PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 5

READING: Textbook, Sections 4.1 - 4.7.

PROBLEMS, due February 15 2018 (in class):

1. Textbook, Problem 4.1 (Page 190).
2. Textbook, Problem 4.2 (Page 190).
3. Show that in normal coordinates

$$R_{\alpha\beta\gamma\delta} + R_{\gamma\beta\alpha\delta} = -3g_{\alpha\gamma,\beta\delta}$$

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4. Show that for coordinate vector fields $\partial_\beta, \partial_\gamma, \partial_\epsilon$ and coordinate one forms dx^α we have

$$R^\alpha_{\beta\gamma\delta;\epsilon} =< dx^\alpha, \nabla_{\partial_\epsilon} R(\partial_\gamma, \partial_\delta)\partial_\beta > ,$$

where the semicolon indicates covariant differentiation.

5. In class I stated the symmetry

$$< R(X,Y)Z,U >=< R(Z,U)X,Y > ,$$

where X, Y, Z and U are vector fields. Prove this relation (it is possible without making use of coordinates).

6. In class I discussed the Newtonian limit of the geodesic equation and derived the relation

$$\Phi = \frac{1}{2}h_{00}$$

between the Newtonian gravitational potential Φ and the 00 component of the perturbed metric $h_{\mu\nu}$. On the other hand, by considering the equation of geodesic deviation I derived the formula

$$\nabla^2\Phi = R_{00}$$

which relates the same Newtonian gravitational potential to the 00 component of the Ricci tensor. Show that these formulas are consistent. Recall that the context for the first equation is a metric of the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

in the weak field and slow motion limit.