Vanish without a trace

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Grisaru Fest, McGill
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1983 review of Superspace

Tackling the supreme symmetry

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Superspace or One Thousand and One Lessons in Supersymmetry.
By S.J. Gates Jr, M.T. Grisaru, M. Roček and W. Siegel.
Benjamin/Cummings: 1983. Pp.548. Hbk $48, £40.80; pbk $25, £21.25.

HAD Scheherazade tried to while away those Arabian nights with *One Thousand and One Lessons in Supersymmetry*, one has the feeling that long before the chapter on quantum superspace Schahriar would have cried "Hey, wait a minute! Where are the references?". The graduate student of

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such a scheme is to avoid the ultraviolet divergences of quantum gravity. The authors present a convincing case that only with the techniques of super-Feynman diagrams (for which they themselves have been largely responsible) can we hope to answer this all-important question.

Superspace is an Aladdin's cave of highly-condensed information. But although it is written with an easy style and admirable clarity, it will require a keen mind and determination, rather than a simple "Open, Sesame", to extract the riches. □

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Abstract

- For fields that are classically non-conformal, such as gravity itself, the total trace of the quantum stress tensor involves a “naive” term in addition to the Weyl anomaly.
- This remains true for the combined trace of generic four-dimensional supergravity plus matter theories such as the MSSM.
- Remarkably, we show to one loop order that those derived from Type II string or M-theory compactification have vanishing naive trace and so behave, in this sense at least, as though they were classically conformal.

Abstract

- Classically, Weyl invariance

$$S(g, \phi) = S(g', \phi')$$

under

$$g'_{\mu\nu}(x) = \Omega(x)^2 g_{\mu\nu}(x) \quad \phi' = \Omega(x)^\alpha \phi$$

implies

$$g^{\mu\nu} T_{\mu\nu} = 0$$

- But in the quantum theory

$$g^{\mu\nu} < T_{\mu\nu} > \neq 0$$

Capper and Duff 1973
Over the period 1973-2012 this Weyl anomaly has found a variety of applications in quantum gravity, black hole physics, inflationary cosmology, string theory and statistical mechanics.

CFTs

- Weyl anomalies appear in their most pristine form when CFTs are coupled to an external gravitational field. In this case
$$g^{\mu\nu}\langle T_{\mu\nu}\rangle = \frac{1}{(4\pi)^2}(cF - aG)$$
where F is the square of the Weyl tensor:
$$F = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2,$$
G is proportional to the Euler density:
$$G = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2,$$
- Note no R^2 term.
- We ignore $\Box R$ terms whose coefficient can be adjusted to any value by adding the finite counterterm
$$\int d^4x\sqrt{g}R^2.$$

Central charges c and a

- In the CFT a and c are the central charges given in terms of the field content by

$$\bar{a} \equiv 720a = 2N_0 + 11N_{1/2} + 124N_1$$

$$\bar{c} \equiv 720c = 6N_0 + 18N_{1/2} + 72N_1$$

where N_s are the number of fields of spin s.

- In the notation of Duff 1977

$$(4\pi)^2 b = c \quad (4\pi)^2 b' = -a$$

Total versus anomalous trace

- For fields that are classically non-conformal, such as gravity itself, the total trace T of the quantum stress tensor

$$T \equiv g_{\mu\nu} \frac{\delta W}{\delta g_{\mu\nu}}$$

involves a "naive" term in addition to the Weyl anomaly.

$$T = T_N + T_A \tag{1}$$

- The anomaly is given by the De Witt b_4 coefficient in the asymptotic exansion of the heat kernel

$$T_A = b_4 = \frac{1}{(4\pi)^2}(c_A F - a_A G + e_A R^2)$$

c_A and a_A are gauge-dependent for spins $3/2$ and 2, though $c_A - a_A$ is not

Euler number

- When $F - G$ vanishes, anomaly reduces to

$$T_A = A_A \frac{1}{32\pi^2} R^{*\,\mu\nu\rho\sigma} R^{*}{}_{\mu\nu\rho\sigma}$$

where

$$360 A_A = \bar{c}_A - \bar{a}_A$$

so that in Euclidean signature

$$\int d^4x \sqrt{g}\, T_A = A\chi(M^4)$$

where $\chi(M^4)$ is the Euler number of spacetime.

Arbitrary spin

- Calculate b_4 for arbitrary (n, m) reps of Lorentz group, then physical anomaly given by

$$A = A(n,m) + A(n-1, m-1) - 2A(n-1/2, m-1/2)$$

so in total

$$A_{total} = 4N_0 + 7N_{1/2} - 52N_1 - 233N_{3/2} + 848N_2$$

where N_s are the number of fields of spin s.

- The b_4 coefficient for chiral reps (1/2,0) (1,0) etc also involve R*R unless we add (0,1/2) (0,1) etc

Christensen and Duff 1978

p-forms and inequivalent anomalies

- Inequivalence:

$$A_A(2-\text{form}) - A_A(\text{scalar}) = 1 \quad A_A(3-\text{form}) = -2$$

  Duff and van Nieuwenhuizen 1980

- Confirmed by string calculations Antoniadis, Gava and Narain 1992
- Can arrange $A_A = 0$ for $\mathcal{N} \geq 3$ Nicolai and Townsend 1980
- But, according to
  Siegel 1980
  Grisaru, Nielson, Siegel and Zanon 1980
  Gates, Grisaru, Siegel and Rocek 1980
  total stress tensors are equivalent.

$$A(2-\text{form}) - A(\text{scalar}) = 0 \quad A(3-\text{form}) = 0$$

ADDED SLIDE

- At this point of the talk a claim was made from the audience that there was a “mistake”. In my view, there are no mistakes. Duff and van Nieuwenhuizen correctly calculated T_A; Grisaru et al correctly calculated T.

- We consider compactification of $(\mathcal{N} = 1, D = 11)$ supergravity on a 7-manifold X^7 with betti numbers $(b_0, b_1, b_2, b_3, b_3, b_2, b_1, b_0)$ and define a generalized mirror symmetry

$$(b_0, b_1, b_2, b_3) \to (b_0, b_1, b_2 - \rho/2, b_3 + \rho/2)$$

under which

$$\rho(X^7) \equiv 7b_0 - 5b_1 + 3b_2 - b_3$$

changes sign

$$\rho \to -\rho$$

Duff and Ferrara 2010

- Generalized self-mirror theories are defined to be those for which $\rho = 0$

Generalized mirror symmetry: IIA-theory on X^6

- M-theory on $X^6 \times S^1$ with betti numbers $(b_0, b_1, b_2, b_3, b_3, b_2, b_1, b_0)$ is equivalent to Type IIA on X^6 with betti numbers $(c_0, c_1, c_2, c_3, c_2, c_1, c_0)$ related by

$$(b_0, b_1, b_2, b_3) = (c_0, c_0 + c_1, c_1 + c_2, c_2 + c_3)$$

and hence

$$\rho(X^6 \times S^1) = \chi(X^6)$$

where $\chi(X^6)$ is the Euler number of X^6

$$\chi(X^6) = 2c_0 - 2c_1 + 2c_2 - c_3$$

- The generalized mirror symmetry transformation then becomes

$$(c_0, c_1, c_2, c_3) \to (c_0, c_1, c_2 - \chi/2, c_3 + \chi)$$

under which χ also changes sign

$$\chi \to -\chi$$

X^6=Calabi-Yau

- Further specializing to X^6=Calabi-Yau with betti numbers:

$$(1, 0, h^{11}, 2 + 2h^{21}, h^{11}, 0, 1)$$

our generalized mirror symmetry reduces to the familiar interchange of hodge numbers

$$h^{11} \leftrightarrow h^{21}$$

.

Total=Naive+Anomalous

	Field	f	A_N	$360A_A$	$360A$	X^7
g_{MN}	$g_{\mu\nu}$	2	-3	848	-232	b_0
	$\mathcal{A}_\mu$	2	0	-52	-52	b_1
	$\mathcal{A}$	1	0	4	4	$-b_1 + b_3$
ψ_M	ψ_μ	2	1	-233	127	$b_0 + b_1$
	χ	2	0	7	7	$b_2 + b_3$
A_{MNP}	$A_{\mu\nu\rho}$	0	2	-720	0	b_0
	$A_{\mu\nu}$	1	-1	364	4	b_1
	A_μ	2	0	-52	-52	b_2
	A	1	0	4	4	b_3
total A_N			0			
total A_A				$-\rho/24$		
total A					$-\rho/24$	

Vanish without a trace!

- Remarkably, we find that the anomalous trace depends on ρ

$$A_A = -\frac{1}{24}\rho(X^7)$$

So the anomaly flips sign under generalized mirror symmetry and vanishes for generalized self-mirror theories. For $X^{(8-\mathcal{N})} \times T^{(\mathcal{N}-1)}$ with $\mathcal{N} \geq 3$ the anomaly vanishes identically (but not the Townsend-Nicolai way)!

- Equally remarkable is that we get the same answer for the total trace

$$A = -\frac{1}{24}\rho(X^7)$$

- So theories derived from Type II string or M-theory compactification have vanishing naive trace and so behave, in this sense at least, as though they were classically conformal.

Type IIA

- In the case of $(\mathcal{N} = 1, D = 11)$ on $X^6 \times S^1$, or equivalently (Type IIA, D=10) on X^6,

$$A = -\frac{1}{24}\chi(X^6)$$

and so in Euclidean signature

$$\int d^4x\sqrt{g}g_{\mu\nu} < T^{\mu\nu} >= -\frac{1}{24}\chi(M^4)\chi(X^6) = -\frac{1}{24}\chi(M^{10})$$

where $\chi(M^4)$ is the Euler number of spacetime.

Four curious supergravities

- Of particular interest are the four cases

$$(b_0, b_1, b_2, b_3) = (1, \mathcal{N} - 1, 3\mathcal{N} - 3, 4\mathcal{N} + 3)$$

  with $\mathcal{N} = 1, 2, 4, 8$, namely the four "curious" supergravities, which enjoy some remarkable properties.
  $\mathcal{N} = 1$, 7 WZ multiplets, $f = 32$,
  $\mathcal{N} = 2$, 3 vector multiplets, 4 hypermultiplets, $f = 64$,
  $\mathcal{N} = 4$, 6 vector mutiplets, $f = 128$,
  $\mathcal{N} = 8$, $f = 256$.
- Reduction of Supergravities for Membranes in $D = 4, 5, 7, 11$

Fano plane

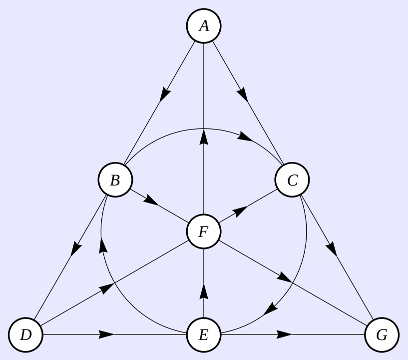


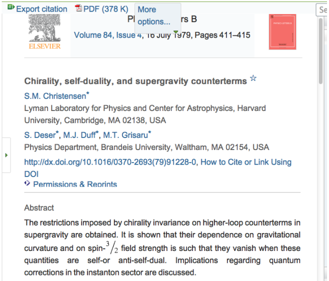
Figure : The Fano plane has seven points and seven lines (the circle counts as a line) with three points on every line and three lines through every point. The truncation from 7 lines to 3 to 1 to 0 corresponds to the truncation from N=8 to N=4 to N=2 to N=1.

O, H, C, R theories

Field	$360A$	**O**	**H**	**C**	**R**
$g_{\mu\nu}$	848	1	1	1	1
B_μ	-52	7	6	0	0
S	4	28	16	10	7
ψ_μ	-233	8	4	2	1
χ	7	56	28	14	7
$A_{\mu\nu\rho}$	-720	1	1	1	1
$A_{\mu\nu}$	364	7	3	1	0
A_μ	-52	21	6	4	0
A	4	35	19	11	7
		$A = 0$	$A = 0$	$A = 0$	$A = 0$

Table : Vanishing anomaly in **O**, **H**, **C** **R** theories.

Much forgotten paper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Chirality, self-duality, and supergravity counterterms ☆

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Abstract

The restrictions imposed by chirality invariance on higher-loop counterterms in supergravity are obtained. It is shown that their dependence on gravitational curvature and on spin-$^3/_2$ field strength is such that they vanish when these quantities are self-or anti-self-dual. Implications regarding quantum corrections in the instanton sector are discussed.

Shall we write another one, Marc?