Tunneling decay of false vortices

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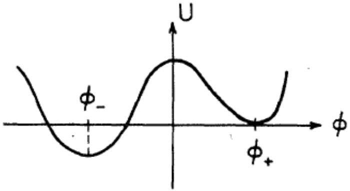
November 12, 2014

- Based on the paper *Tunneling decay of false vortices* by Richard Mackenzie, Manu Paranjape and others (hep-th 1308.3501)
- I worked on this during the summer of 2013
- Some modifications to the paper
- Review of the paper *Fate of the false vacuum: Semiclassical theory* by Sidney Coleman (Phys.Rev. D15 (1977) 2929-2936)

Overview

- Theory in 2+1 dimensions
- Universe is in the false vacuum (phase transition)
- Vacuum bubbles can form and decay
- Vortices are present (disk of true vacuum)
- Decay of vortices useful or not?

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - U(\phi)$$



ϕ_+ = false vacuum
ϕ_- = true vacuum

- Universe in ϕ_+ (phase transition)
- Locally (bubbles), $\phi_+ \to \phi_-$ can happen because of QM, with probability:

$$\Gamma/V = A\,e^{-\frac{B}{\hbar}}(1 + O(\hbar))$$

- Interesting because age of the universe $< \infty$
- Consider t such that Γ/V times 4-volume of past light cone is of order 1
  - If $t \ll 1$ year: inapplicable (too hot)
  - If $t \sim 1$ year: secondary Big Bang
  - If $t \sim 10^9$ years: we should worry!

- We compute only B (A is tougher)
- Start from QM, but I skip to field theory directly

$$B = S_E = \int d\tau d^3x \, \mathcal{L}_E$$

- E means Euclidean ($\tau = it$) and we solve with the conditions:

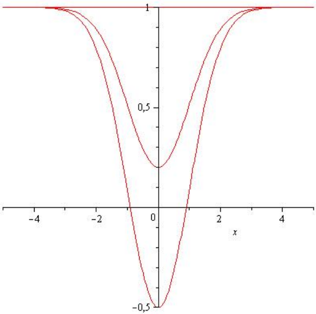
$$\lim_{\tau \to \pm\infty} \phi(\tau, \vec{x}) = \phi_+$$

$$\frac{\partial \phi}{\partial \tau}(0, \vec{x}) = 0$$

$$\lim_{|\vec{x}| \to \infty} \phi(\tau, \vec{x}) = \phi_+ \quad \text{(finite energy)}$$

The bounce:

- Not physical
- Not unique (lowest one counts)



We suppose that ϕ is invariant under $O(4)$

$$\phi(x) = \phi(\rho), \qquad \rho = \sqrt{\tau^2 + |\vec{x}|^2}$$

The equation of motion is:

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = U'(\phi)$$

With the conditions:

$$\lim_{\rho\to\infty} \phi(\rho) = \phi_+$$

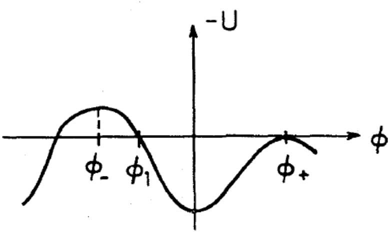
$$\left.\frac{d\phi}{d\rho}\right|_{\rho=0} = 0 \qquad \text{(regular at the origin)}$$

$$\implies B = 2\pi^2 \int_0^\infty \rho^3 d\rho \left(\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2 + U(\phi)\right)$$

Equation of motion of a particle in the potential $-U$ with damping

If released far from ϕ_-
$\rightarrow$ undershoot

If released near ϕ_-
$\rightarrow$ overshoot



$\implies$ There is a solution (Supposedly with the lowest action of all)

We consider a complex scalar field ϕ with a gauge field A_μ.

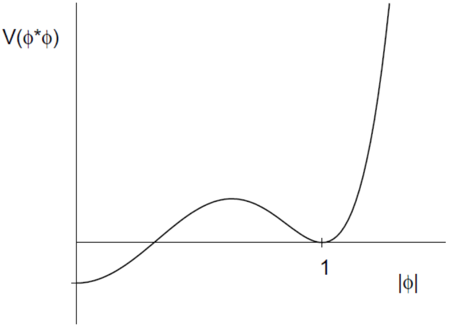
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(|\phi|)$$

with $D_\mu = \partial_\mu - ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

The potential that we use is, after rescaling:

$$V(|\phi|) = (|\phi|^2 - \epsilon)(|\phi|^2 - 1)^2$$

This theory is invariant under a local $U(1)$ transformation



- True vacuum at $\phi = 0$
- Circle of symmetry breaking false vacua at $|\phi| = 1$

We want rotationally symmetric solutions of the form

$$\phi(t,r,\theta) = f(t,r)e^{in\theta}, \qquad A_i(t,r,\theta) = -\frac{n}{e}\frac{\epsilon^{ij} r_j}{r^2} a(t,r)$$

The energy is then

$$E = 2\pi \int_0^\infty r\,dr \left(\frac{n^2(\dot{a}^2 + a'^2)}{2e^2 r^2} + \dot{f}^2 + f'^2 + \frac{n^2}{r^2}(1-a)^2 f^2 + (f^2-\epsilon)(f^2-1)^2 \right)$$

Must have $f \to 1$ and $a \to 1$ as $r \to \infty$ for finite energy

Must have $f \to 0$ and $a \to 0$ as $r \to 0$ for continuity

This solution is a VORTEX

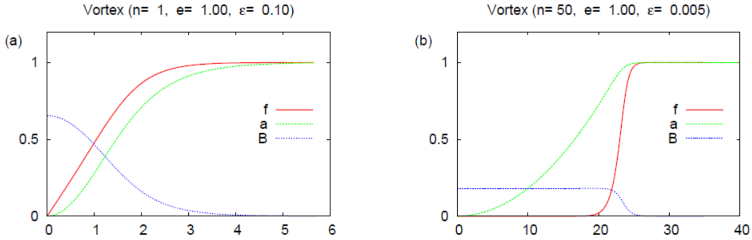
- Metastable for certain parameters
- Unstable for other parameters
- Topological defect of winding number n
- Section of cosmic string
- Quantized magnetic flux

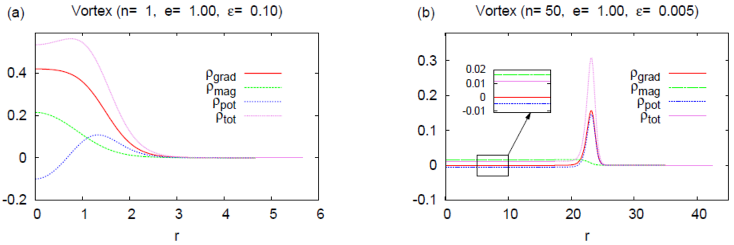
We solve numerically the static EOM

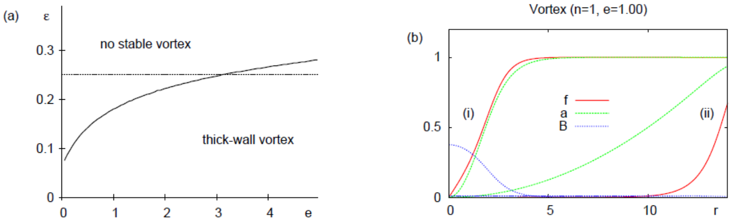
$$f'' + \frac{f'}{r} - \frac{n^2}{r^2}(1-a)^2 f - (f^2-1)(3f^2-(1+3\epsilon))f = 0$$

$$a'' - \frac{a'}{r} + 2e^2(1-a)f^2 = 0$$

With the vortex boundary conditions and a set of parameters (n, e, ϵ)




(a)
Vortex (n= 1, e= 1.00, ε= 0.10)
ρgrad
ρmag
ρpot
ρtot
r
(b)
Vortex (n= 50, e= 1.00, ε= 0.005)
ρgrad
ρmag
ρpot
ρtot
r


(a)
ε
no stable vortex
thick-wall vortex
e
(b)
Vortex (n=1, e=1.00)
(i)
(ii)
f
a
B
r

We generalize Coleman for the decay of the vortices

Must find the bounce, which respects:

- Vortex as $\tau \to -\infty$
- Turning point at $\tau = 0$
- Vortex as $\tau \to \infty$

Really tough so we restrict to a one parameter family of deformations

We parametrize by the radius R of the vortex to get a bigger action

We must find the extremal solution to

$$S_E = \int d\tau \, (T + E)$$

$$T = 2\pi \int_0^\infty r\, dr \left(\dot{f}^2 + \frac{n^2 \dot{a}^2}{2e^2 r^2} \right)$$

$$E = 2\pi \int_0^\infty r\, dr \left(\frac{n^2 a'^2}{2e^2 r^2} + f'^2 + \frac{n^2}{r^2}(1-a)^2 f^2 + (f^2 - \epsilon)(f^2 - 1)^2 \right)$$

With the conditions of the bounce

Thin wall case ($n \gg 1$) most interesting: separate static energy

$$E(R) = E_{int} + E_{wall} + E_{ext}$$

Interior ($r < R - \delta/2$):

- $f = 0$
- $a = \left(\frac{r}{R}\right)^2$

$$\Rightarrow E_{int} = \frac{2\pi n^2}{e^2 R^2} - \epsilon \pi R^2$$

Exterior ($r > R + \delta/2$):

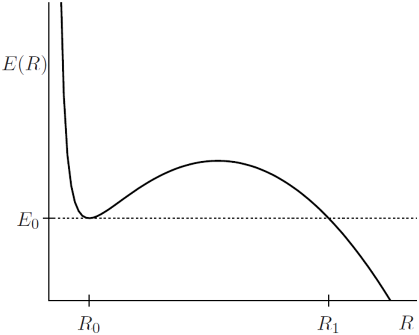
- $f = a = 1$

$$\Rightarrow E_{ext} = 0$$

Wall ($R - \delta/2 < r < R + \delta/2$):

- $r \sim R \gg 1$
- $f \simeq 1$
- $1 - a = \frac{1}{R}$

$\Rightarrow E_{wall} = \pi R$



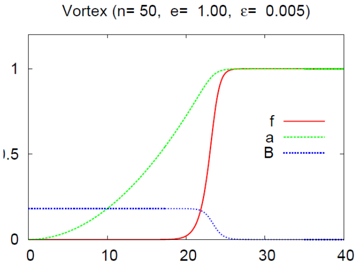
No stable solution for $\epsilon > 0.24\left(\frac{e}{2n}\right)^{2/3} \equiv \epsilon_c$ because R_0 disappears

At first order in ϵ:

$$R_0 = \left(\frac{2n}{e}\right)^{2/3} \quad , \qquad E_0 = \frac{3\pi}{2}\left(\frac{2n}{e}\right)^{2/3} \quad , \qquad R_1 = \frac{1}{\epsilon}$$

Decay of vortex: $R_0 \to R_1$ quantum mechanically and $R_1 \to \infty$ classically

Recall the solution for $n = 50$, $e = 1$ and $\epsilon = 0.005$:



	Approximation	Numerical
E_0	101.5	92.5
R_0	21.5	22
ϵ_c	0.0109	0.0105

$\Longrightarrow$ Good approximation!!

To have the bounce we need T, with $R = R(\tau)$

$$T(R) = T_{int} + T_{wall} + T_{ext}$$

Interior ($r < R - \delta/2$):

- $f = 0$
- $a = \left(\frac{r}{R}\right)^2$

$$\Rightarrow T_{int} = \frac{\pi n^2}{e^2}\frac{\dot{R}^2}{R^2}$$

Exterior ($r > R + \delta/2$):

- $f = a = 1$

$$\Rightarrow T_{ext} = 0$$

Wall ($R - \delta/2 < r < R + \delta/2$):

- $f = f(r - R)$

$$\Rightarrow T_{wall} = \frac{\pi R}{2}\dot{R}^2$$

Must shift energy by E_0 to fit with Coleman

$$S_E^{thin} = \int d\tau \left(B(R)\dot{R}^2 + E(R) - E_0 \right)$$

with $R(-\infty) = R(\infty) = R_0$ and $\dot{R}(0) = 0$

We find the first integral

$$B(R)\dot{R}^2 - E(R) + E_0 = 0$$

$$\Longrightarrow S_E^{thin} = 2 \int_{R_0}^{R_1} dR \sqrt{B(R)(E(R) - E_0)}$$

After some approximations we find (for small ϵ)

$$S_E^{thin} \simeq \frac{4\sqrt{2}\pi}{15\epsilon^2}\left(1 - \frac{45\epsilon}{8}\left(\frac{2n}{e}\right)^{2/3}\right)$$

and we get a lower bound

$$\Gamma^{thin} = A^{thin}\left(\frac{S_E^{thin}}{2\pi}\right)^{1/2} e^{-S_E^{thin}}$$

Decay rate of vacuum $\phi = 1$ and A_μ not excited

We already saw the starting point, with $\phi(\tau, \vec{x}) = \phi(\rho)$ and in (2+1) d:

$$S_E^{vac} = 4\pi \int_0^\infty d\rho \rho^2 (\phi'^2 + V(\phi))$$

$$\phi'' + \frac{2}{\rho}\phi' = V'(\phi)$$

with $\phi(\infty) = 1$ (so $\phi(0) \simeq 0$) and $\phi'(0) = 0$

If ϵ is small, the friction gives

$$\phi(\rho) = \begin{cases} 0 & \text{for } \rho \lesssim \rho_0 \\ \phi_k & \text{for } \rho \gtrsim \rho_0 \end{cases}$$

with ϕ_k respecting (neglecting ϵ in the potential)

$$\phi_k' = \phi_k(1 - \phi_k^2)$$

$$\Longrightarrow S_E^{vac}(\rho_0) = \pi\left(\rho_0^2 - \frac{4}{3}\epsilon\rho_0^3\right)$$

The minimal action is

$$S_E^{vac} = \frac{\pi}{12\epsilon^2}$$

and the decay rate for a volume Ω is about

$$\Gamma^{vac} = \Omega A^{vac} \left(\frac{S_E^{vac}}{2\pi} \right)^{1/2} e^{-S_E^{vac}}$$

Two effects of the vortices:

- Replace existing false vacuum by true vacuum (core)
- Can decay themselves

We compare:

- Universe (volume Ω) in false vacuum
- Universe with N non-interacting vortices (same volume, neglect false vacuum between them)

$$\frac{\Gamma^{vac}}{N\Gamma^{thin}} \sim exp(S_E^{thin} - S_E^{vac}) = exp\left(\frac{\pi}{\epsilon^2}\left(\frac{4\sqrt{2}}{15} - \frac{3\sqrt{2}\epsilon}{2}\left(\frac{2n}{e}\right)^{2/3} - \frac{1}{12}\right)\right)$$

For $n = 50$ and $e = 1$ we get (remember $\epsilon_c = 0.01$)

- For $\epsilon < 0.006$, $\frac{\Gamma^{vac}}{N\Gamma^{thin}} > 1 \Rightarrow$ vortices work against the decay
- For $\epsilon > 0.006$, $\frac{\Gamma^{vac}}{N\Gamma^{thin}} < 1 \Rightarrow$ vortices help the decay

Generically $\epsilon \sim \epsilon_c$ helps the decay

Thank you!