Thermal Conformal Blocks

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Outline

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Setup

CFT = QFT with scale invariance

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Symmetry generators:

- Translations P_μ, Rotations $M_{\mu\nu}$
- Dilatations D, Special Conformal Transformations K_μ

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Irreps of conformal group = Conformal families (HW reps)

- Primary $|\mathcal{O}\rangle$: $K_\mu |\mathcal{O}\rangle = 0$, $D |\mathcal{O}\rangle = \Delta_\mathcal{O} |\mathcal{O}\rangle$, spin
- Descendants $|\mathcal{O}, n\rangle \sim P^n |\mathcal{O}\rangle$, $D |\mathcal{O}, n\rangle = (\Delta_\mathcal{O} + n) |\mathcal{O}, n\rangle$

Definitions

Study scalar field $\phi(x)$ in thermal state with $T = 1/\beta$

$$\langle\phi(x)\rangle_\beta = \mathrm{Tr}[\phi(x)e^{-\beta H}] = \mathrm{Tr}[\phi(x)e^{-\beta D}] = \sum_i q^{\Delta_i} \langle i|\,\phi(x)\,|i\rangle$$

$$= \sum_{\text{primaries } \mathcal{O}} q^{\Delta_{\mathcal{O}}} \sum_n \langle\mathcal{O}, n|\,\phi(x)\,|\mathcal{O}, n\rangle\, q^n$$

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Conformal symmetry relates $\langle\mathcal{O},n|\,\phi\,|\mathcal{O},n\rangle$ to $\langle\mathcal{O}|\,\phi\,|\mathcal{O}\rangle$ so define

$$\langle\phi\rangle_\beta = \sum_{\text{primaries } \mathcal{O}} \langle\mathcal{O}|\,\phi\,|\mathcal{O}\rangle\, q^{\Delta_\mathcal{O}} F(\Delta_\phi, \Delta_\mathcal{O}, \ell_\mathcal{O}; q)$$

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Purely kinematic object, focus on $\ell_\mathcal{O}$

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Tools

- Conformal algebra:

$$[D, P_\mu] = P_\mu \,, \quad [D, K_\mu] = -K_\mu \,, \quad [K_\mu, P_\nu] = 2\delta_{\mu\nu} D - 2M_{\mu\nu}$$

$$[M_{\mu\nu}, P_\rho] = \delta_{\nu\rho} P_\mu - \delta_{\mu\rho} P_\nu \quad , \quad [M_{\mu\nu}, K_\rho] = \delta_{\nu\rho} K_\mu - \delta_{\mu\rho} K_\nu$$

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- For scalars:

$$[D, \phi(x)] = (\Delta_\phi + x^\mu \partial_\mu)\phi(x)\,, \quad [P_\mu, \phi(x)] = \partial_\mu \phi(x)$$

$$[K_\mu, \phi(x)] = (2x_\mu \Delta_\phi + 2x_\mu x^\nu \partial_\nu - x^2 \partial_\mu)\phi(x)$$

$$[M_{\mu\nu}, \phi(x)] = (x_\nu \partial_\mu - x_\mu \partial_\nu)\phi(x)$$

Example at level 1

Using $\langle\mathcal{O}|\,\phi(x)\,|\mathcal{O}\rangle \sim (x^2)^{-\Delta_\phi/2}$, get

$$\langle\mathcal{O}|\,K_\mu\phi(x)P_\nu\,|\mathcal{O}\rangle = \left(2\Delta\delta_{\mu\nu} - \Delta_\phi\delta_{\mu\nu} + \Delta_\phi^2\frac{x_\mu x_\nu}{x^2}\right)\langle\mathcal{O}|\,\phi(x)\,|\mathcal{O}\rangle$$

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$$\langle\mathcal{O}|\,K_\mu P_\nu\,|\mathcal{O}\rangle = 2\Delta_O\delta_{\mu\nu}$$

$$\Rightarrow \sum_{\mu,\nu}\langle K_\mu\phi(x)P_\nu\rangle\left(\langle K_\mu P_\nu\rangle\right)^{-1} = d + \frac{\Delta_\phi(\Delta_\phi - d)}{2\Delta_\mathcal{O}}$$

Result

For example in $3d$ we find

$$F(\Delta_\phi, \Delta_{\mathcal{O}}; q) = a_0 + a_1 q + a_2 q^2 + \ldots$$

with

$$a_0 = 1$$

$$a_1 = 3 + \frac{\Delta_\phi(\Delta_\phi - 3)}{2\Delta_{\mathcal{O}}}$$

$$a_2 = 6 + \frac{\Delta_\phi(\Delta_\phi - 3)[\Delta_{\mathcal{O}}\Delta_\phi(\Delta_\phi - 3) + 2\left(8\Delta_{\mathcal{O}}^2 + \Delta_{\mathcal{O}} - 2\right)]}{4(\Delta_{\mathcal{O}} + 1)(\Delta_{\mathcal{O}})(2\Delta_{\mathcal{O}} - 1)}$$

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Can't resum...

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Angular potential

Focus on 3d so rotations are $SO(3)$

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Need to add potential for angular momentum:

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Casimir operator of conformal group (in 3d)

$$C = D(D-3) + J^2 - P_\mu K^\mu$$

has eigenvalue $\Delta(\Delta - 3)$ on irrep.

Differential equation

Insert Casimir into trace

$$\Rightarrow \mathrm{Tr}_{\mathcal{O}}[C\phi q^{D} y^{J_z}] \sim \Delta_{\mathcal{O}}(\Delta_{\mathcal{O}}-3)F(\Delta_{\phi},\Delta_{\mathcal{O}};q,y)$$

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Convert to differential equation using algebra and

$$D \sim q\partial_q\,,\ \ J_z \sim y\partial_y$$

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Can't solve exactly...

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Anti de Sitter space

Euclidean AdS space is like a cylinder

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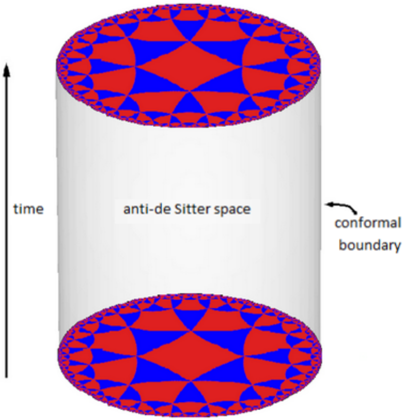
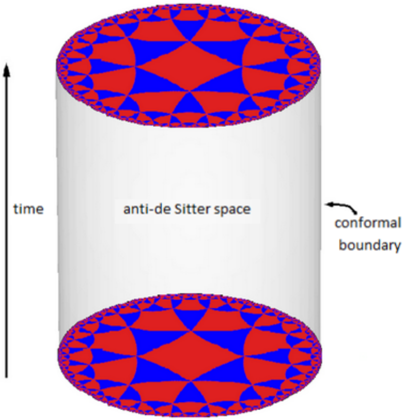


Anti de Sitter space

Euclidean AdS space is like a cylinder

Isometry group of AdS_{d+1}
$=$ Conformal group on $\mathbb{R}^d$



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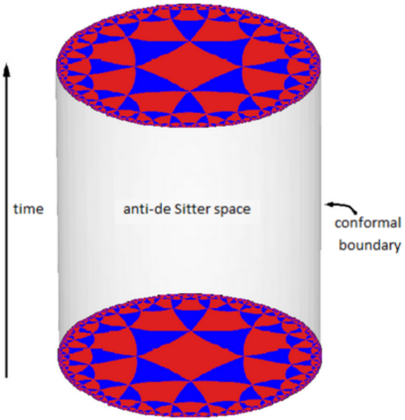
$\Rightarrow$ Consider CFT on boundary and compute using bulk



Witten diagram

Thermal state so periodic in time ($t \sim t + \beta$)

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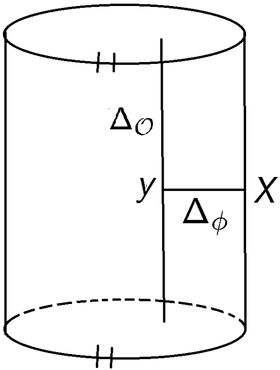

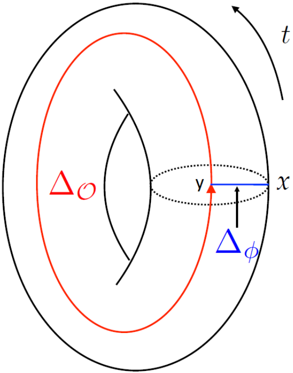

Final result

Explicitely (without angular potential)

$$F(\Delta_\phi, \Delta_{\mathcal{O}}; q) = \int_{AdS} dy \sqrt{g}\, G^{\Delta_\phi}_{bulk-bdy}(y, x)\, G^{\Delta_{\mathcal{O}}}_{bulk-bulk}(y, y_{thermal})$$

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Can actually do the integral

$$\Rightarrow F(\Delta_\phi, \Delta_\mathcal{O}; q) = \frac{1}{(1-q)^{2\Delta_\mathcal{O}}} \times$$

$${}_3F_2\left[\frac{1-d}{2} + \Delta_\mathcal{O}, \Delta_\mathcal{O} - \frac{\Delta_\phi}{2}, \frac{\Delta_\phi - d}{2} + \Delta_\mathcal{O}; \Delta_\mathcal{O}, -d + 2\Delta_\mathcal{O} + 1; -\frac{4q}{(q-1)^2}\right]$$

Conclusion and future work

Computed conformal blocks for thermal 1-point function of scalar operator with 3 different methods.

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- Higher point functions
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Thank you!