PHYS 743 Very Early Universe 2020
Problem Set 2

1. Assume that the energy scale of inflation is 10^{15}GeV. How long (in units of H^{-1} does the period of inflation have to be
a) in order to solve the horizon problem
b) in order to solve the flatness problem
c) in order to solve the structure formation problem.
Assume that H is constant during the period of inflation.

2. Based on the action principle, derive the equation of motion of a real scalar field in a Friedmann-Robertson-Walker-Lemaitre universe. Hint: apply the Einstein equivalence principle to find the action in curved space-time from the usual action in flat space-time.

3. Consider the Klein-Gordon equation in a FRWL background, and consider linearlized fluctuations about a homogeneous solution $\varphi_0(t)$. Each Fourier mode of the fluctuations evolves independently. Find approximate solutions for sub-Hubble ($k \gg H$) and super-Hubble ($k \ll H$) modes.

4. Derive the energy-momentum tensor of a real scalar field in a FRWL background, and derive the expressions for the energy density and pressure. Note: you will need to use the following Lemma about the variation of the determinant of the metric:

$$\delta\sqrt{-g} \;=\; \tfrac{1}{2}\sqrt{-g}g^{\alpha\beta}\delta g_{\alpha\beta}$$

Note: those of you who have not taken a class on General Relativity can skip this problem.

5. Consider the new inflationary scenario with potential

$$V(\varphi) \;=\; \tfrac{1}{4}(\varphi^2 - \eta^2)^2$$

. For which values of $|\varphi|$ are the slow-roll criteria satisfied?

6. Consider the same problem for a chaotic inflation model with potential

$$V(\varphi) \;=\; \tfrac{1}{2}m^2\varphi^2.$$