Pulsar Activities

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1. Estimating a Pulsar's Distance

The Main Idea

In astronomy, determining distances to stars and other bodies is extremely difficult. Some of the most famous astronomers in history earned that fame by inventing new and innovative ways of figuring out how far away things in the universe really are.

When we need to measure distances on the Earth, we can just lay a ruler down, use a pedometer, or or even use radar or sonar. The task isn't that difficult because we can actually send *something* to the distant location, whether that be a person, a car, or a radar signal. Radar works well within our solar system as well, because light travels quickly enough to reach the moon and nearby planets in a reasonable amount of time. However, even the closest stars are way, way too far away to send even a radar signal to. As mentioned earlier, astronomers have had to invent creative techniques to measure the distance to the stars.

One method in particular is relevant for pulsars, and it involves two key things: first is the **dispersion measure** of a pulsar, and the second is a model for **electron density** in our galaxy.

Recall that dispersion measure, or DM, is a measure of the number of electrons that the pulsar signal encounters on its way to Earth. The units of DM are pc/cm^3, or parsecs (a unit of distance that astronomers use) per cubic centimeter. Now, here is the key—*if* we know the average density of electrons from some other means, we can find the distance to the pulsar by using the following formula:

$$\text{distance} = \frac{\text{DM}}{\text{average electron density}}$$

You can see for yourself that the units work out correctly. So the question becomes, do we know the average density of electrons between the pulsar and Earth? The simple answer to this question is yes. The more complete answer is that we have a very good model that can give us an estimate, but it won't be an exact answer.

The Procedure

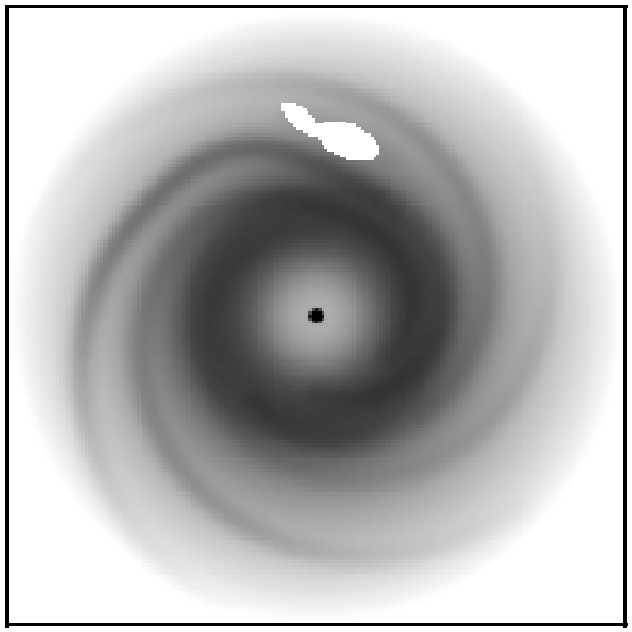

Fig. 1.— A pictorial representation of the NE2001 Model of our galaxy. Dark areas indicate regions of high electron density, and light areas indicate regions of low electron density. The spiral structure of our galaxy is clearly visible in this model as regions of higher density. The white oval shape is the region around the Sun known as the local bubble, which has a particularly low electron density.

The model was compiled by astronomers Jim Cordes and Joe Lazio and is called the NE2001 Galactic Free Electron Density Model. Figure 1 is a pictorial representation of the model. There is a website you can visit that will do most of the work for you. All you need is the DM and the coordinates (the **right ascension** and **declination** of your pulsar. The right ascension and declination are important because the density of electrons is different for different parts of the galaxy. Figures 2 and 3 are screen captures of the website you'll be using.

1. Go to the website `http://rsd-www.nrl.navy.mil/7213/lazio/ne_model/` or Google "NE2001" and click on the first link labeled "Galactic Free Electron Density Model—NE2001".

2. Scroll down to the second-to-last section, labeled "Arbitrary Line of Sight (Text Interface)".

3. Change the drop down menu next to "Line of sight" to "Right ascension-declination (J2000 hh mm ss.s dd mm ss.s)"

4. Enter the right ascension and declination of your pulsar in the spaces below. You should leave out the colons (:) and replace them with a space. For example, if your RA is 19:39:38.6 and your DEC is +21:34:59.1, you would enter "19 39 38.6" and " +21 34 59.1".

5. Leave the next drop down menu unchanged, so that it says "dispersion measure (DM; pc/cmˆ3)".

6. Enter the dispersion measure of your pulsar in the space next to this menu.

7. Optionally, you can can enter your observing frequency, which for the Drift Scan Survey is 0.35 GHz.

8. Hit "Submit Query".

9. The next page has a number of predicted quantities, but the two that interest us are the first and second bulleted points. The first is the DM you entered, and should be in bold face font. The second is the **predicted distance** in units of kiloparsecs (1 kiloparsec is equal to 1000 parsecs). That is the quantity you are after! Below that, in parentheses, are the distances for a DM that is 20% lower and 20% higher than the one you input. These can be treated like the uncertainty in the distance measurement.

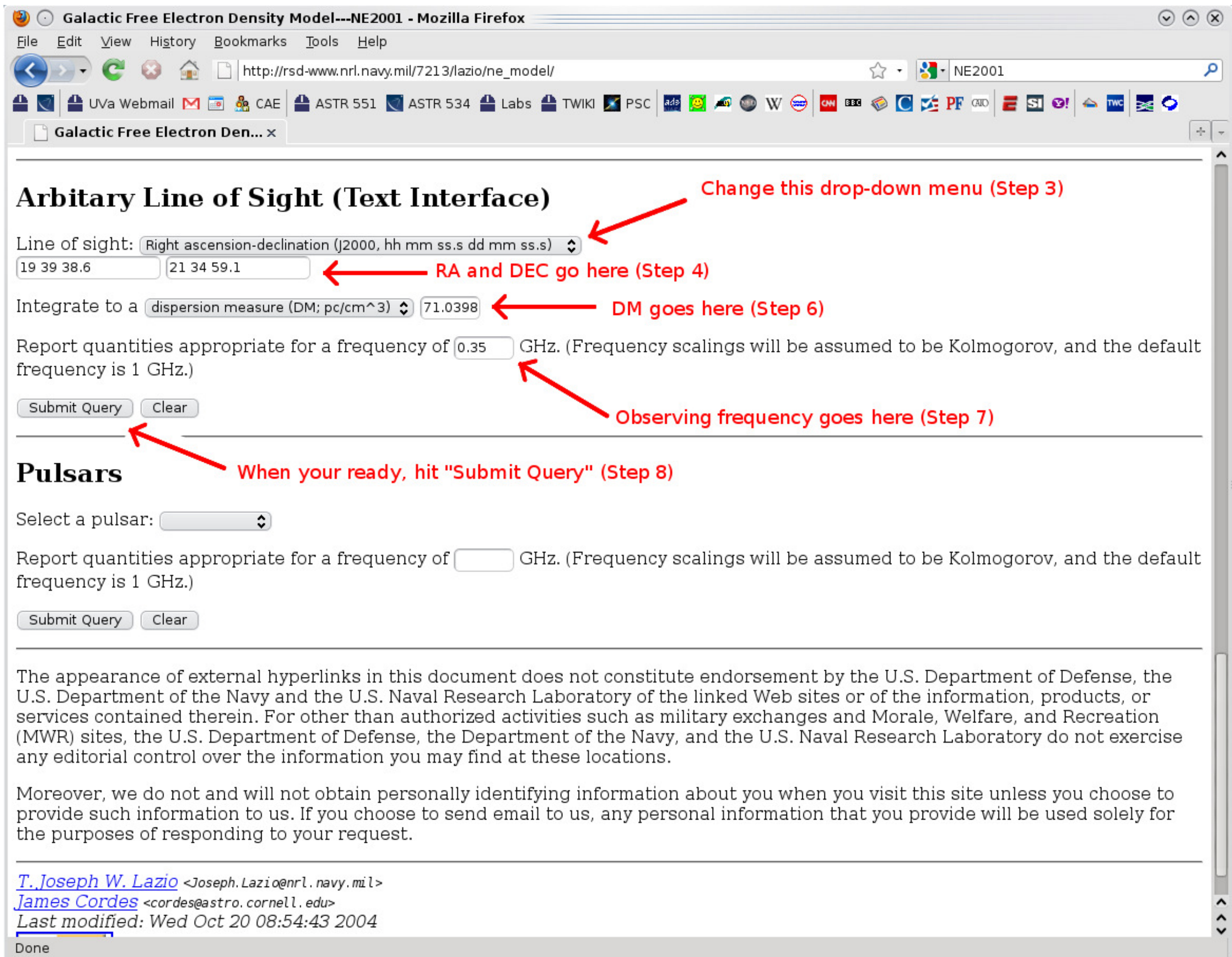

Fig. 2.— A screen capture of the NE2001 website, showing the parts you will be using. Everything is labeled, along with the corresponding step in the procedure.

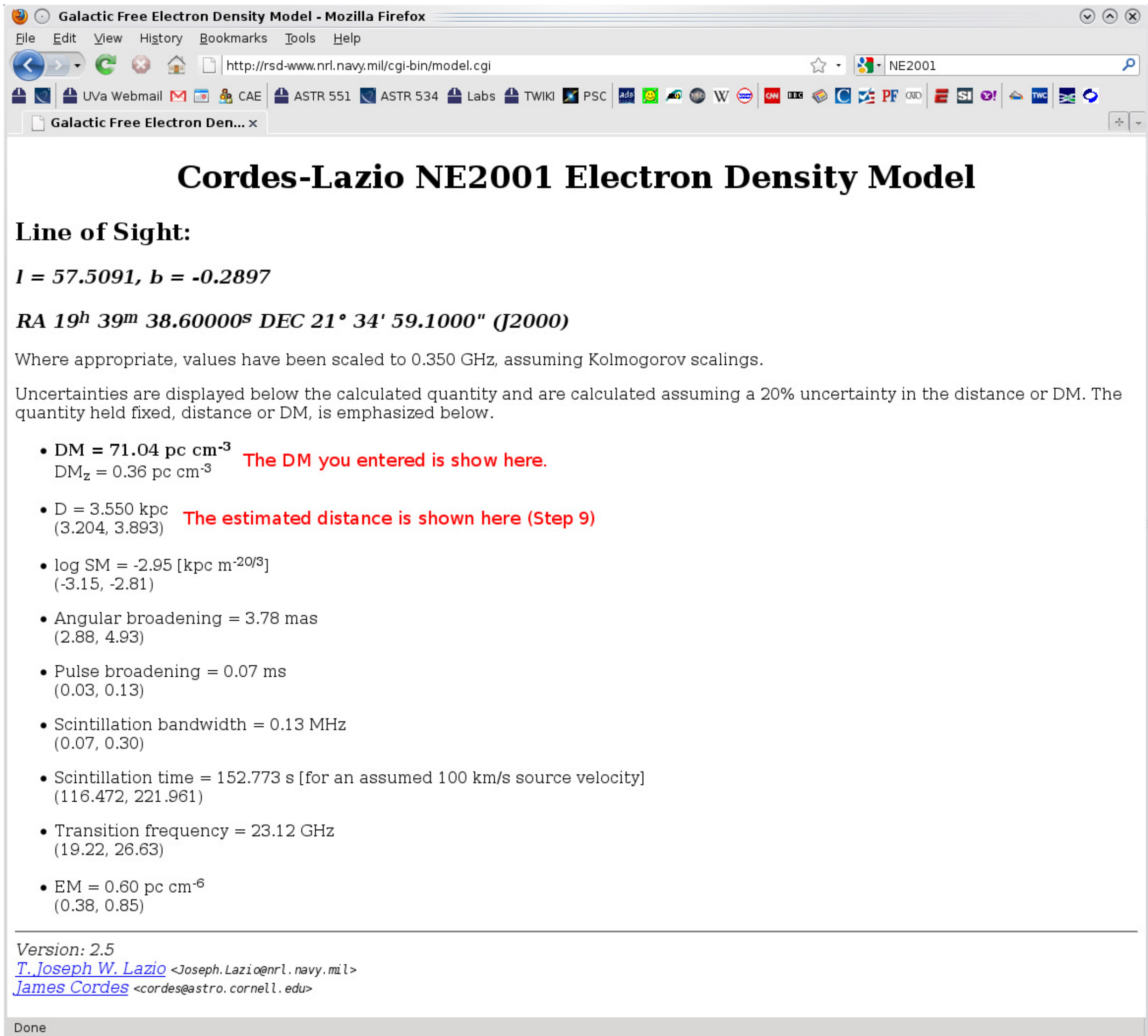

Fig. 3.— A screen capture of the NE2001 results page. You'll be interested in the first two quantities.

2. Locating a Pulsar in the Galaxy

The Main Idea

Finding the distance to a pulsar is important, but this is only one coordinate in space. To locate an object, we need three coordinates (one for each spatial dimension). There are any number of **coordinate systems** one could use to specify the other two dimensions. You are already familiar with right ascension and declination, which mimic lines of longitude and latitude on the Earth. This is called the **equatorial coordinate system** and is appropriate when we want to easily describe where an object is in the night sky, but it is cumbersome to use when talking about an object's location in the galaxy. So astronomers have defined the **galactic coordinate system**.

The galactic coordinate system is centered on the Sun. The imaginary line connecting the galactic center with the Sun plays the role of a prime meridian, and longitude is represented by ℓ. The plane of the galaxy plays the role of an equator and is represented by b.

So, if an object has $0° \leq \ell \leq 180°$, you can think of it as being "west" of the galactic center, while an object with $180° \leq \ell \leq 360°$ can be thought of as "east" of the galactic center. Similarly, an object with $0 < b \leq 90°$ is "north" and an object with $0° > b \geq -90°$ is "south".

If you know the right ascension and declination of an object, you can transform it to galactic coordinates. When combined with the distance, this gives us the full 3-D position of an object in the galaxy.

The Procedure

The transformation from right ascension (α) and declination (δ) to ℓ and b is a bit tricky. The formulas are

$$\begin{aligned}
\sin b &= \sin\delta \sin(27°\,08') - \cos\delta\cos(27°\,8')\sin(\alpha - 18^{\mathrm{h}}\,51.4^{\mathrm{m}}) \\
\cos(\ell - 32.93°)\cos b &= \cos(\alpha - 18^{\mathrm{h}}\,51.4^{\mathrm{m}})\cos\delta \\
\sin(\ell - 32.93°)\cos b &= \sin\delta\cos(27°\,08') + \cos\delta\sin(27°\,08')\sin(\alpha - 18^{\mathrm{h}}\,51.4^{\mathrm{m}}).
\end{aligned}$$

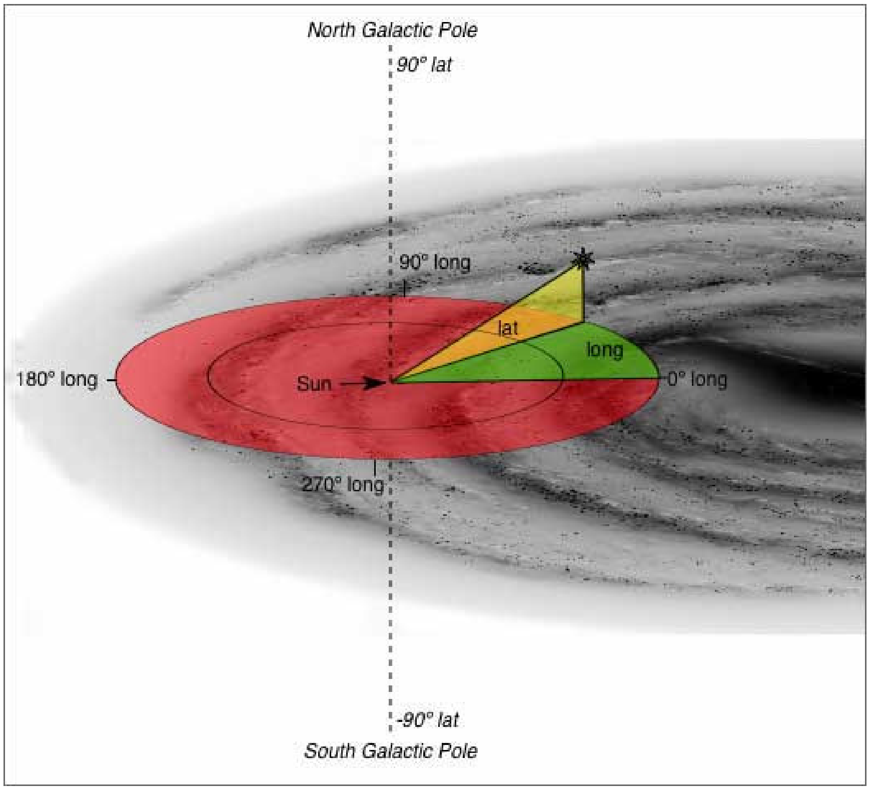

Fig. 4.— The galactic coordinate system. It is centered on the Sun but the galaxy is used as a reference. Longitude and latitude are represented by ℓ and b, both measured in degrees.

An easier way of doing this is to use an online tool.

1. Go to the website `http://fuse.pha.jhu.edu/support/tools/eqtogal.html` (or simply search for "equatorial to galactic coordinates" and click on the first link).
2. Enter the right ascension and declination in the spaces provided, being sure not to include any colons (:), or degree markers.
3. Change the "Epoch" to "J2000.0".
4. Click on "convert coordinates".

3. The P-$\dot{P}$ Diagram

The Main Idea

As you know, pulsars spin and the time it takes to complete one rotation is called the period (P). But the period of a pulsar isn't constant. Most pulsars derive the majority of their energy from their rotation, and as the pulsar emits that energy in the form of particle winds and radiation, the rotation has to slow down. This is a simple example of the conservation of energy. This change in period is very small but it is something we can observe over the course of many months and years. We use the symbol $\dot{P}$ (pronounced "p dot") to represent the change in period. The $\cdot$ is mathematical notation for something that changes over time[1] We typically use units of s/s, or seconds per second, for $\dot{P}$. As an example, a $\dot{P}$ of 2×10^{-17} s/s indicates that every second, the period of the pulsar gets 2×10^{-17} s longer.

One might wonder if there is any relationship between P and $\dot{P}$. An easy way to test for one would be to plot the P and $\dot{P}$ for all the pulsars we know of and to look for trends in the data. Figure 5 shows just such a plot, and it is easy to see that a relationship does indeed exist. Such a plot is called a P-$\dot{P}$ diagram and is commonly used by pulsars astronomers. Look at it closely. What do you notice?

One striking fact is that there is a tendency for shorter period pulsars to have smaller $\dot{P}$, while longer period pulsars have larger $\dot{P}$. A second observation is that there is a distinct grouping around $P \sim 0.4$ s and $\dot{P} \sim 10^{-15}$ s/s. What may not be quite as obvious is the existence of second, smaller group centered around $P \sim 0.003$ s and $\dot{P} \sim 10^{-20}$ s/s. These are the millisecond pulsars. There is also a sparsely populated region between the two groups.

It can be useful to determine where a pulsar that you find falls on the P-$\dot{P}$ diagram. If it falls within a well populated region, then it is probably similar in some regards to other known pulsars. If it is in a less populated region of the diagram, it could be an indication that the pulsar is different and interesting. Unfortunately it takes about a year to accurately measure $\dot{P}$, so you can only precisely place a pulsar you find if it is already known and well studied. However, if you find a new pulsar, you can still get some estimate of what its $\dot{P}$ should be by using the diagram.

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[1] If you're familiar with calculus, you may recognize that $\dot{P} = dP/dt$, or the time derivative of P.

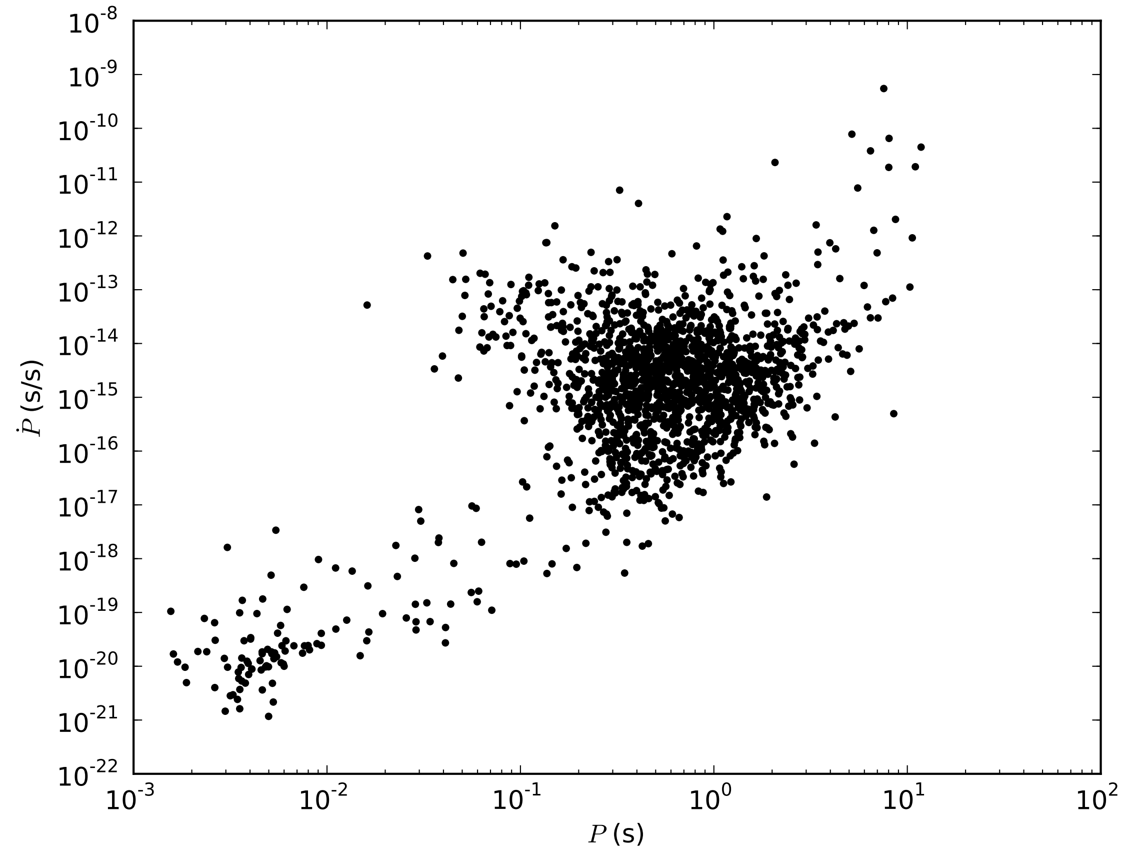

Fig. 5.— The P-$\dot{P}$ diagram. There is a clear trend in the data showing that shorter period pulsars tend to have smaller $\dot{P}$. Most pulsars have longer periods and larger $\dot{P}$.

The Procedure

If you find a known pulsar, it is fairly easy to place it on the P-$\dot{P}$ diagram using information in the ATNF pulsar database.

1. When you log into the ATNF pulsar database, check the box labeled "P1" to display $\dot{P}$, which is directly next to "P0", which displays the period.

2. When you find your pulsar in the database, make a note of both the period and $\dot{P}$.

3. Find the location on the diagram that corresponds to the measured P and $\dot{P}$ and make a mark there. A ruler may be useful for this.

If you find a new pulsar, you should be very excited! It will take time to measure $\dot{P}$ but you can make a rough prediction of what it is likely to be, assuming that it is similar to other pulsars with roughly the same period.

1. Find the period that corresponds to your new pulsar on the x-axis of the diagram.
2. Draw a vertical line through the plot at this location.
3. Your vertical line should go through a region on the plot with many other pulsars that have similar periods. Draw two horizontal lines that enclose the pulsars with similar periods to your own. The horizontal lines should go all the way over to the y-axis.
4. You can now predict that the $\dot{P}$ of your new pulsar will lie somewhere between these two horizontal lines.

4. Estimating the Age and Magnetic Field Strength of a Pulsar

Knowing the period and $\dot{P}$ of a pulsar is interesting for many reasons. Two of the coolest reasons are that it lets us estimate the age and magnetic field strength of the pulsar. Understanding how we can use the period and $\dot{P}$ to estimate these quantities requires understanding something about the models we use to describe pulsar spin-down, as well as some calculus. At this point, you needn't worry about these details, but those who are interested in a challenge can look at some descriptions online. For now, all you need to worry about are the results. The estimated age of a pulsar is given by

$$\begin{aligned} \tau_{\rm c} &\equiv \frac{P}{2\dot{P}} \\ &\simeq 15.8\ \mathrm{Myr}\left(\frac{P}{s}\right)\left(\frac{\dot{P}}{10^{-15}\ \mathrm{s/s}}\right)^{-1} \end{aligned} \tag{1}$$

where the second line tells us that this is equivalent to 15.8 million years times the period in seconds divided by $\dot{P}$ measured in 10^{-15} s/s. It is important to note that this is not necessarily (and probably won't be) the true age of the pulsar. It is an estimate that we arrived at after making some important assumptions about the spin-down of the pulsar. We call this the **characteristic age**.

In a way, calculating the characteristic age is similar to estimating the age of a person simply by looking at them. We all know that as a person gets older, they get taller, their hair grays, and their skin starts to wrinkle. We might look at a person with hair that is just starting to gray and skin that is just starting to wrinkle and guess that they are about 60 years old

(Warning: exercise caution when trying this out with people you know. When in doubt, always use good manners and report an age on the low end of your estimate!). Of course, there are many factors that can make a person look older or younger than they really are. In addition to age, diet, lifestyle, and individual genetic factors all play a role in our physical appearance. Similarly, the characteristic age of a pulsar may differ from its true age very significantly depending on the specifics of that individual pulsar. As long as we remember this caveat, we can use the characteristic age as an important estimate of the true age of the pulsar.

Similarly, if we assume a relationship between the loss of rotational energy and the magnetic field of the pulsar, we can assign a **characteristic magnetic field**, measured at the surface of the pulsar:

$$B_{\rm S} \equiv \sqrt{\frac{3c^3}{8\pi^2}\frac{I}{R^6\sin^2\alpha}P\dot{P}} \tag{2}$$

where c is the speed of light, I is the **moment of inertia** of the pulsar, R is the radius of the pulsar, and α is the angle between the rotation axis and magnetic poles. Typically, we don't know I, R, or α, but we can make estimates for a "typical" neutron star of $I = 10^{45}$ g cm^2, $R = 10$ km, and $\alpha = 90°$. In this case, we find

$$\begin{aligned} B_{\rm S} &\simeq 3.2 \times 10^{19}\ {\rm G}\ \sqrt{P\dot{P}} \\ &\simeq 10^{12}\ {\rm G}\ \left(\frac{P}{\rm s}\right)^{1/2}\left(\frac{\dot{P}}{10^{-15}\ {\rm s/s}}\right)^{1/2}. \end{aligned} \tag{3}$$

Remember that the characteristic magnetic field is an estimate based upon a specific model.

Notice that the characteristic age and characteristic magnetic field depend only on the period and $\dot{P}$. This means that we can assign a $\tau_{\rm c}$ and $B_{\rm S}$ anywhere on the P-$\dot{P}$ diagram. Figure 6 shows the P-$\dot{P}$ diagram with lines of constant $tau_{\rm c}$ and $B_{\rm S}$. When we do this, it is easy to see that the millisecond pulsars have lower $B_{\rm S}$ and larger $\tau_{\rm c}$. In other words, millisecond pulsars are older and have weaker magnetic fields than normal pulsars. The reason for this has to do with how millisecond pulsars are made. Instead of forming directly from supernovae, a millisecond pulsar is thought to be a "dead" pulsar that was resurrected when a companion star donates mass to it. This weakened the magnetic field.

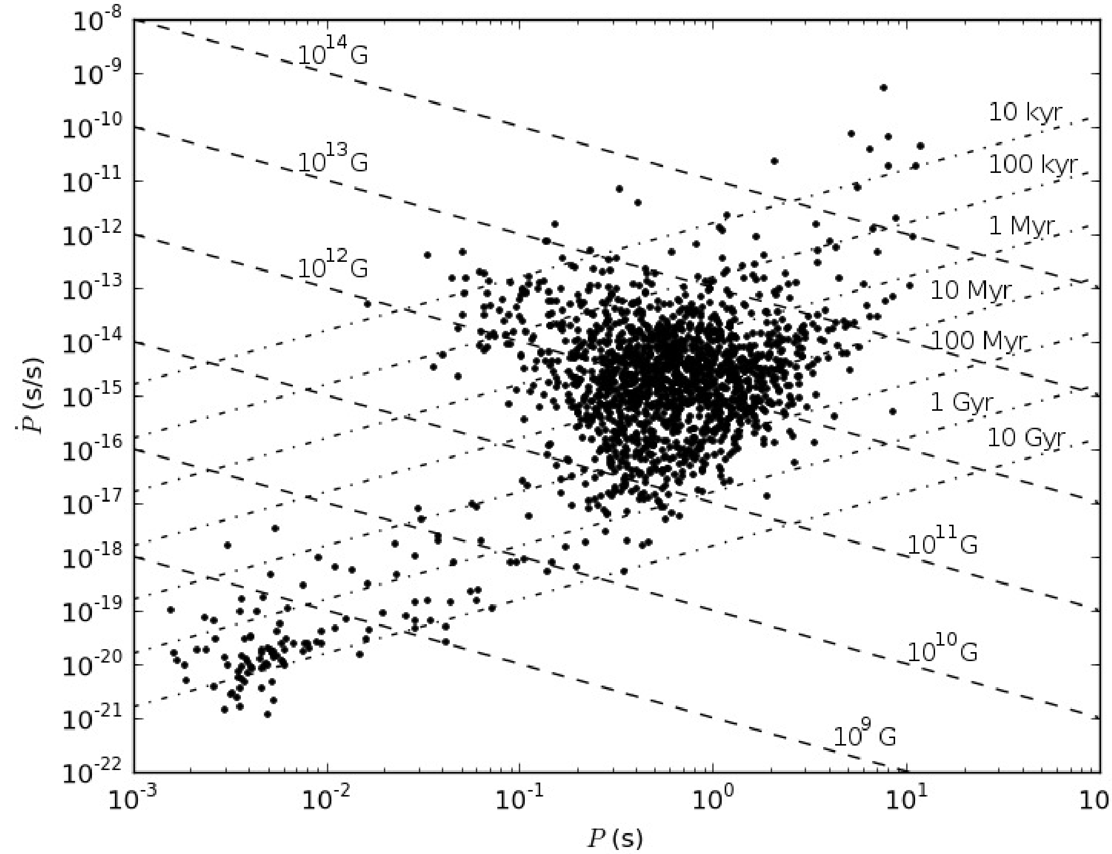

Fig. 6.— The P-$\dot{P}$ diagram with lines of constant τ_c (dot-dashed lines) and B_S (dashed-dashed lines).