

**PHYS 743    Very Early Universe    2019**  
**Problem Set 3 - for Week 3, due Oct. 1 in class**

1. In class I discussed the basic equations which describe matter and gravitational potential fluctuations (Equation 4 in the reading). Verify that the resulting equation for linearized fluctuations about a Minkowski background is given by Equation 5.
2. Do the corresponding problem for Newtonian fluctuations about an expanding background cosmology. Namely, starting with Eqs. 4 and 11 of the reading, derive Eq. 13.
3. Consider matter fluctuations in the radiation-dominated phase of Standard Big Bang cosmology. If  $\delta_m$  denotes the fractional matter density fluctuation, and  $\delta_r$  the fractional radiation density fluctuations, derive the equation of motion which  $\delta_m$  obeys. In this analysis, consider matter to have vanishing pressure. Study how the dominant mode of the solution scales in time.
4. When discussing classical relativistic fluctuations, I discussed the linearized Einstein equations (57). Evaluate these equations in the case of the matter source being a real scalar field. Demonstrate how the equations can be combined to yield Eq. 62.
5. Consider large field inflation with potential

$$V(\varphi) = \frac{1}{2}m^2\varphi^2.$$

Compute the power spectrum of cosmological perturbations as a function of  $m$ , and determine the value of  $m$  by requiring that the amplitude of the power spectrum match the observed value (you can take the value to be  $10^{-8}$ ).