PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 1

READING: Textbook, Chapter 1 (Sections 1 - 7), Chapter 2 (Sections 1 - 3)

PROBLEMS (due Jan. 16, 2018, in class):

N.B. These problems are intended to make sure you have an adequate background in special relativity.

1. Carroll, Chapter 1, Problem 1.
2. Carroll, Chapter 1, Problem 3.
3. Carroll, Chapter 1, Problem 5.

4. a) When a photon scatters off a charged particle which is moving with a speed very nearly that of light, the photon is said to have undergone an *inverse Compton* scattering. Consider an inverse Compton scattering in which a charged particle of rest mass m and total mass-energy (as seen in the lab frame) $E \gg m$ collides head-on with a photon of frequency ν ($\nu \ll m$, in units where Planck's constant is set to 1). What is the maximum energy which the particle can transfer to the photon?
b) If space is filled with black-body radiation of temperature 3^oK and contains cosmic ray protons of energies up to 10^{20}eV, how much energy can a proton of energy 10^{20}eV transfer to a 3^oK photon?

5. Derive a formula for the mass M of a decaying particle in terms of the masses m_1 and m_2, energies E_1 and E_2, momenta p_1 and p_2 of the decay products, and the opening angle θ between the two tracks in the laboratory frame.

6. The action of a non-relativistic point particle is

$$S_{nr} = \int \frac{1}{2} m v^2 dt$$

where m is the mass of the particle, v is its three velocity, and t is time.
a) Using the variational principle, derive the resulting equation of motion.
b) Consider now the following action for a point particle moving along a world line C

$$S = m \int d\tau$$

where as before m is the mass, but now τ is the proper time along the world line, and the integral is along the world line. Derive the resulting equation of motion and show that in the non-relativistic limit $v \ll 1$ you recover the action and equation of motion in the non-relativistic case.