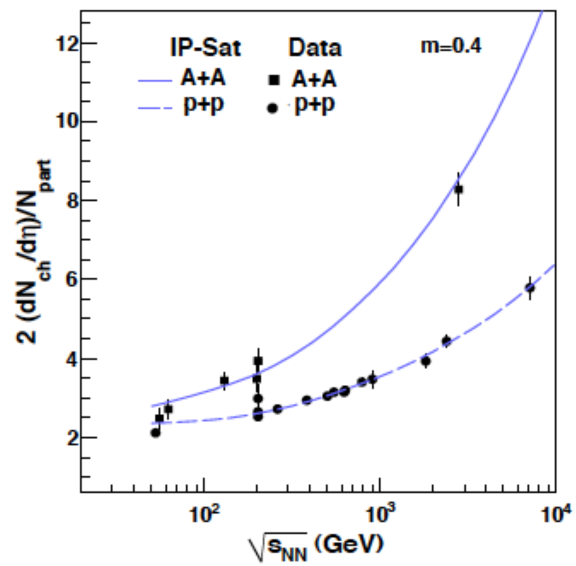
Dusling,Epelbaum,Gelis,RV, in progress
Blaizot, Liao, McLerran

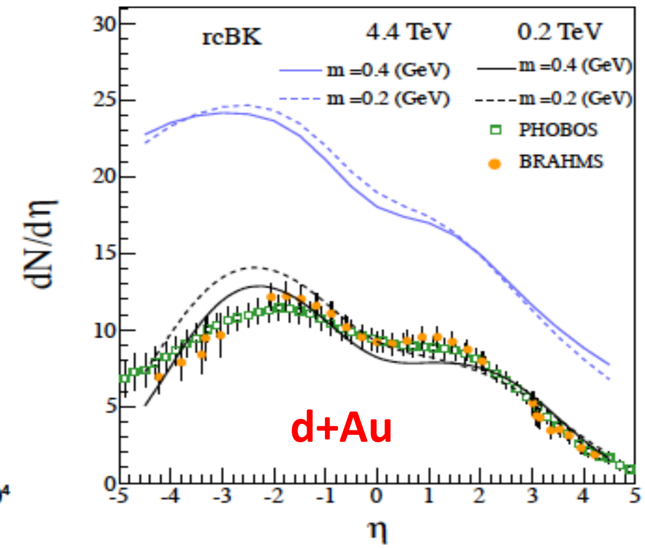
CGC based models and bulk distributions

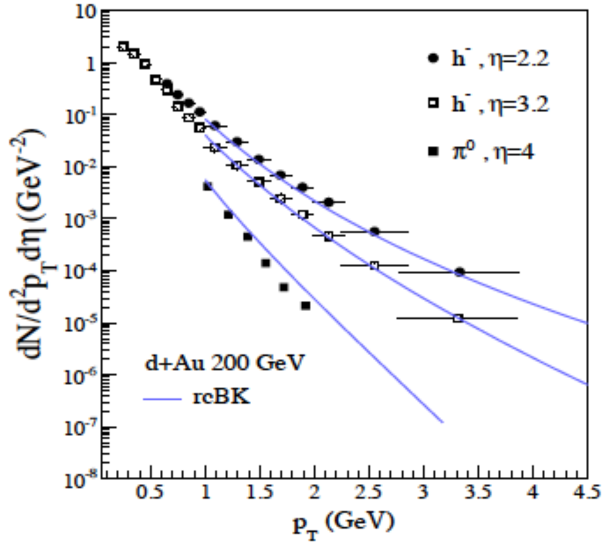
Kowalski, Motyka, Watt
Tribedy, RV: 1112.2445

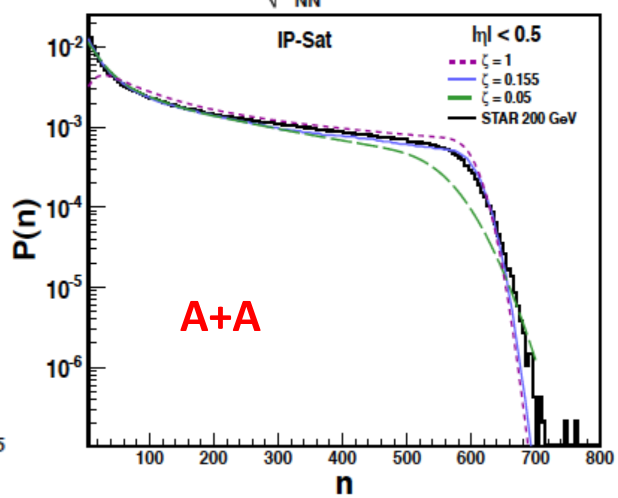
e+p constrained fits give good description of hadron data











Also:

Collimated long range Rapidity correlations "the ridge"

Di-hadron d+A correlations