PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 6

READING: Textbook, Sections 5.1 - 5.5.

PROBLEMS, due Feb. 27 , 2018 (in class):

1. The metric for the three-sphere in coordinates (ψ, θ, ϕ) is

$$ds^2 = d\psi^2 + sin^2(\psi)(d\theta^2 + sin^2(\theta)d\phi^2)$$

a) Calculate the Christoffel symbols.
b) Calculate the Riemann tensor, Ricci tensor and Ricci scalar.

2. Do the same calculation using the tetrad basis.

3. Consider 3-dimensional (i.e. 2+1) gravity.
a) How many degrees of freedom are in the Riemann tensor?
b) How many degrees of freedom are in the Ricci tensor?
c) The Riemann tensor can be decomposed into the Ricci tensor and the Weyl tensor (see textbook, Page 130). Using this fact, how many degrees of freedom are in the Weyl tensor?
d) Are there gravity waves (gravity waves are fluctuations of space-time without associated matter perturbations)?

4. Find the solution to Einstein's field equations for a massive point particle (at rest) in 2+1 dimensional gravity. Hint: Write the stress-tensor for a point particle, make an ansatz for the metric making use of the symmetries of the problem (diagonal, static, polar symmetry), and then find the resulting Riemann tensor. Then solve for the metric.

5. In class I sketched the derivation of the Einstein tensor for the spherically symmetric metric

$$ds^2 = e^{2a(r)}dt^2 - [e^{2b(r)}dr^2 + r^2 d\Omega^2]$$

using the tetrad formalism. Complete the derivation.

6. Derive the Einstein tensor for the metric of the above problem, this time using the coordinate approach. After solving this problem you should be convinced that it is easier to use the tetrad formalism.