Constraints on the spectrum of W-algebras

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Work in progress

Outline

Conformal symmetry in d=2

A Conformal Field Theory is a QM system invariant under local rescaling

$$\eta'_{\mu\nu} = \Lambda(x)\eta_{\mu\nu}$$

Rewrite 2d flat space (Euclidean) in complex coordinates $z = x^0 + ix^1$, $\bar{z} = x^0 - ix^1$:

$$ds^2 = (dx^0)^2 + (dx^1)^2 = dz d\bar{z}$$

Conformal transformations are holomorphic functions $z \to f(z)$

$$dz d\bar{z} \to \left|\frac{df}{dz}\right|^2 dz d\bar{z}$$

Virasoro algebra

Expand infinitesimal transformation in Laurent modes to get generators

$$z' = z + \epsilon(z) \Rightarrow \ell_n = -z^{n+1}\partial_z$$

$$\bar{z}' = \bar{z} + \bar{\epsilon}(\bar{z}) \Rightarrow \bar{\ell}_n = -\bar{z}^{n+1}\bar{\partial}_{\bar{z}}$$

Two *independent* copies of the Witt algebra
$[\ell_m, \ell_n] = (m-n)\ell_{m+n}$

Algebra allows for central extension: Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

Realized in quantized theory

Energy-momentum tensor

In 2d, conformal invariance dictates the form of the energy-momentum tensor (in complex coordinates):

$$\begin{pmatrix} T(z) & 0 \\ 0 & \bar{T}(\bar{z}) \end{pmatrix}$$

Turns out that Virasoro generators are Laurent modes of $T(z)$

$$T(z) = \sum_{n\in\mathbb{Z}} z^{-n-2} L_n$$

If extra conserved operator, symmetry algebra is bigger $\Rightarrow$ $\mathcal{W}$-algebra

W(2,4) algebra

Consider extension of Virasoro with modes of spin 4 chiral operator

$$W(z) = \sum_{n\in\mathbb{Z}} z^{-n-4} W_n$$

Algebra becomes

$$[L_m, W_n] = (3m - n)W_{m+n}$$

$$\begin{aligned}[W_m, W_n] =& aL_{m+n} + bW_{m+n} + d\mathcal{N}(TT)_{m+n} + e\mathcal{N}(T\partial^2 T)_{m+n} + \\ & f\mathcal{N}(\mathcal{N}(TT)T)_{m+n} + g\mathcal{N}(WT)_{m+n} + \frac{c}{4}\binom{m+3}{7}\delta_{m+n,0}\end{aligned}$$

Coefficients are complicated functions of m, n, c, some imaginary for certain values of c

Highest weight representation

Hilbert space of theory is representation of this algebra

Start with primary state $|h, w\rangle$ with:

- $L_0 |h, w\rangle = h |h, w\rangle$ and $W_0 |h, w\rangle = w |h, w\rangle$
- $h > 0$ is conformal dimension, w is W-charge
- L_n and W_n with $n > 0$ lower dimension and kill primary states
- L_n and W_n with $n < 0$ raise dimension and create new states

Levels form orthogonal subspaces with fixed dimension, but not well defined charge

CFT=sum of Verma modules

Want to know what primaries $|h, w\rangle$ are allowed in unitary CFTs

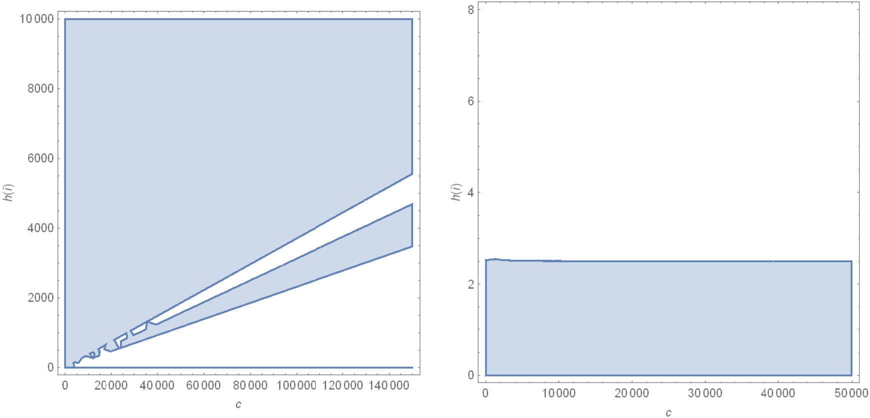
Don't want states with negative/complex norm

- Level 1: L_{-1}, W_{-1}
- Level 2: $L_{-2}, W_{-2}, L_{-1}L_{-1}, W_{-1}W_{-1}, L_{-1}W_{-1}$

Ask that norm matrix have only positive real eigenvalues

Easier to study determinant, but weaker constraint

$w = 0$

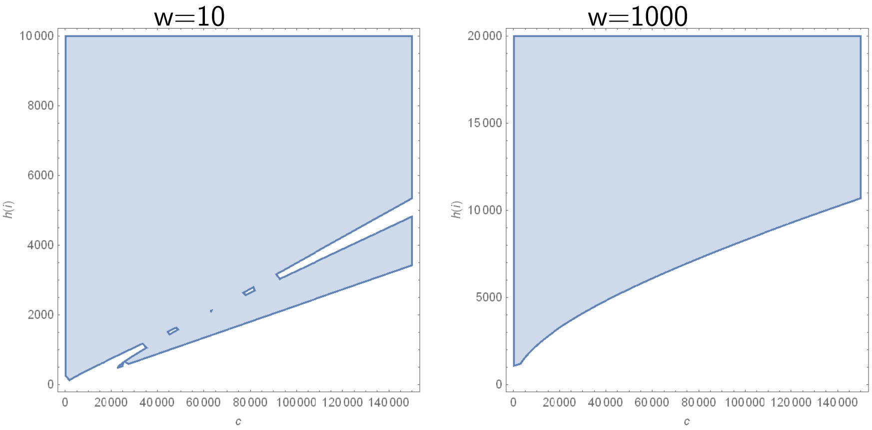

10 000
8000
6000
4000
2000
0
0
20 000
40 000
60 000
80 000
100 000
120 000
140 000
h(i)
c
8
6
4
2
0
0
10 000
20 000
30 000
40 000
50 000
h(i)
c

Results for $w = 0$

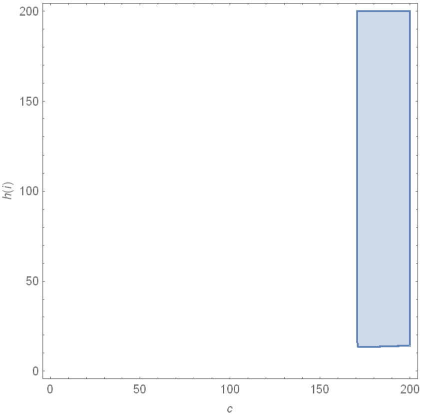
Some interesting results:

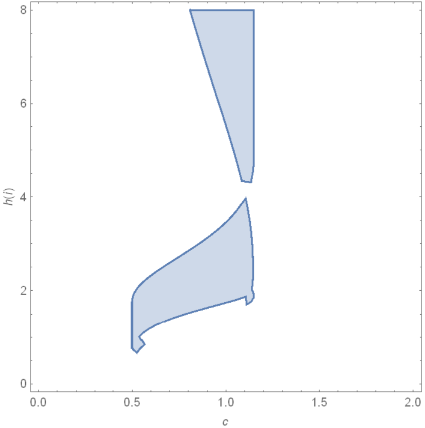
- Vacuum is included
- $h > \frac{c}{43} + O(1)$ at large c
- Almost no light states in semi-classical limit

Finite w


w=10
10 000
8000
6000
4000
2000
0
h(i)
0
20 000
40 000
60 000
80 000
100 000
120 000
140 000
c
w=1000
20 000
15 000
10 000
5000
0
h(i)
0
20 000
40 000
60 000
80 000
100 000
120 000
140 000
c

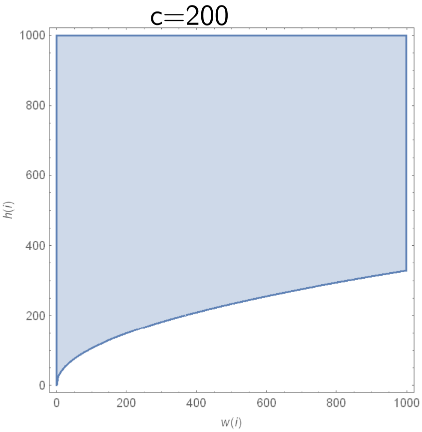
w=1 with small c

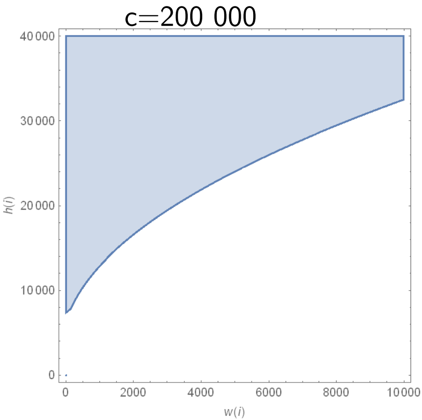




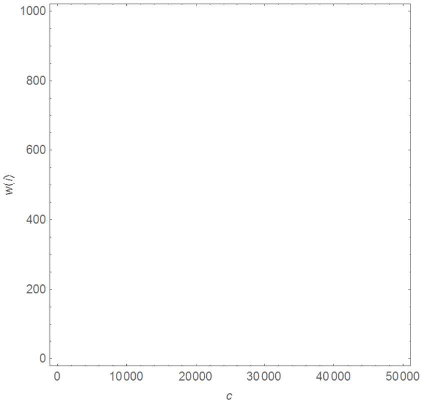
Region excluded because algebra not real for these values of c

Fixed central charge





h=0



$h = 0$ allowed only for very small values of w

Conclusion

Found constraints on $\mathcal{W}(2,4)$ representations

Study $\mathcal{W}(2,3)$, $\mathcal{W}(2,3,4)$ and $\mathcal{W}(2,3,4,5)$

Application to AdS/CFT for higher spin gravity

Thank you!