Introduction to QCD and Jet III

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Outline

1. Dihadron Correlations
   - Breaking down of the k_t factorization in di-jet production
   - Probing two fundamental gluon distributions
   - Gluon+Jet in *pA*
2. NLO Forward Hadron Production in *pA* Collisions
   - LO Forward Hadron production in *pA* collisions
   - NLO Forward Hadron Production in *pA* Collisions
3. Conclusion

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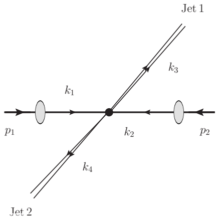
K_t Factorization "expectation"

Consider the inclusive production of two high-transverse-momentum back-to-back particles in hadron-hadron collisions, i.e., in the process:

$$H_1 + H_2 \to H_3 + H_4 + X.$$



The standard k_t factorization "expectation" is:

$$E_3 E_4 \frac{d\sigma}{d^3 p_3 d^3 p_4} = \sum \int d\hat{\sigma}_{i+j\to k+l+X} f_{i/1} f_{j/2} d_{3/k} d_{4/l} + \cdots$$

- Convolution of $d\hat{\sigma}$ with $f(x, k_\perp)$ and $d(z)$.
- Factorization ⇔ Factorization formula + Universality
- Only Drell-Yan process is proved for factorization in hadron-hadron collisions. [Bodwin; 85, 86], [Collins, Soper, Sterman; 85, 88].

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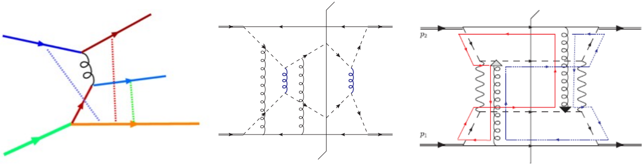
Breaking down of the k_t factorization in di-hadron production

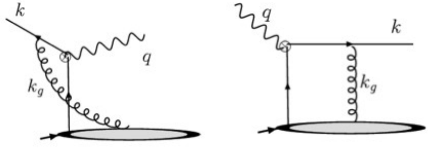
- [Bacchetta, Bomhof, Mulders and Pijlman; 04-06] Wilson lines approach
  Studies of Wilson-line operators show that the TMD parton distributions are not generally process-independent due to the complicated combinantion of initial and final state interactions. TMD PDFs admit process dependent Wilson lines.
- [Collins, Qiu; 07], [Collins; 07], [Vogelsang, Yuan; 07] and [Rogers, Mulders; 10]
  Scalar QED models and its generalization to QCD (Counterexample to Factorization)



- $\mathcal{O}(g^2)$ calculation shows non-vanishing anomalous terms with respect to standard factorization.
- Remarks: k_t factorization is violated in di-jet production; TMD parton distributions are non-universal.
- Things get worse: For pp and AA collisions, no factorization formula at all for dijet production.

Why is the di-jet production process special?

Initial state interactions and/or final state interactions



- In Drell-Yan process, there are only initial state interactions.

$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ - i\epsilon} A^+(k_g) = \int_0^{-\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

  Eikonal approximation $\Longrightarrow$ gauge links.

- In DIS, there are only final state interactions.

$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ + i\epsilon} A^+(k_g) = \int_0^{+\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

  Eikonal approximation $\Longrightarrow$ gauge links.

- However, there are both initial state interactions and final state interactions in the di-jet process.

McLerran-Venugopalan Model

In QCD, the McLerran-Venugopalan Model describes high density gluon distribution in a relativistic large nucleus ($A \gg 1$) by solving the classical Yang-Mills equation:

$$[D_\mu, F^{\mu\nu}] = gJ^\nu \quad \text{with} \quad J^\nu = \delta^{\nu +}\rho_a(x^-, x_\perp)T^a, \quad \text{COV gauge} \Rightarrow -\nabla_\perp^2 A^+ = g\rho.$$

To solve the above equation, we define the Green's function

$$\nabla_{z_\perp}^2 G(x_\perp - z_\perp) = \delta^{(2)}(x_\perp - z_\perp) \quad \Rightarrow \quad G(x_\perp - z_\perp) = -\int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \frac{e^{ik_\perp \cdot (x_\perp - z_\perp)}}{k_\perp^2}$$

MV model assumes that the density of color charges follows a Gaussian distribution

$$W[\rho] = \exp\left[-\int \mathrm{d}z^- \mathrm{d}^2 z_\perp \frac{\rho_a(z^-, z_\perp)\rho_a(z^-, z_\perp)}{2\mu^2(z^-)}\right].$$

With such a weight, average of two color sources is

$$\langle \rho_a \rho_b \rangle = \int \mathcal{D}[\rho] W[\rho] \rho_a(x^-, x_\perp)\rho_b(y^-, y_\perp) = \mu^2(x^-)\delta_{ab}\delta(x^- - y^-)\delta(x_\perp - y_\perp).$$

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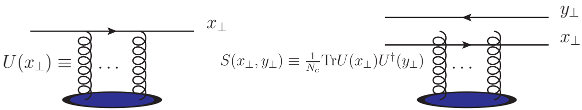
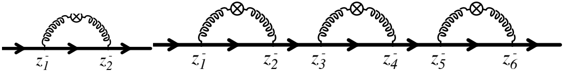
Dipole amplitude in MV model

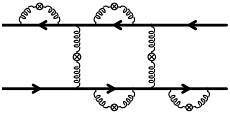
The Wilson line [F. Gelis, A. Peshier, 01]

$$U(x_\perp) = \mathcal{P} \exp\left[-ig^2 \int dz^- d^2 z_\perp G\left(x_\perp - z_\perp\right) \rho\left(z^-, z_\perp\right)\right]$$



Use gaussian approximation to pair color charges:





$$\Rightarrow S(x_\perp, y_\perp) \simeq \exp\left\{-\frac{\mu_s^2}{4}\int d^2 z_\perp \left[G\left(x_\perp - z_\perp\right) - G\left(y_\perp - z_\perp\right)\right]^2\right\}$$

$$\simeq \exp\left[-\frac{1}{4}Q_s^2\left(x_\perp - y_\perp\right)^2\right] \Leftarrow \text{GBW model}$$

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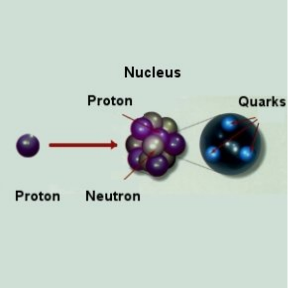
- Quadrupoles $\frac{1}{N_c}\mathrm{Tr}U_1 U_2^\dagger U_3 U_4^\dagger$ and Sextupoles $\frac{1}{N_c}\mathrm{Tr}U_1 U_2^\dagger U_3 U_4^\dagger U_5 U_6^\dagger$...

Forward observables at pA collisions



Why pA collisions?

- For pA (dilute-dense system) collisions, there is an effective k_t factorization.

$$\frac{d\sigma^{pA\to qfX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p,\mu^2) x_A f(x_A, q_\perp^2) \frac{1}{\pi}\frac{d\hat{\sigma}}{d\hat{t}}.$$

- For dijet processes in pp, AA collisions, there is no k_t factorization[Collins, Qiu, 08],[Rogers, Mulders; 10].

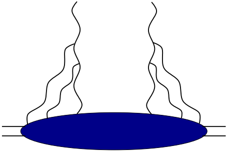
Why forward?

- At forward rapidity y, $x_p \propto e^y$ is large, while $x_A \propto e^{-y}$ is small.
- Ideal place to find gluon saturation in the target nucleus.

A Tale of Two Gluon Distributions

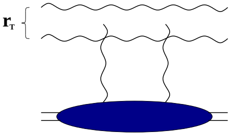
In small-x physics, two gluon distributions are widely used:[Kharzeev, Kovchegov, Tuchin; 03]
I. Weizsäcker Williams gluon distribution ([KM, 98'] and MV model):

$$xG^{(1)} = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \times \int \frac{d^2 r_\perp}{(2\pi)^2} \frac{e^{-ik_\perp \cdot r_\perp}}{r_\perp^2} \left(1 - e^{-\frac{r_\perp^2 Q_{sg}^2}{2}}\right) \Leftarrow$$



II. Color Dipole gluon distributions:

$$xG^{(2)} = \frac{S_\perp N_c}{2\pi^2 \alpha_s} k_\perp^2 \times \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} e^{-\frac{r_\perp^2 Q_{sq}^2}{4}} \Leftarrow$$



Remarks:

- The WW gluon distribution simply counts the number of gluons.
- The Color Dipole gluon distribution often appears in calculations.
- Does this mean that gluon distributions are non-universal? Answer: Yes and No!

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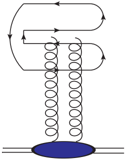
A Tale of Two Gluon Distributions

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution

$$xG^{(1)} = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \times \int \frac{d^2 r_\perp}{(2\pi)^2} \frac{e^{-ik_\perp \cdot r_\perp}}{r_\perp^2} \left(1 - e^{-\frac{r_\perp^2 Q_s^2}{2}}\right) \Leftarrow$$



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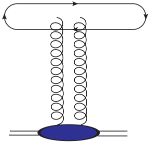
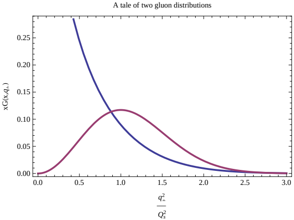




A Tale of Two Gluon Distributions

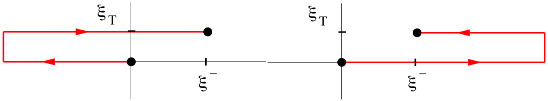
In terms of operators (known from TMD factorization), we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution:

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle .$$

II. Color Dipole gluon distributions:

$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle .$$



$\mathcal{U}^{[-]}$ $\mathcal{U}^{[+]}$

Remarks:

- The WW gluon distribution is the conventional gluon distributions. In light-cone gauge, it is the gluon density. (Only final state interactions.)
- The dipole gluon distribution has no such interpretation. (Initial and final state interactions.)
- Both definitions are gauge invariant.
- Same after integrating over $q_\perp$.

A Tale of Two Gluon Distributions

In terms of operators, we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, 11]

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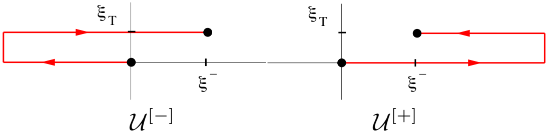
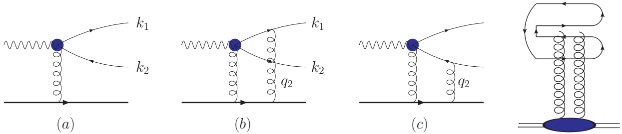


Questions:

- Can we distinguish these two gluon distributions? Yes, We Can.
- How to measure $xG^{(1)}$ directly? DIS dijet.
- How to measure $xG^{(2)}$ directly? Direct γ+Jet in pA collisions.
  For single-inclusive particle production in pA up to all order.
- What happens in gluon+jet production in pA collisions? It's complicated!

DIS dijet

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]



$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} \propto N_c\alpha_{em}e_q^2\int\frac{\mathrm{d}^2x}{(2\pi)^2}\frac{\mathrm{d}^2x'}{(2\pi)^2}\frac{\mathrm{d}^2b}{(2\pi)^2}\frac{\mathrm{d}^2b'}{(2\pi)^2}e^{-ik_{1\perp}\cdot(x-x')}$$
$$\times e^{-ik_{2\perp}\cdot(b-b')}\sum\psi_T^*(x-b)\psi_T(x'-b')$$
$$\underbrace{\left[1+S_{x_g}^{(4)}(x,b;b',x')-S_{x_g}^{(2)}(x,b)-S_{x_g}^{(2)}(b',x')\right]}_{-u_iu'_j\frac{1}{N_c}\langle\mathrm{Tr}\left[\partial^iU(v)\right]U^\dagger(v')\left[\partial^jU(v')\right]U^\dagger(v)\rangle_{x_g}\Rightarrow\text{Operator Def}},$$

- Eikonal approximation ⇒ Wilson Line approach [Kovner, Wiedemann, 01].
- In the dijet correlation limit, where $u=x-b\ll v=zx+(1-z)b$
- $S_{x_g}^{(4)}(x,b;b',x')=\frac{1}{N_c}\left\langle\mathrm{Tr}U(x)U^\dagger(x')U(b')U^\dagger(b)\right\rangle_{x_g}\neq S_{x_g}^{(2)}(x,b)S_{x_g}^{(2)}(b',x')$
- Quadrupoles are generically different objects and only appear in dijet processes.

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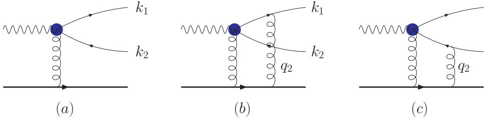
DIS dijet

The dijet production in DIS.



TMD factorization approach:

$$\frac{d\sigma^{\gamma_T^* A \to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} = \delta(x_{\gamma^*} - 1) x_g G^{(1)}(x_g, q_\perp) H_{\gamma_T^* g \to q\bar{q}},$$

Remarks:

- Dijet in DIS is the only physical process which can measure Weizsäcker Williams gluon distributions.
- Golden measurement for the Weizsäcker Williams gluon distributions of nuclei at small-x. The cross section is directly related to the WW gluon distribution.
- EIC and LHeC will provide us a perfect machine to study the strong gluon fields in nuclei. Important part in EIC and LHeC physics.

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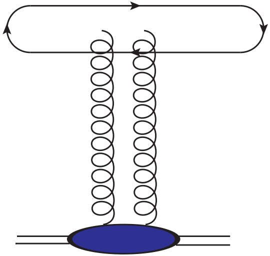
γ+Jet in pA collisions

The direct photon + jet production in pA collisions. (Drell-Yan follows the same factorization.)
TMD factorization approach:

$$\frac{d\sigma^{(pA\to\gamma q+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_f x_1 q(x_1,\mu^2) x_g G^{(2)}(x_g,q_\perp) H_{qg\to\gamma q}.$$

Remarks:

- Independent CGC calculation gives the identical result in the correlation limit.
- Direct measurement of the Color Dipole gluon distribution.



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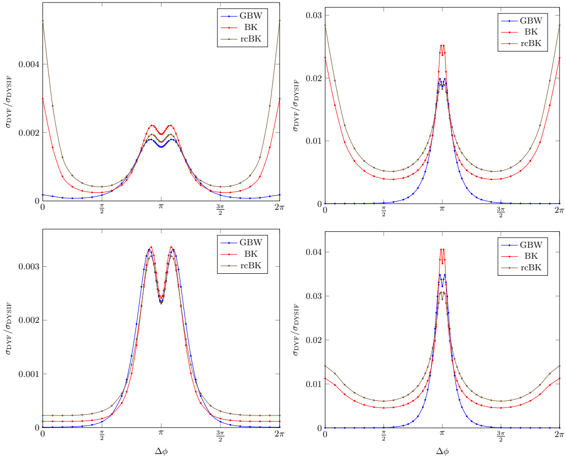
DY correlations in pA collisions

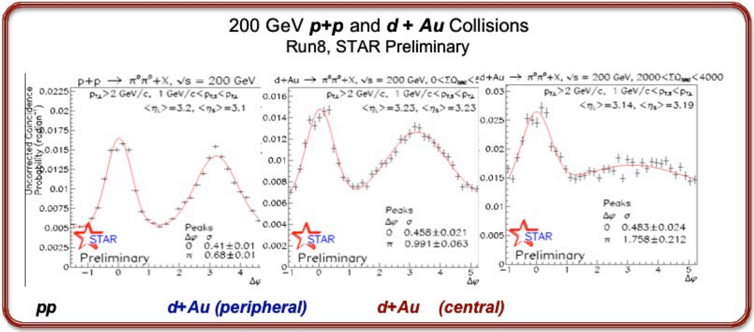
[Stasto, BX, Zaslavsky, 12]



$M = 0.5, 4\text{GeV}, Y = 2.5$ at RHIC dAu. $M = 4, 8\text{GeV}, Y = 4$ at LHC pPb.

- Partonic cross section vanishes at $\pi \Rightarrow$ Dip at π.
- Prompt photon calculation [J. Jalilian-Marian, A. Rezaeian, 12]

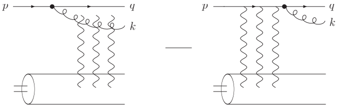
STAR measurement on di-hadron correlation in *dA* collisions

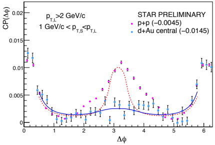


- There is no sign of suppression in the $p + p$ and $d + Au$ peripheral data.
- The suppression and broadening of the away side jet in $d + Au$ central collisions is due to the multiple interactions between partons and dense nuclear matter (CGC).
- Probably the best evidence for saturation.

First calculations on dijet production

Quark+Gluon channel [Marquet, 07] and [Albacete, Marquet, 10]





- Prediction of saturation physics.
- All the framework is correct, but over-simplified 4-point function.
- Improvement [F. Dominguez, C. Marquet, BX and F. Yuan, 11.]

$$S^{(4)}_{x_g}(x_1,x_2;x_2',x_1') \simeq e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)+\Gamma(x_2'-x_1')]}$$

$$-\frac{F(x_1,x_2;x_2',x_1')}{F(x_1,x_2';x_2,x_1')}\left(e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)+\Gamma(x_2'-x_1')]}-e^{-\frac{C_F}{2}[\Gamma(x_1-x_1')+\Gamma(x_2'-x_2)]}\right)$$

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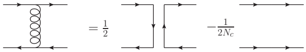
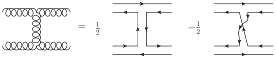
Dijet processes in the large N_c limit

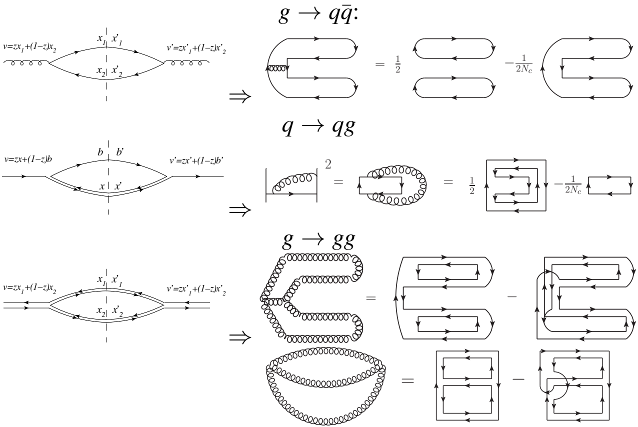
The Fierz identity:



and



Graphical representation of dijet processes



The Octupole and the Sextupole are suppressed.

Gluon+quark jets correlation

Including all the $qg \to qg$, $gg \to gg$ and $gg \to q\bar{q}$ channels, a lengthy calculation gives

$$\frac{d\sigma^{(pA\to \text{Dijet}+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_q x_1 q(x_1,\mu^2)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{qg}^{(1)}H_{qg}^{(1)} + \mathcal{F}_{qg}^{(2)}H_{qg}^{(2)}\right]$$
$$+x_1 g(x_1,\mu^2)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{gg}^{(1)}\left(H_{gg\to q\bar{q}}^{(1)} + \frac{1}{2}H_{gg\to gg}^{(1)}\right)\right.$$
$$\left.+\mathcal{F}_{gg}^{(2)}\left(H_{gg\to q\bar{q}}^{(2)} + \frac{1}{2}H_{gg\to gg}^{(2)}\right) + \mathcal{F}_{gg}^{(3)}\frac{1}{2}H_{gg\to gg}^{(3)}\right],$$

with the various gluon distributions defined as

$$\mathcal{F}_{qg}^{(1)} = xG^{(2)}(x,q_\perp), \quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)}\otimes F,$$
$$\mathcal{F}_{gg}^{(1)} = \int xG^{(2)}\otimes F, \quad \mathcal{F}_{gg}^{(2)} = -\int \frac{q_{1\perp}\cdot q_{2\perp}}{q_{1\perp}^2}xG^{(2)}\otimes F,$$
$$\mathcal{F}_{gg}^{(3)} = \int xG^{(1)}(q_1)\otimes F\otimes F,$$

where $F = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \text{Tr}U(r_\perp)U^\dagger(0)\right\rangle_{x_g}$.

Remarks:

- Only the term in NavyBlue color was known before.
- This describes the dihadron correlation data measured at RHIC in forward dAu collisions.

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Illustration of gluon distributions

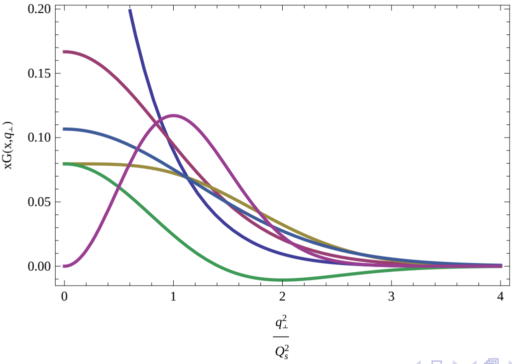
The various gluon distributions:

$$xG^{(1)}_{\mathrm{WW}}(x,q_\perp),\quad \mathcal{F}^{(1)}_{qg} = xG^{(2)}(x,q_\perp),$$

$$\mathcal{F}^{(1)}_{gg} = \int xG^{(2)} \otimes F,\quad \mathcal{F}^{(2)}_{gg} = -\int \frac{q_{1\perp}\cdot q_{2\perp}}{q^2_{1\perp}} xG^{(2)} \otimes F\,,$$

$$\mathcal{F}^{(3)}_{gg} = \int xG^{(1)}(q_1) \otimes F \otimes F\,,\quad \mathcal{F}^{(2)}_{qg} = \int xG^{(1)} \otimes F$$

6 different gluon distributions



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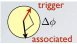
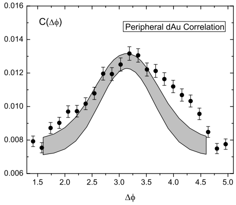
Comparing to STAR and PHENIX data

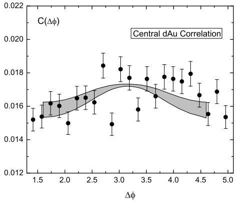


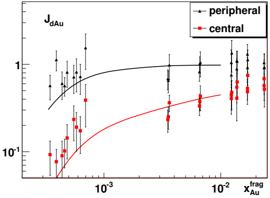
Physics predicted by C. Marquet. Further calculated in[A. Stasto, BX, F. Yuan, 11]
For away side peak in both peripheral and central *dAu* collisions

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{pA\to h_1 h_2}}{dy_1 dy_2 d^2 p_{1\perp} d^2 p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA\to h_1}}{dy_1 d^2 p_{1\perp}}}$$

$$J_{dA} = \frac{1}{\langle N_{\rm coll}\rangle} \frac{\sigma_{dA}^{\rm pair}/\sigma_{dA}}{\sigma_{pp}^{\rm pair}/\sigma_{pp}}$$







- Using: $Q_{sA}^2 = c(b)A^{1/3}Q_s^2(x)$.
- Physical picture: Dense gluonic matter suppresses the away side peak.

Conclusion and Outlook

Conclusion:

- DIS dijet provides direct information of the WW gluon distributions. Perfect for testing CGC, and ideal measurement for EIC and LHeC.
- Modified Universality for Gluon Distributions:

	Inclusive	Single Inc	DIS dijet	γ +jet	g+jet
$xG^{(1)}$	$\times$	$\times$	$\surd$	$\times$	$\surd$
$xG^{(2)}$, F	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

$\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

- Two fundamental gluon distributions. Other gluon distributions are just different combinations and convolutions of these two.
- The small-x evolution of the WW gluon distribution, a different equation from Balitsky-Kovchegov equation;[Dominguez, Mueller, Munier, Xiao, 11]
- Dihadron correlation calculation agrees with the RHIC STAR and PHENIX data.

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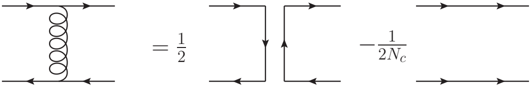
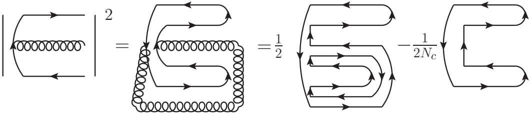
Outlook

[Dominguez, Marquet, Stasto, BX, in preparation] Use Fierz identity:



- The three-jet (same rapidity) production processes in the large N_c limit:



$q\bar{q}g$-jet

- In the large N_c limit at small-x, the dipole and quadrupole amplitudes are the only two fundamental objects in the cross section of multiple-jet production processes at any order in terms α_s.
- Other higher point functions, such as sextupoles, octupoles, decapoles and duodecapoles, etc. are suppressed by factors of $\frac{1}{N_c^2}$.

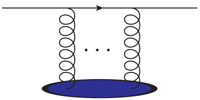
Forward hadron production in pA collisions

Consider the inclusive production of inclusive forward hadrons in pA collisions, i.e., in the process: [Dumitru, Jalilian-Marian, 02]

$$p + A \to H + X.$$

The leading order result for producing a hadron with transverse momentum $p_\perp$ at rapidity y_h

$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p)\mathcal{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z)\right].$$



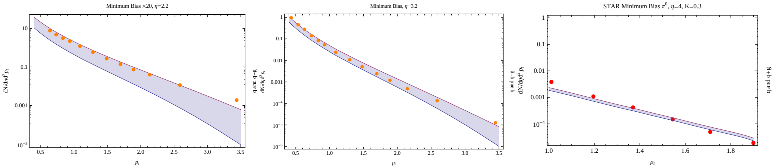
$$\Rightarrow \qquad U(x_\perp) = \mathcal{P}\exp\left\{ig_S\int_{-\infty}^{+\infty} \mathrm{d}x^+\, T^c A_c^-(x^+, x_\perp)\right\},$$

$$\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp).$$

- $p_\perp = zk_\perp$, $x_p = \frac{p_\perp}{z\sqrt{s}}e^{y_h}$ (large), $\tau = zx_p$ and $x_g = \frac{p_\perp}{z\sqrt{s}}e^{-y_h}$(small).
- $S_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle_Y$ with $Y \sim \ln 1/x_g$.
- The gluon channel with $\tilde{\mathcal{F}}(k_\perp)$ defined in the adjoint representation.
- Classical $p_\perp$ broadening calculation, no divergences, no evolution.

Issues with the leading order calculation

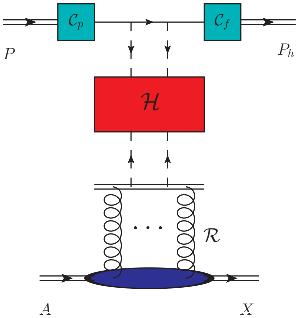
The comparison between the leading order calculation and the RHIC data:



Comments: Why do we need NLO calculations?

- LO calculation is order of magnitude estimate. Normally, we need to introduce the artificial K factor to fix the normalization. Fails to describe large $p_\perp$ data.
- There are large theoretical uncertainties due to renormalization/factorization scale dependence in $xf(x)$ and $D(z)$. Choice of the scale at LO requires information at NLO.
- In general, higher order in the perturbative series in α_s helps to increase the reliability of QCD predictions.
- NLO results reduce the scale dependence and may distort the shape of the cross section. $K = \frac{\sigma_{\text{LO}}+\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$ is not a good approximation.
- NLO is vital in terms of establishing the QCD factorization in saturation physics. Fun!

The overall picture



The QCD factorization formalism for this process reads as,

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu) D_{h/c}(z,\mu) \int [dx_\perp] S^Y_{a,c}([x_\perp])\mathcal{H}_{a\to c}(\alpha_s,\xi,[x_\perp]\mu) \, .$$

- For UGD, the rapidity divergence cannot be canceled between real and virtual gluon emission due to different restrictions on $k_\perp$.
- Subtractions of the divergences via renormalization$\Rightarrow$ Finite results for hard factors.

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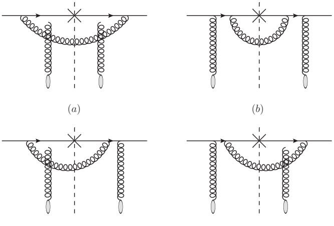
The real contributions in the coordinate space

Computing the real diagrams with a quark ($b_\perp$) and a gluon ($x_\perp$) in the final state in the dipole model in the coordinate space: [G. Chirilli, BX and F. Yuan, 11;12]



$$
\begin{aligned}
\frac{d\sigma^{qA\to qgX}}{d^3k_1 d^3k_2} &= \alpha_S C_F \delta(p^+ - k_1^+ - k_2^+) \int \frac{\mathrm{d}^2 x_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 x'_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 b'_\perp}{(2\pi)^2} \\
&\quad \times e^{-ik_{1\perp}\cdot(x_\perp - x'_\perp)} e^{-ik_{2\perp}\cdot(b_\perp - b'_\perp)} \sum_{\lambda\alpha\beta} \psi^{\lambda *}_{\alpha\beta}(u'_\perp)\psi^{\lambda}_{\alpha\beta}(u_\perp) \\
&\quad \times \Big[S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp) + S_Y^{(2)}(v_\perp, v'_\perp) \\
&\qquad - S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) - S_Y^{(3)}(v_\perp, x'_\perp, b'_\perp) \Big],
\end{aligned}
$$

with $u_\perp = x_\perp - b_\perp$ and $v_\perp = (1-\xi)x_\perp + \xi b_\perp$.

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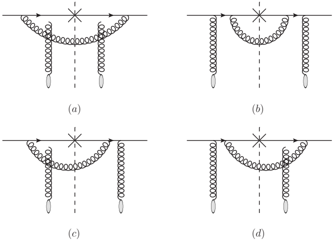
The real contributions in the coordinate space

Computing the real diagrams with a quark ($b_\perp$) and a gluon ($x_\perp$) in the final state in the dipole model in the coordinate space: [G. Chirilli, BX and F. Yuan, 11;12]



$$S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp) = \frac{1}{C_F N_c} \left\langle \mathrm{Tr}\left(U(b_\perp)U^\dagger(b'_\perp)T^d T^c\right)\left[W(x_\perp)W^\dagger(x'_\perp)\right]^{cd}\right\rangle_Y,$$

$$S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) = \frac{1}{C_F N_c} \left\langle \mathrm{Tr}\left(U(b_\perp)T^d U^\dagger(v'_\perp)T^c\right) W^{cd}(x_\perp)\right\rangle_Y.$$

- By integrating over the gluon momentum, we identify $x_\perp$ to $x'_\perp$ which simplifies $S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp)$ to $S^{(2)}(b_\perp, b'_\perp)$.
- $S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) = \frac{N_c}{2C_F}\left[S_Y^{(4)}(b_\perp, x_\perp, v'_\perp) - \frac{1}{N_c^2} S_Y^{(2)}(b_\perp, v'_\perp)\right]$

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The real contributions in the momentum space

By integrating over the gluon $(k_1^+, k_{1\perp})$, we can cast the real contribution into

$$\frac{\alpha_s}{2\pi^2}\int\frac{dz}{z^2}D_{h/q}(z)\int_{\tau/z}^1 d\xi\frac{1+\xi^2}{1-\xi}xq(x)\left\{C_F\int d^2k_{g\perp}\mathcal{I}(k_\perp,k_{g\perp})\right.$$
$$\left.+\frac{N_c}{2}\int d^2k_{g\perp}d^2k_{g1\perp}\mathcal{J}(k_\perp,k_{g\perp},k_{g1\perp})\right\},$$

where $x=\tau/z\xi$ and $\mathcal{I}$ and $\mathcal{J}$ are defined as

$$\mathcal{I}(k_\perp,k_{g\perp}) = \mathcal{F}(k_{g\perp})\left[\frac{k_\perp-k_{g\perp}}{(k_\perp-k_{g\perp})^2}-\frac{k_\perp-\xi k_{g\perp}}{(k_\perp-\xi k_{g\perp})^2}\right]^2,$$

$$\mathcal{J}(k_\perp,k_{g\perp},k_{g1\perp}) = \left[\mathcal{F}(k_{g\perp})\delta^{(2)}(k_{g1\perp}-k_{g\perp})-\mathcal{G}(k_{g\perp},k_{g1\perp})\right]\frac{2(k_\perp-\xi k_{g\perp})\cdot(k_\perp-k_{g1\perp})}{(k_\perp-\xi k_{g\perp})^2(k_\perp-k_{g1\perp})^2}$$

$$\text{with}\quad \mathcal{G}(k_\perp,l_\perp) = \int\frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4}e^{-ik_\perp\cdot(x_\perp-b_\perp)-il_\perp\cdot(b_\perp-y_\perp)}S_Y^{(4)}(x_\perp,b_\perp,y_\perp).$$

Three types of divergences:

- $\xi\to 1 \Rightarrow$ Rapidity divergence.
- $k_{g\perp}\to k_\perp \Rightarrow$ Collinear divergence associated with parton distributions.
- $k_{g\perp}\to k_\perp/\xi \Rightarrow$ Collinear divergence associated with fragmentation functions.

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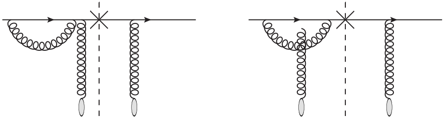
The virtual contributions in the momentum space

Now consider the virtual contribution



(a) (b)

$$-2\alpha_s C_F \int \frac{d^2v_\perp}{(2\pi)^2}\frac{d^2v'_\perp}{(2\pi)^2}\frac{d^2u_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(v_\perp - v'_\perp)} \sum_{\lambda\alpha\beta} \psi^{\lambda *}_{\alpha\beta}(u_\perp)\psi^{\lambda}_{\alpha\beta}(u_\perp)$$

$$\times \left[S_Y^{(2)}(v_\perp, v'_\perp) - S_Y^{(3)}(b_\perp, x_\perp, v'_\perp)\right]$$

$$\Rightarrow \quad -\frac{\alpha_s}{2\pi^2}\int \frac{dz}{z^2} D_{h/q}(z) x_p q(x_p) \int_0^1 d\xi \frac{1+\xi^2}{1-\xi}$$

$$\times \left\{ C_F \int d^2q_\perp \mathcal{I}(q_\perp, k_\perp) + \frac{N_c}{2}\int d^2q_\perp d^2k_{g1\perp} \mathcal{J}(q_\perp, k_\perp, k_{g1\perp}) \right\}.$$

Three types of divergences:

- $\xi \to 1 \Rightarrow$ Rapidity divergence.
- Collinear divergence associated with parton distributions and fragmentation functions.

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The subtraction of the rapidity divergence

We remove the rapidity divergence from the real and virtual diagrams by the following subtraction:

$$\mathcal{F}(k_\perp) = \mathcal{F}^{(0)}(k_\perp) - \frac{\alpha_s N_c}{2\pi^2}\int_0^1 \frac{d\xi}{1-\xi}\int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \times \frac{(x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2(y_\perp - b_\perp)^2}\left[S^{(2)}(x_\perp, y_\perp) - S^{(4)}(x_\perp, b_\perp, y_\perp)\right].$$

Comments:

- This divergence removing procedure is similar to the renormalization of parton distribution and fragmentation function in collinear factorization.
- Splitting functions becomes $\frac{1+\xi^2}{(1-\xi)_+}$ after the subtraction.
- Rapidity divergence disappears when the $k_\perp$ is integrated.
  Unique feature of unintegrated gluon distributions.

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The subtraction of the rapidity divergence

$$\mathcal{F}(k_\perp) = \mathcal{F}^{(0)}(k_\perp) - \frac{\alpha_s N_c}{2\pi^2}\int_0^1 \frac{d\xi}{1-\xi}\int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \times \frac{(x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2(y_\perp - b_\perp)^2}\left[S^{(2)}(x_\perp, y_\perp) - S^{(4)}(x_\perp, b_\perp, y_\perp)\right].$$

This is equivalent to the Balitsky-Kovchegov equation:

$$\frac{\partial}{\partial Y} S_Y^{(2)}(x_\perp, y_\perp) = -\frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2b_\perp\,(x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2(y_\perp - b_\perp)^2}\left[S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp)\right].$$

- Recall that $\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S^{(2)}(x_\perp, y_\perp)$.
- Renormalize the soft gluon into the gluon distribution function of the target nucleus through the BK evolution equation.

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 NLO Forward Hadron Production in pA Collisions

The subtraction of the collinear divergence

Let us take the following integral as an example:

$$\begin{aligned} I_1(k_\perp) &= \int \frac{d^2 k_{g\perp}}{(2\pi)^2} \mathcal{F}(k_{g\perp}) \frac{1}{(k_\perp - k_{g\perp})^2} , \\ &= \frac{1}{4\pi} \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} S_Y^{(2)}(x_\perp, y_\perp) \left(-\frac{1}{\hat{\epsilon}} + \ln \frac{c_0^2}{\mu^2 r_\perp^2} \right) , \end{aligned}$$

where $c_0 = 2e^{-\gamma_E}$, γ_E is the Euler constant and $r_\perp = x_\perp - y_\perp$.

- Use dimensional regularization ($D = 4 - 2\epsilon$) and the $\overline{\text{MS}}$ subtraction scheme ($\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$).
- $\int \frac{d^2 k_{g\perp}}{(2\pi)^2} \Rightarrow \mu^{2\epsilon} \int \frac{d^{2-2\epsilon} k_{g\perp}}{(2\pi)^{2-2\epsilon}}$ where μ is the renormalization scale dependence coming from the strong coupling g.
- The terms proportional to the collinear divergence $\frac{1}{\hat{\epsilon}}$ should be factorized either into parton distribution functions or fragmentation functions.

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The subtraction of the collinear divergence

Remove the collinear singularities by redefining the quark distribution and the quark fragmentation function as follows

$$q(x,\mu) = q^{(0)}(x) - \frac{1}{\hat{\epsilon}}\frac{\alpha_s(\mu)}{2\pi}\int_x^1 \frac{d\xi}{\xi} C_F \mathcal{P}_{qq}(\xi) q\left(\frac{x}{\xi}\right),$$

$$D_{h/q}(z,\mu) = D^{(0)}_{h/q}(z) - \frac{1}{\hat{\epsilon}}\frac{\alpha_s(\mu)}{2\pi}\int_z^1 \frac{d\xi}{\xi} C_F \mathcal{P}_{qq}(\xi) D_{h/q}\left(\frac{z}{\xi}\right),$$

with

$$\mathcal{P}_{qq}(\xi) = \underbrace{\frac{1+\xi^2}{(1-\xi)_+}}_{\text{Real Sub}} + \underbrace{\frac{3}{2}\delta(1-\xi)}_{\text{Virtual Sub}}.$$

Comments:

- Reproducing the DGLAP equation for the quark channel. Other channels will complete the full equation.
- The emitted gluon is collinear to the initial state quark $\Rightarrow$ Renormalization of the parton distribution.
- The emitted gluon is collinear to the final state quark $\Rightarrow$ Renormalization of the fragmentation function.

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Hard Factors

For the $q \to q$ channel, the factorization formula can be written as

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S_Y^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right] + \int \frac{d^2b_\perp}{(2\pi)^2}S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\}$$

with $\mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)$ and

$$\mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right) - 3C_F\delta(1-\xi)e^{-ik_\perp\cdot r_\perp}\ln\frac{c_0^2}{r_\perp^2k_\perp^2} - (2C_F-N_c)\,e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21}-\left(\frac{\left(1+\xi^2\right)\ln(1-\xi)^2}{1-\xi}\right)_+\right]$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2} -\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2}-\delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right.$$

where $$\widetilde{I}_{21} = \int\frac{d^2b_\perp}{\pi}\left\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\left[\frac{b_\perp\cdot(\xi b_\perp-r_\perp)}{b_\perp^2\,(\xi b_\perp-r_\perp)^2}-\frac{1}{b_\perp^2}\right]+e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\right\}.$$

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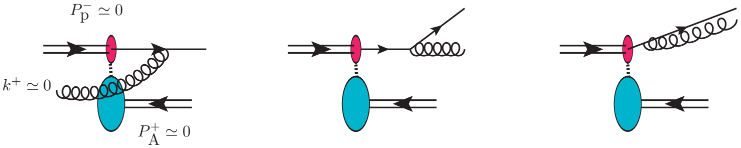
 NLO Forward Hadron Production in pA Collisions

What have we learnt so far?

- Achieve a systematic factorization for the $p + A \to H + X$ process by systematically remove all the divergences!
- Gluons in different kinematical region give different divergences. 1.soft, collinear to the target nucleus; 2. collinear to the initial quark; 3. collinear to the final quark.



Rapidity Divergence   Collinear Divergence (P)   Collinear Divergence (F)

- Large N_c limit simplifies the calculation quite a lot.
- Consistent check: take the dilute limit, $k_\perp^2 \gg Q_s^2$, the result is consistent with the leading order collinear factorization formula. Good large $p_\perp$ behavior!
- The NLO prediction and test of saturation physics now is not only conceivable but also practicable!
- The other three channels follows accordingly.

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Conclusion

- We calculate inclusive hadron productions in pA collisions in the small-x saturation formalism at one-loop order.
- The rapidity divergence with small-x dipole gluon distribution of the nucleus is factorized into the BK evolution of the dipole gluon distribution function.
- The collinear divergences associated with the incoming parton distribution of the nucleon and the outgoing fragmentation function of the final state hadron are factorized into the well-known DGLAP equation.
- The hard coefficient function, which is finite and free of divergence of any kind, is evaluated at one-loop order.
- Now we have a systematic NLO description of inclusive forward hadron productions in pA collisions which is ready for making reliable predictions and conducting precision test. Phenomenological applications are promising for both RHIC and LHC (upcoming pA run) experiments.

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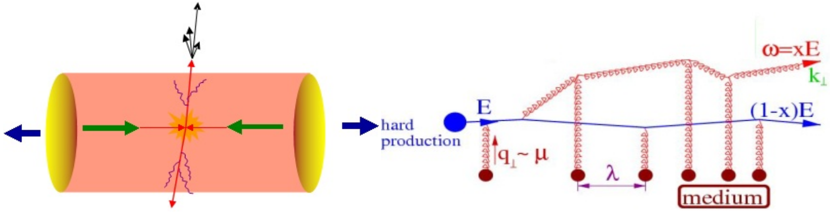
AA Collisions and Energy Loss



- Productions in Collisions. Factorization?
- Energy loss. Higher order?
- $p_\perp$ broadening. Higher order?